A New Approach to Multiple Antenna Systems

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Abstract — **Using an isomorphism between the multiuser detection problem and transmission of information over MIMO channels coupled to recent results on optimal signature waveform construction, we provide a new approach to information transfer in multiple antenna systems. The approach is based on a multicarrier modulation scheme at each transmit antenna with sinusoids as carriers, which leads to a matrix model of the MIMO channel. A set of bits is transmitted over the MIMO channel in parallel by assigning different codewords to each bit in the set similar to the way codewords are assigned to different users in a CDMA system. Optimal codewords can be obtained by application of interference avoidance methods, and the corresponding optimal linear receiver structure is a set of matched filters. An analysis of the MIMO channel is performed using singular value decomposition and the dimensionality of the problem is reduced only to those dimensions of the signal space that actually carry information.**

I. Introduction

The increasing demand for high data rate wireless communications has motivated a strong research effort in the area of multiple-input multiple-output (MIMO) channels. This type of channel is associated primarily with communication systems that have multiple antennas at the receiver and/or transmitter.

The use of multiple antennas in wireless communication systems provides spatial diversity to improve system performance by mitigating the effects of multipath fading. While traditionally spatial diversity was implemented at only one side of the communication system (mainly at the receiver) recent research indicates that one could significantly improve performance by using spatial diversity both at the transmitter and at the receiver.

Performance of multiple antenna systems in fading environments has been analyzed in several papers which have shown a potentially large increase in capacity. Since standard approaches are not close in performance to the theoretical limits [2, 4], new modulation schemes have been proposed and analyzed for multiple antenna systems [1, 3]. It has also been shown that presence of multipath can improve performance with an appropriate multiple antenna structure [6].

Our approach is based on the equivalence between the problems of multiuser detection and transmission of information over dispersive channels, and uses recent results on construction of optimal signature waveforms through interference avoidance methods. The theoretical framework described in [5] for representation of signals as vectors in signal space and for modeling of dispersive communication channels is used. The communication channel between one transmitter and the corresponding receiver is described by a diagonal matrix containing the channel eigenvalues. Under the assumption that the communication interval is large relative the durations of all the channel impulse responses, the channel eigenfunctions will all be approximately sinusoidal "tonebursts", which implies that a form of multicarrier modulation is used for transmitting information. This approach leads to a matrix representation for the MIMO channel which can be decomposed in terms of subblocks corresponding to each receiver/transmitter pair, which is similar to the matrix channel model presented in [6]. However, while in our approach elements of a subblock correspond to orthogonal partitions of a dispersive channel in terms of its eigenfunctions, in the spatio-temporal model in [6] subblocks correspond to convolution matrices of distinct channels.

A set of bits is transmitted over the MIMO channel in parallel by assigning codewords to each bit in a way similar to the way codewords are assigned to different users in a CDMA system. Formulation of the MIMO channel problem in this framework allows direct application of interference avoidance techniques [5, 8] to determine optimal codewords corresponding to each bit of the input set. Codewords are optimal in the sense that the shared signal-to-interference plus nois-ratio (SINR) for all bits is maximized, as well as in the information theoretic sense of maximizing the mutual information between received signal vector and transmitted bit set (sum capacity) [8, 11].

II. Problem Statement

Let us consider the MIMO system consisting of L transmit antennas and K receive antennas depicted in figure 1. The communication channel between transmit antenna l and receive antenna k of this MIMO system is characterized by the causal impulse response $h_{kl}(t)$ assumed stable (time-invariant) over the duration of the communication interval $\mathcal{T}$. The duration of the communication interval $\mathcal{T}$ is assumed much larger than the duration of all $h_{kl}(t)$. This assumption implies that eigenfunctions for all channels will be approximately sinusoidal "tonebursts" which leads to the same eigen-decomposition for all channels. Let us denote by N the dimension of the signal space induced by this eigen-decomposition.

Decomposition of each channel into orthogonal subchannels implies that a form of multicarrier modulation [5] is used to send information on each pair of transmit/receive antennas. Specifically, N-dimensional input/output vectors corresponding to transmit antenna l and receive antenna k are related by

$$\mathbf{r}_k = \Lambda_{lk}^{1/2}\mathbf{x}_l + \mathbf{n}_k \qquad l = 1, \ldots, L,\ k = 1, \ldots, K \qquad (1)$$

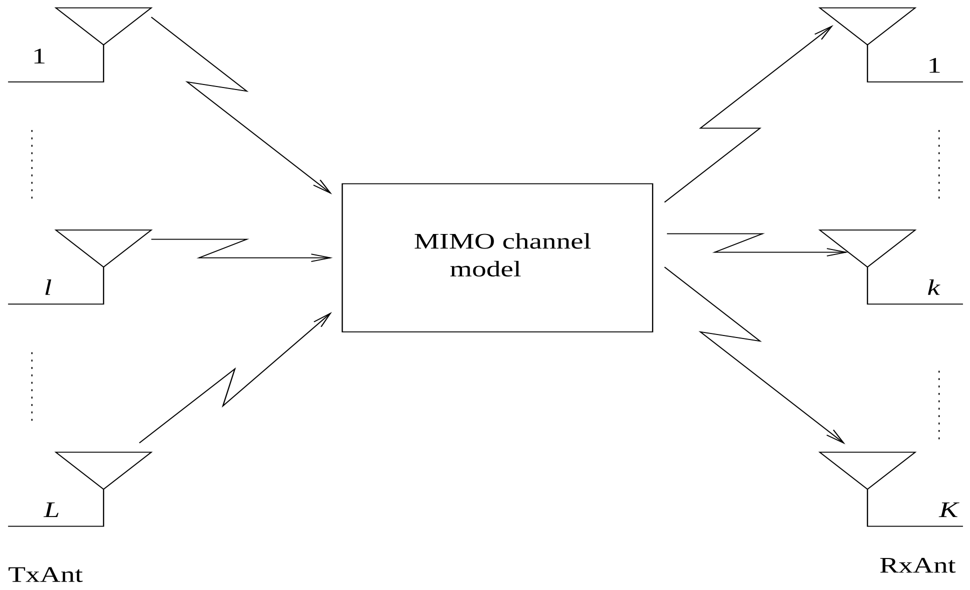

Figure 1: MIMO channel consisting of multiple transmit and receive antennas

where $\mathbf{n}_k$ is the additive noise vector that corrupts the received signal at antenna k assumed colored with uncorrelated components with diagonal covariance matrix $E[\mathbf{n}_k\mathbf{n}_k^\top] = \mathbf{W}_k$, and Λ_{lk} is the $N \times N$ matrix containing the channel eigenvalues.

By stacking together all received signal vectors from all receive antennas we can write

$$\mathbf{r} = \begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_k \\ \vdots \\ \mathbf{r}_K \end{bmatrix} = \mathbf{H}\mathbf{x} + \mathbf{n} = \begin{bmatrix} \Lambda_{11}^{1/2} & \dots & \Lambda_{l1}^{1/2} & \dots & \Lambda_{L1}^{1/2} \\ \vdots & & \vdots & & \vdots \\ \Lambda_{1k}^{1/2} & \dots & \Lambda_{lk}^{1/2} & \dots & \Lambda_{Lk}^{1/2} \\ \vdots & & \vdots & & \vdots \\ \Lambda_{1K}^{1/2} & \dots & \Lambda_{lK}^{1/2} & \dots & \Lambda_{LK}^{1/2} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_l \\ \vdots \\ \mathbf{x}_L \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_k \\ \vdots \\ \mathbf{n}_K \end{bmatrix} \quad (2)$$

where we have denoted by $\mathbf{H}$ the $NK \times NL$ matrix containing channel eigenvalue matrices of all channels, $\mathbf{x}$ the NL-dimensional channel input vector, and $\mathbf{n}$ the NK-dimensional noise vector that corrupts the received signal at the output of the channel. Under the assumption that noise vectors at different antennas are independent we have

$$E[\mathbf{n}\mathbf{n}^\top] = \mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & & \\ & \ddots & \\ & & \mathbf{W}_K \end{bmatrix}$$

The NL-dimensional input signal $\mathbf{x}$ of the MIMO channel described by equation (2) is generated by a linear superposition of codeword column vectors $\mathbf{s}_m$ for each of the M bits $\mathbf{b} = [b_1 \dots b_M]^\top$ that will be sent during the communication interval $\mathcal{T}$. That is, we have a dimension $NL \times M$ codeword matrix $\mathbf{S}$

$$\mathbf{S} = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1 & \mathbf{s}_2 & \cdots & \mathbf{s}_M \\ | & | & & | \end{bmatrix} \quad (3)$$

so that

$$\mathbf{x} = \mathbf{S}\mathbf{b} \quad (4)$$

which implies that equation (2) can be rewritten as

$$\mathbf{r} = \mathbf{H}\mathbf{S}\mathbf{b} + \mathbf{n} \quad (5)$$

This approach of sending the M bits in parallel during the communication interval $\mathcal{T}$ by assigning different codewords to different bits can be regarded as a form of CDMA, as if each bit corresponded to a different user.

Equation (5) is similar to the single user dispersive channel problem formulated in [5], and as will be seen, interference avoidance methods can be applied in a similar way to determine an optimal codeword matrix.

III. Orthogonal Input/Output Representation and Interference Avoidance

Because of the different number of transmit and receive antennas one can no longer relate the inputs and the outputs of the MIMO channel through a one-to-one mapping. For example, it could be that some received signals are impossible through any possible excitation of the transmit antennas or that an infinite number of transmit signals correspond to a single received signal. Singular value decomposition (SVD) of the MIMO channel matrix $\mathbf{H}$ provides a means of relating different subspaces of the input and output signal spaces. The SVD of the channel matrix $\mathbf{H}$ is [9]

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^\top \quad (6)$$

where matrix $\mathbf{U}$ of dimension $NK \times NK$ has as columns the eigenvectors of $\mathbf{H}\mathbf{H}^\top$, matrix $\mathbf{V}$ of dimension $NL \times NL$ has as columns the eigenvectors of $\mathbf{H}^\top\mathbf{H}$, and matrix $\mathbf{D}$ of dimension $NK \times NL$ contains the singular values of $\mathbf{H}$ on the main diagonal and zero elsewhere.

Any vector in the NL-dimensional input space of the MIMO channel can then be represented in terms of the orthonormal set of vectors $\{\mathbf{v}_i\}$ representing the columns of $\mathbf{V}$. Similarly, any vector in the NK-dimensional output space of the MIMO channel is representable in terms of the orthonormal set of vectors $\{\mathbf{u}_i\}$ representing the columns of $\mathbf{U}$. Furthermore, because these sets of vectors come from the SVD decomposition (6) we have

$$\mathbf{v}_i^\top \mathbf{v}_j = \delta_{ij} \ \Rightarrow \mathbf{v}_i^\top \mathbf{H}^\top \mathbf{H} \mathbf{v}_j = d_i^2 \delta_{ij} \quad (7)$$

Therefore, energy at the input of the MIMO channel should only be put into those vectors $\mathbf{v}_i$ that correspond to non-zero singular values $d_i \neq 0$.

Let us denote the rank of the MIMO channel matrix, equal to the number of non-zero singular values, by ρ. It is obvious that

$$\rho = \operatorname{rank}(\mathbf{H}) \leq \min(NK, NL) \quad (8)$$

Then, the dimension of the column space of matrix $\mathbf{H}$ will be equal to ρ. Also, the dimension of the null space of $\mathbf{H}$ is $NL - \rho$ and the dimension of the left null space is $NK - \rho$. Because there are only ρ non-zero singular values and we are interested only in their corresponding eigenvectors, we can partition matrix $\mathbf{D}$ containing the singular values as

$$\mathbf{D} = \begin{bmatrix} \bar{\mathbf{D}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (9)$$

with a $\rho \times \rho$ diagonal matrix $\bar{\mathbf{D}}$ which contains the nonzero singular values and zero matrices of appropriate dimensions.

Returning to equation (5) in which we apply the SVD for matrix $\mathbf{H}$ we obtain

$$\mathbf{r} = \mathbf{U}\mathbf{D}\mathbf{V}^\top \mathbf{S}\mathbf{b} + \mathbf{n} \quad (10)$$

We can premultiply by $\mathbf{U}^\top$

$$\tilde{\mathbf{r}} = \mathbf{U}^\top \mathbf{r} = \mathbf{D}\mathbf{V}^\top \mathbf{S}\mathbf{b} + \mathbf{U}^\top \mathbf{n} \quad (11)$$

By defining $\tilde{\mathbf{S}} = \mathbf{V}^\top \mathbf{S}$ and $\tilde{\mathbf{n}} = \mathbf{U}^\top \mathbf{n}$ we have

$$\tilde{\mathbf{r}} = \mathbf{D}\tilde{\mathbf{S}}\mathbf{b} + \tilde{\mathbf{n}} \quad (12)$$

Note that because both $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices they preserve norms of vectors. Thus, columns of $\tilde{\mathbf{S}}$ are also unit norm as were the columns of $\mathbf{S}$. Also, if the noise vector $\mathbf{n}$ is white, then $\tilde{\mathbf{n}}$ will remain white. However, when noise is colored with uncorrelated components, the transformation induced by multiplication with $\mathbf{U}^\top$ will correlate its components. This is not of too much concern because the resulting noise covariance matrix is already in diagonal decomposition with $\mathbf{U}^\top$ as eigenvectors $\tilde{\mathbf{W}} = E[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^\top] = \mathbf{U}^\top \mathbf{W}\mathbf{U}$.

The partition of (9) on $\mathbf{D}$ induces the following partition of $\tilde{\mathbf{S}}$

$$\tilde{\mathbf{S}} = \begin{bmatrix} \tilde{\mathbf{S}}_1 \\ \tilde{\mathbf{S}}_2 \end{bmatrix} \quad (13)$$

with $\tilde{\mathbf{S}}_1$ of dimension $\rho \times M$ and $\tilde{\mathbf{S}}_2$ of dimension $(NL-\rho) \times M$.

In light of the partitions in equations (9) and (13) we can ignore the $NK - \rho$ dimensions which are not going to be observed at the output and reduce dimensionality of the problem to the rank of the channel matrix ρ. This is equivalent to removing the ρ non-zero components of the received vector

$$\bar{\mathbf{r}} = [\mathbf{I}_\rho \ \ \mathbf{0}]\tilde{\mathbf{r}} = \bar{\mathbf{D}}\bar{\mathbf{S}}\mathbf{b} + \bar{\mathbf{n}} \quad (14)$$

with $\bar{\mathbf{S}} = \tilde{\mathbf{S}}_1$ and $\bar{\mathbf{n}} = [\mathbf{I}_\rho \ \ \mathbf{0}]\tilde{\mathbf{n}}$. The covariance matrix of the "new" noise vector is $\bar{\mathbf{W}} = E[\bar{\mathbf{n}}\bar{\mathbf{n}}^\top] = [\mathbf{I}_\rho \ \ \mathbf{0}]\tilde{\mathbf{W}}[\mathbf{I}_\rho \ \ \mathbf{0}]^\top$.

Equation (14) is identical to the single user dispersive channel equation [5] and the eigen-algorithm for interference avoidance can be applied to determine optimal codeword matrix $\bar{\mathbf{S}}$ as follows:

- Define the equivalent "clear space" problem [5] by premultiplying with $\bar{\mathbf{D}}^{-1}$

$$\mathbf{r}_{clr} = \bar{\mathbf{D}}^{-1}\bar{\mathbf{r}} = \bar{\mathbf{S}}\mathbf{b} + \bar{\mathbf{D}}^{-1}\bar{\mathbf{n}} \quad (15)$$

- For each bit m calculate the covariance matrix of the corresponding interference (from the other bits) plus noise

$$\mathbf{R}_m = \bar{\mathbf{S}}\bar{\mathbf{S}}^\top - \bar{\mathbf{s}}_m\bar{\mathbf{s}}_m^\top + \bar{\mathbf{D}}^{-1}\bar{\mathbf{W}}\bar{\mathbf{D}}^{-1} \quad (16)$$

- Replace $\bar{\mathbf{s}}_m$ by the minimum eigenvalue eigenvector of $\mathbf{R}_m$
- Iterate until convergence

We note that the existence of such codeword sets $\{\mathbf{s}_k\}$ is guaranteed by the convergence to the minimum TSC of greedy interference avoidance [7] as well as by constructive algorithms [10, 11].

With the matrix $\bar{\mathbf{S}}$ yielded by the eigen-algorithm for interference avoidance one can obtain the full dimension codeword matrix

$$\mathbf{S} = \mathbf{V}\begin{bmatrix} \bar{\mathbf{S}} \\ \mathbf{0} \end{bmatrix} \quad (17)$$

so that each input codeword vector is a linear combination of only those $\mathbf{v}_i$ which actually appear at the channel output.

IV. The Sum Capacity of the MIMO Channel

The sum capacity of the MIMO channel defined as the maximum mutual information between the received vector and the transmitted bit set is [5]

$$C = \frac{1}{2}\log[\det(\mathbf{H}\mathbf{S}\mathbf{S}^\top\mathbf{H}^\top + \mathbf{W})] - \frac{1}{2}\log(\det \mathbf{W}) \quad (18)$$

By applying the SVD (6) the expression in equation (18) is completely equivalent to

$$C = \frac{1}{2}\log[\det(\mathbf{D}\tilde{\mathbf{S}}\tilde{\mathbf{S}}^\top\mathbf{D}^\top + \tilde{\mathbf{W}})] - \frac{1}{2}\log(\det \tilde{\mathbf{W}}) \quad (19)$$

Taking into account the partition of matrix $\mathbf{D}$ in equation (9) and the fact that only the first ρ elements of $\tilde{\mathbf{r}}$ in equation (12) carry information, reducing the dimensionality of the received vector from NK to ρ does not affect mutual information. Thus, sum capacity remains unchanged and can be written in terms of the reduced order problem in equation (14) as

$$C = \frac{1}{2}\log[\det(\bar{\mathbf{D}}\bar{\mathbf{S}}\bar{\mathbf{S}}^\top\bar{\mathbf{D}} + \bar{\mathbf{W}})] - \frac{1}{2}\log(\det \bar{\mathbf{W}}) \quad (20)$$

It is known that interference avoidance monotonically decreases the total square correlation (TSC) [8] defined as

$$\text{TSC} = \text{Trace}\left[(\bar{\mathbf{S}}\bar{\mathbf{S}}^\top + \bar{\mathbf{W}})^2\right] \quad (21)$$

and that minimization of TSC is equivalent to maximizing the (sum) capacity [7] defined in equation (20). Also, from [8] we know that in a colored noise background the eigen-algorithm for interference avoidance *water fills* those dimensions with minimum background noise energy. Hence, the eigen-algorithm for interference avoidance yields an optimal codeword matrix that maximizes the capacity of the MIMO channel.

V. Conclusions

A new approach to multiple antenna systems has been proposed in the paper. Each transmit antenna uses a multicarrier modulation scheme with sinusoids as carriers. For sufficiently long transmission intervals sinusoids are (approximately) eigenfunctions of all channels in the multiple antenna link. This leads to a matrix model for the MIMO channel based only on eigenvalues associated with the eigenfunctions for all channels in the link.

A bit set is transmitted over the MIMO channel by assigning different codewords to each bit in the set analogous to a CDMA system where each bit corresponded to a different user. Optimal codewords that maximize the SINR for all bits are then derived using interference avoidance methods. Resulting codeword ensembles are also optimal in an information theoretic sense since they also maximize the sum capacity of the system. The receiver structure uses matched filters which are the optimal linear receivers for such an optimal ensemble of codewords [11].

The paper also presents a novel analysis of the matrix model of the MIMO channel. Rather than using the number of transmit and receive antennas, the analysis is done using the SVD of the channel matrix. This allows straightforward identification of those signal space dimensions that actually carry information, and reduces the dimensionality of the problem to only those dimensions.

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