The Density of the Universe

"The whole universe is expanding, so why be surprised that we're drifting apart?"

Because space is nearly flat today, we know the average density is close to the critical density.

$$\rho_{\mathrm{crit}} = \frac{3\,\mathrm{H}_0^{\;2}}{8\pi\,\mathrm{G}} = 10^{-26}\ \mathrm{kg/m^3}$$

The density can be provided by either matter or energy:

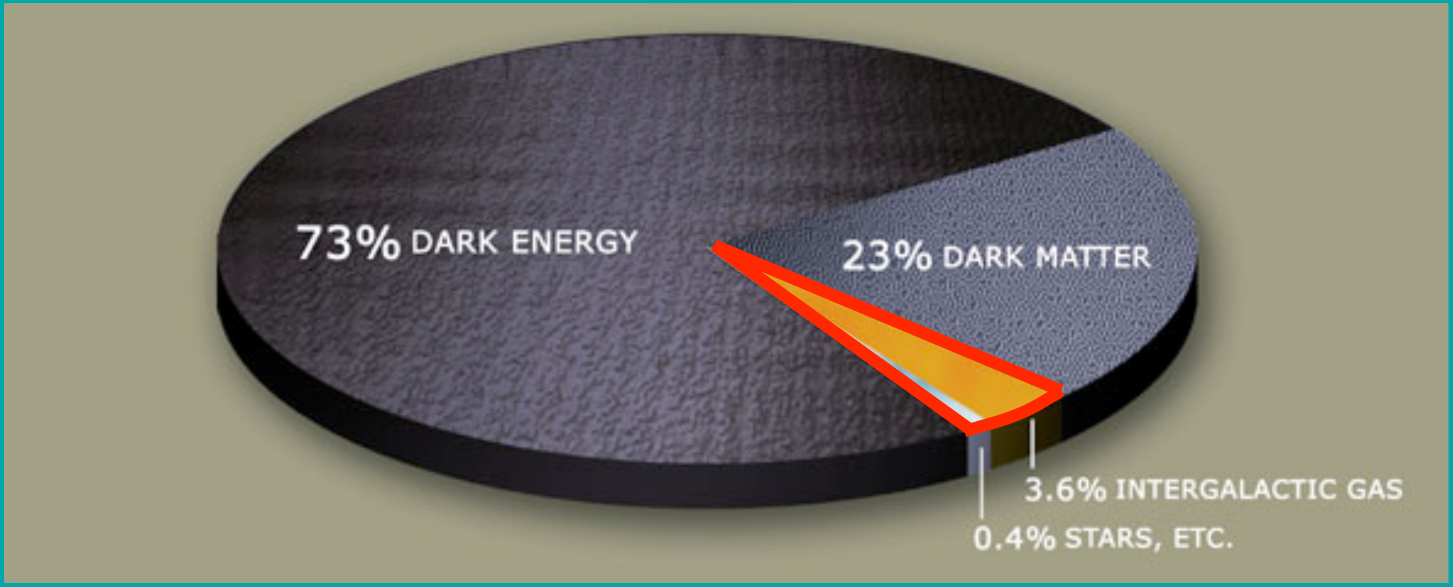
mass density:

$$\rho_{\text{crit}} = 10^{-26}\ \text{kg/m}^3$$

energy density:

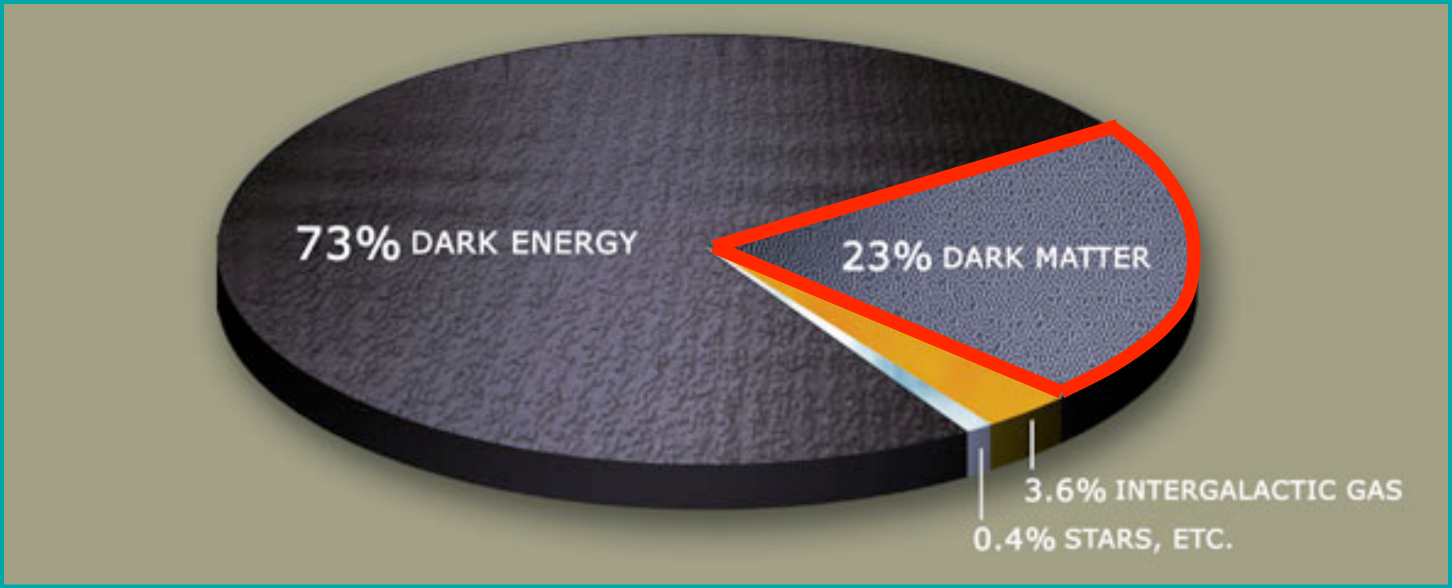
$$\rho_{\text{crit}} c^2 = 9 \times 10^{-9}\ \text{joules/m}^3$$

Right now, only **4%** of the density is provided by **ordinary matter** (protons, neutrons, electrons).



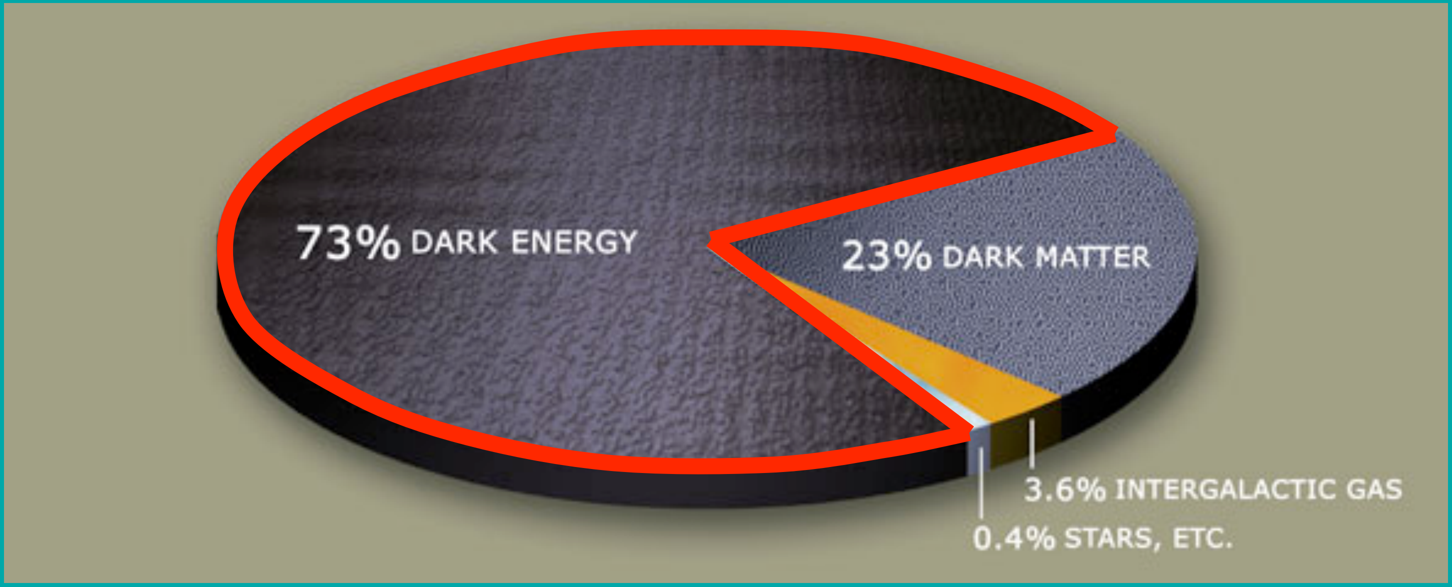
Some ordinary matter is in stars, but most is in low-density intergalactic gas.

Right now, **23%** of the density is provided by **dark matter** (WIMPs, neutrinos).



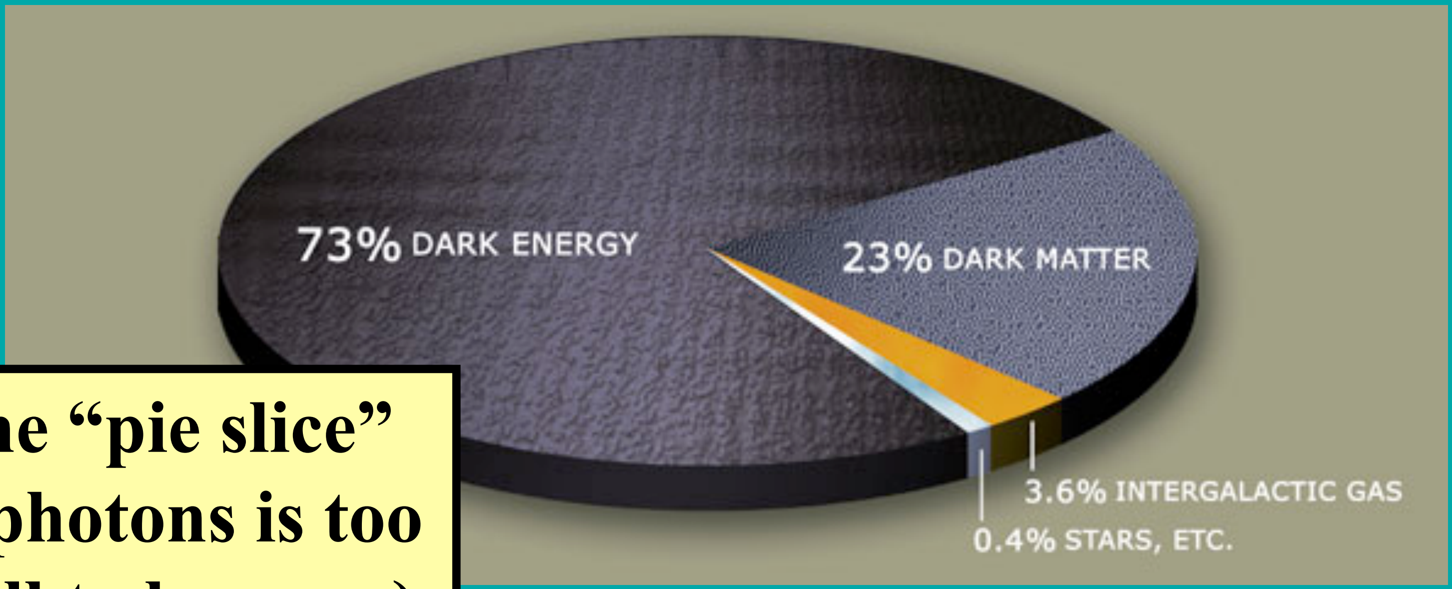
WIMPs, neutrinos, protons, neutrons, & electrons are particles with mass.

Right now, **73%** of the density is provided by **dark energy**.



Dark energy is an energy field, and is not made of massive particles.

Right now, **0.005%** of the density is provided by **photons**.



(The "pie slice" for photons is too small to be seen.)

Photons are particles with **energy**, but not mass.

On the futility of stars.

Stars have been converting H to He for 13 billion years. However, most helium was created at t ≈ 3 minutes.

Stars have been making photons for 13 billion years. However, most light is left over from t ≈ 400,000 years.

The total density of the universe was greater in the past than it is now.

hot
dense
→
cold
tenuous

The density of different components (photons, matter, dark energy) varied at different rates as space expanded.

Dark energy remains **constant** in density as the universe expands.

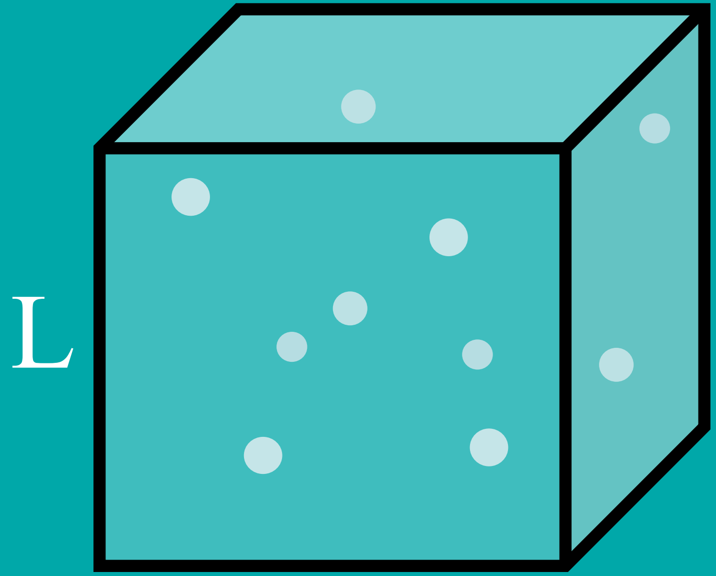
Current density of dark energy =

$$0.73\ \rho_{crit}\ c^2$$

$$= 0.73\ (9 \times 10^{-9}\ \text{joules/m}^3)$$

$$= 6.6 \times 10^{-9}\ \text{joules/m}^3$$

How does density of **matter** evolve with time?



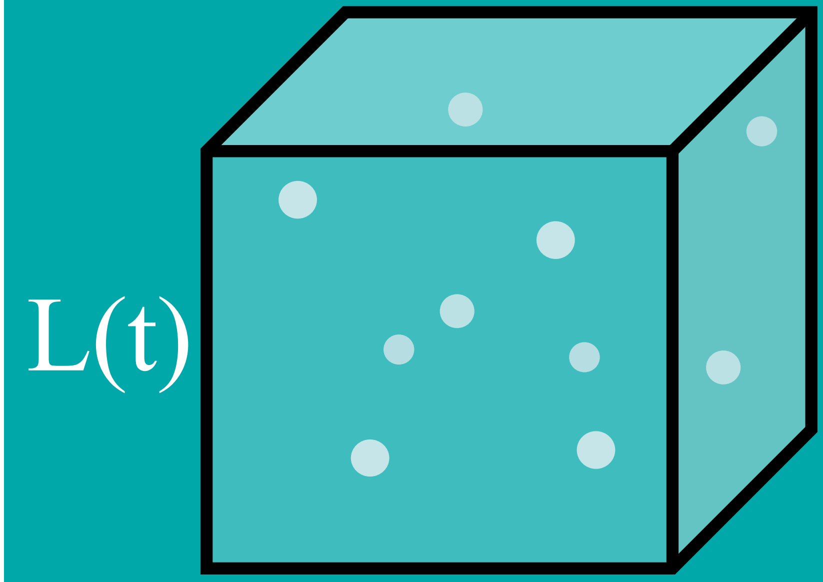
Consider a cube with sides of length L.

Volume of cube = L^3

Mass of particles in cube = M

Density of particles in cube (ρ)= M/L^3

Let's suppose the cube is expanding along with the universe.

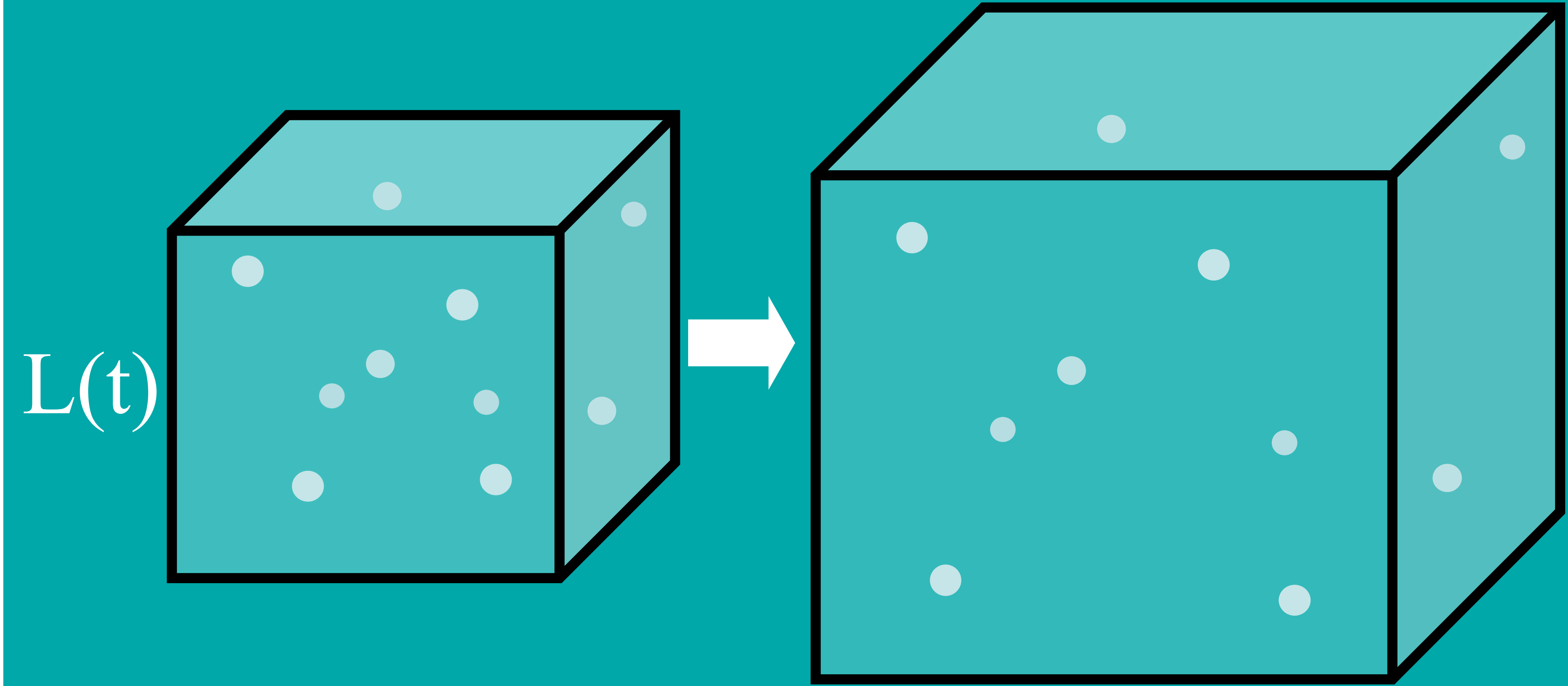


Length of side of cube $= L(t) = L_0 \times a(t)$

L_0 = Current length of side

$a(t)$ = scale factor of universe

As the cube expands, the number of particles is constant. The mass per particle is constant.



Thus, the total mass M within the cube is **constant**.

The density of matter (ρ) in an expanding universe:

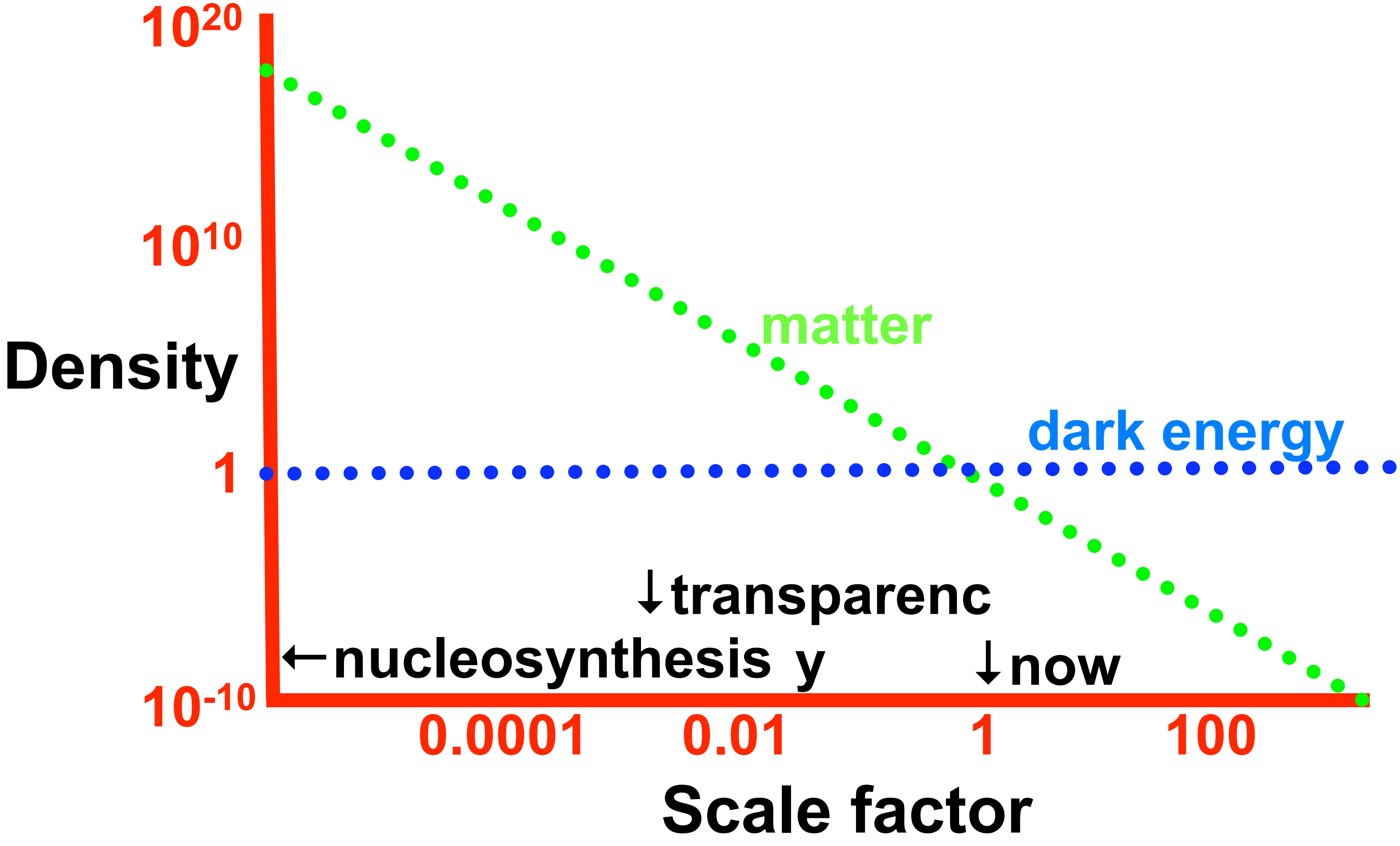
$$\rho(t) = \frac{M}{L(t)^3} = \frac{M}{L_0^3 a(t)^3} = \frac{\rho_0}{a(t)^3}$$

ρ_0 = Current density (0.27×10^{-26} kg/m^3)

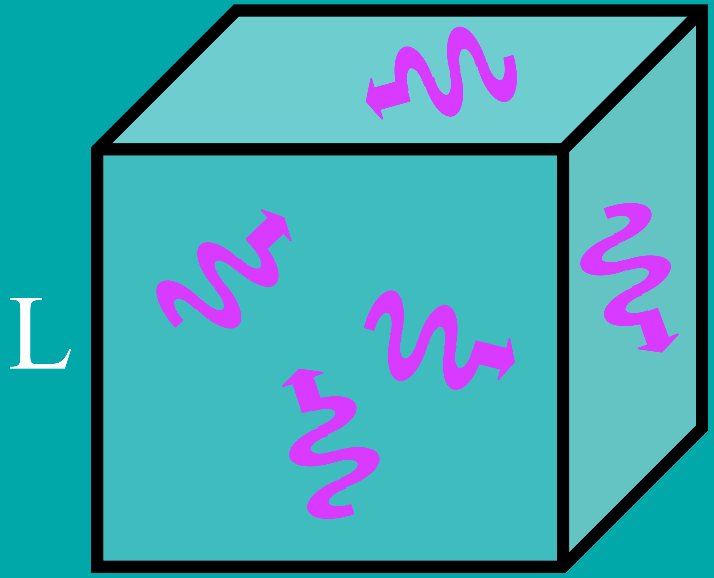
$a(t)$ = scale factor of universe

$\rho(t)$ = Density at time t

As the universe expands, a(t) increases. Thus, the density of matter, proportional to $1/a(t)^3$, **decreases**.



How does energy density of **photons** evolve with time?



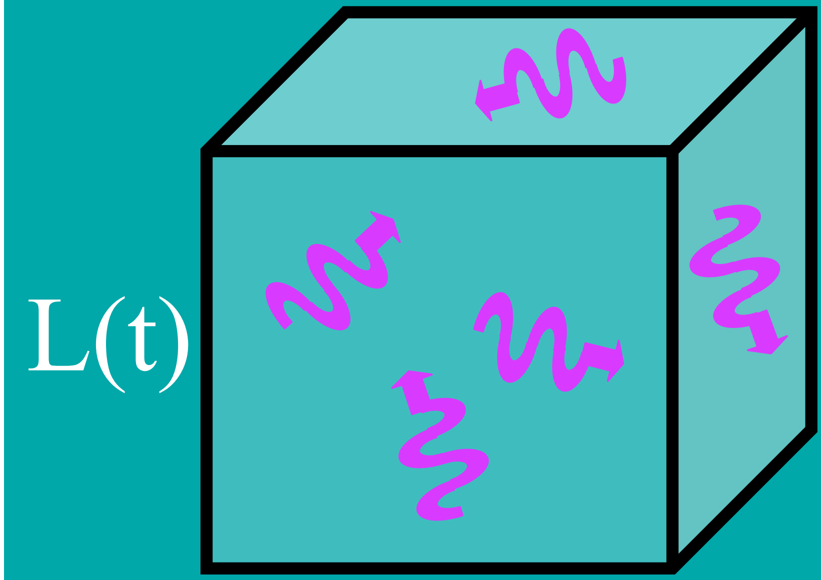
Consider a cube with volume L^3.

Number of photons in cube = N

Energy per photon = E

Energy density of photons in cube (ρc^2)= $N \times E / L^3$

Let’s suppose the cube is expanding along with the universe.

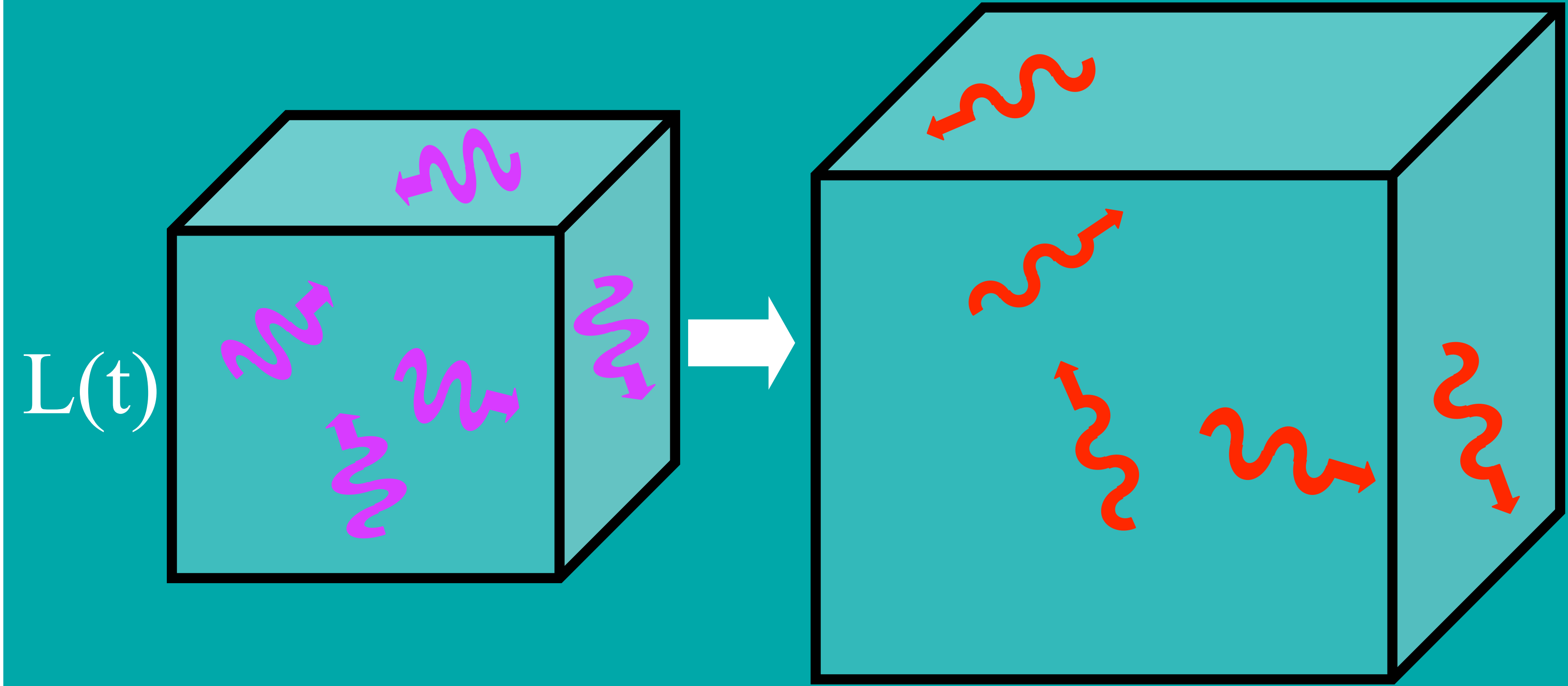


Length of side of cube $= L(t) = L_0 \times a(t)$

L_0 = Current length of side

$a(t)$ = scale factor of universe

As the cube expands, the number of photons is roughly **constant** (remember the futility of stars!)



However, the energy E of each photon is **not** constant.

As space expands, the wavelength of light expands.
Longer wavelength → lower frequency → lower photon energy.

Wavelength: $\lambda(t) = \lambda_0 \times a(t)$

Frequency: $f(t) = f_0 / a(t)$

Photon energy: $E(t) = E_0 / a(t)$

The energy density of photons (ρc^2) in an expanding universe:

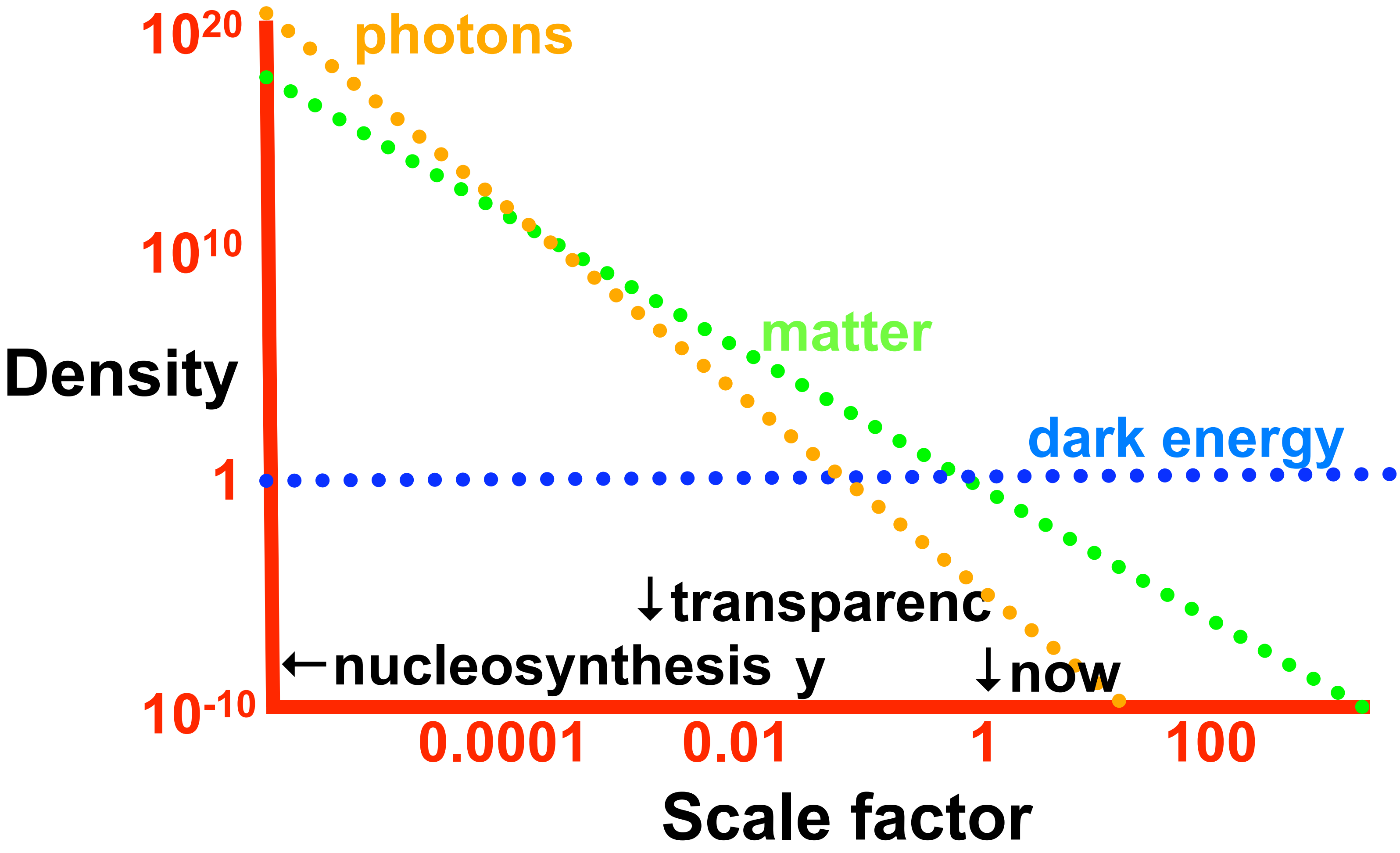
$$\rho(t)c^2 = \frac{N\,E(t)}{L(t)^3} = \frac{N\,E_0 / a(t)}{L_0^3 a(t)^3} = \frac{\rho_0 c^2}{a(t)^4}$$

$\rho_0 c^2$ = Current density (0.00005 $\rho_{crit}c^2$)

$a(t)$ = scale factor of universe

$\rho(t)c^2$ = Density at time t

As the universe expands, energy density of photons decreases as $1/a(t)^4$.



When the scale factor was **$a < 0.0003$**, & age of the universe was **$t < 70,000$ years**, the universe was **radiation-dominated**.

"Radiation-dominated" simply means that photons provided most of the density.

When **$0.0003 < a < 0.7$**, &

70,000 years < t < 10 billion years,

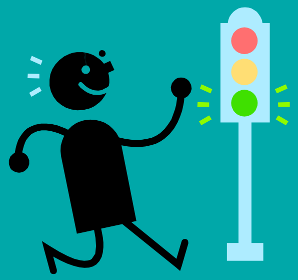
the universe was **matter-dominated**.

“Matter-dominated” means that ordinary & dark matter provided most of the density.

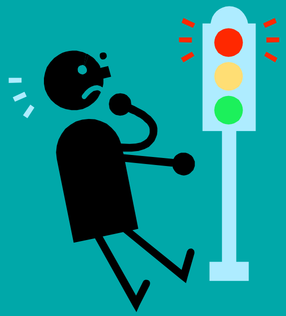
Now that **$a > 0.7$ & $t > 10$ billion years**, the universe is **dark-energy-dominated**.

Photons & matter have finally been diluted to the point where dark energy provides most of the density.

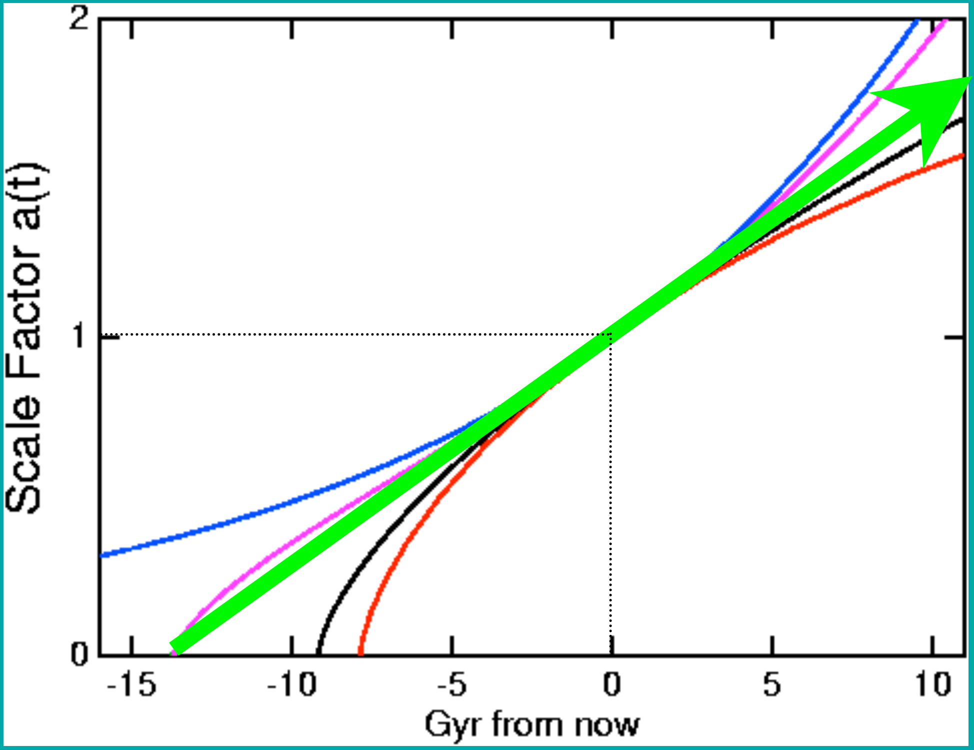
We have just reached the stage where expansion is **speeding up** (under the influence of dark energy).



At earlier times, gravity acting on photons & matter caused the expansion to **slow down**.



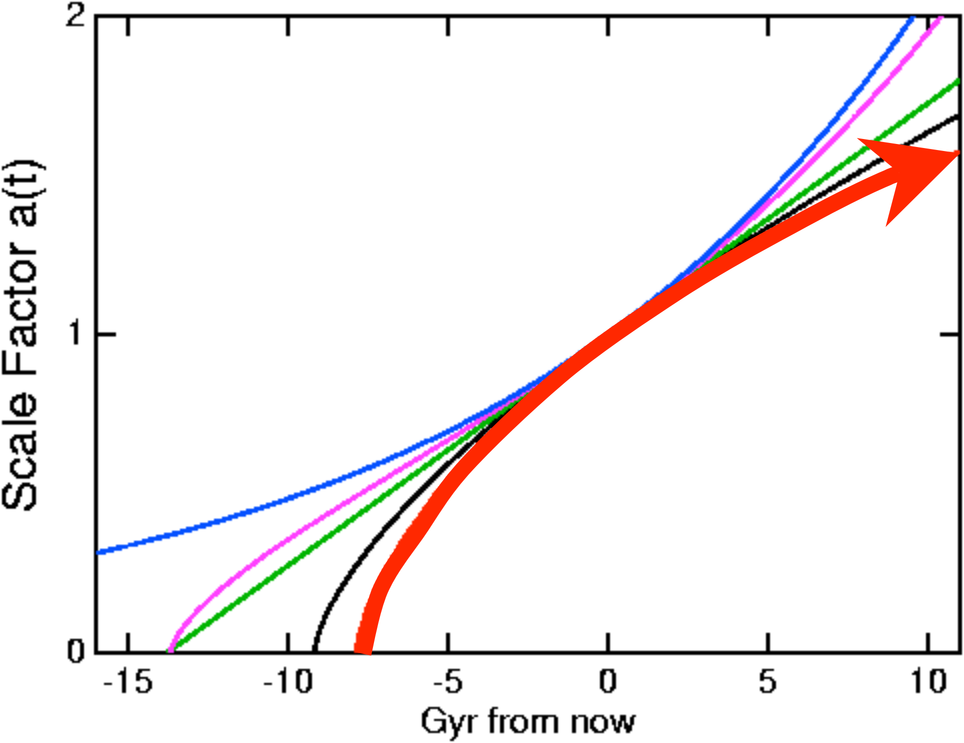
In an empty universe ($\rho=0$), the age of the universe is exactly equal to the Hubble time.



Gyr =
gigayear =
1 billion years

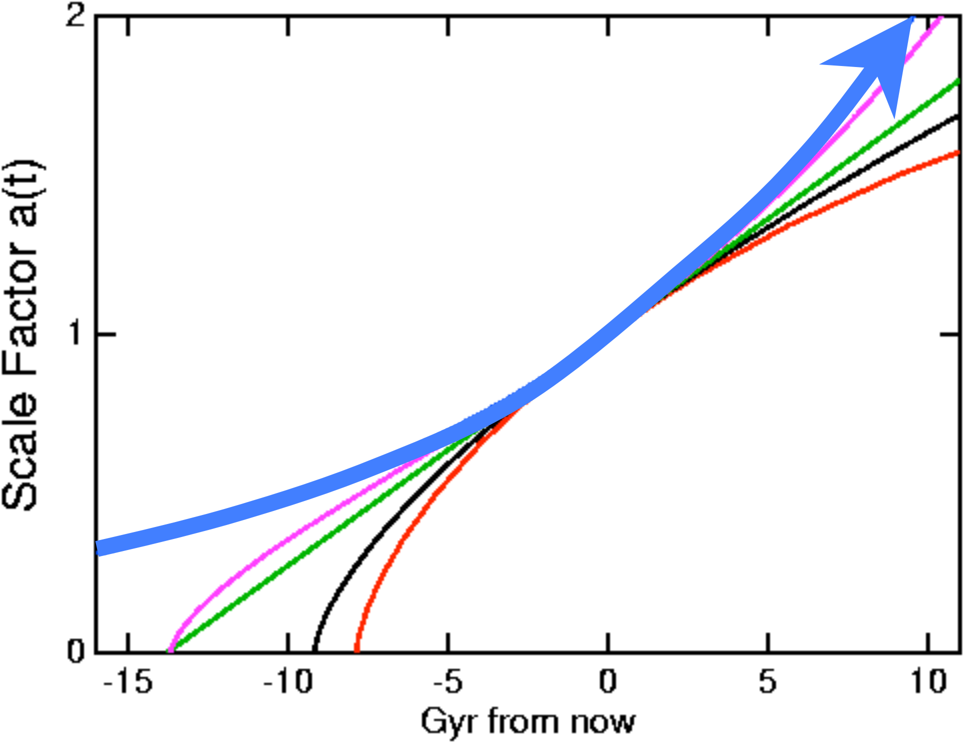
The expansion of the universe is coasting in this case – relative speed of any 2 points is constant.

In a flat universe containing only photons & matter, the age of the universe is less than the Hubble time.



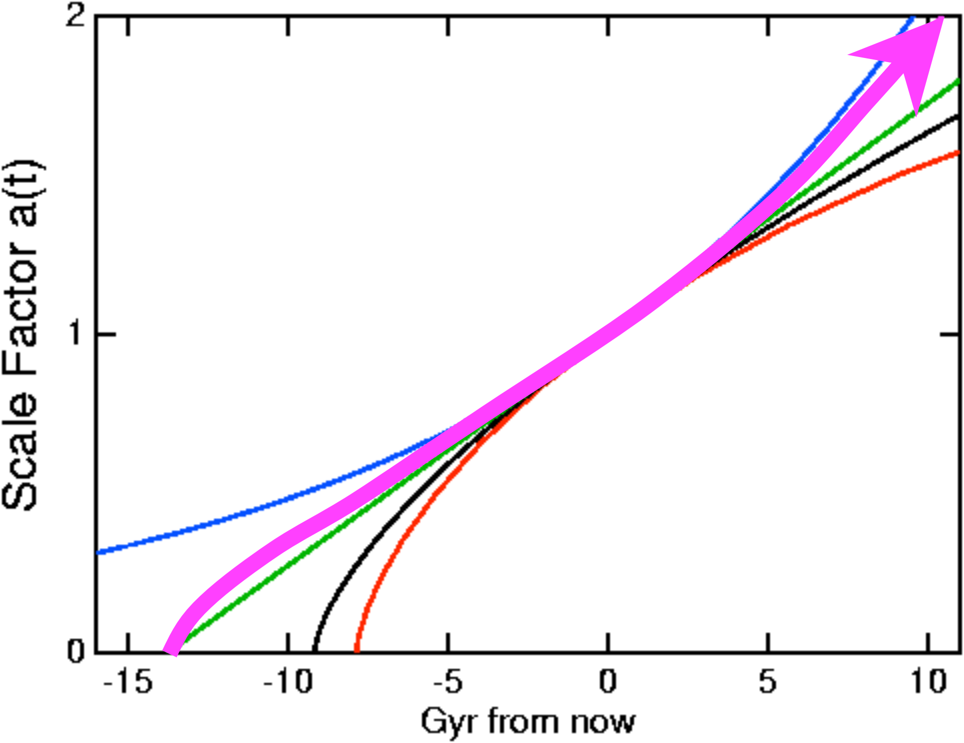
Expansion is slowing down in this case – relative speed of any 2 points was faster in the past.

In a flat universe containing lots of dark energy, the age of the universe is greater than the Hubble time.



Expansion is speeding up in this case – relative speed of any 2 points was slower in the past.

What about the real universe? Amusingly, the early slow-down almost exactly balances the later speed-up.

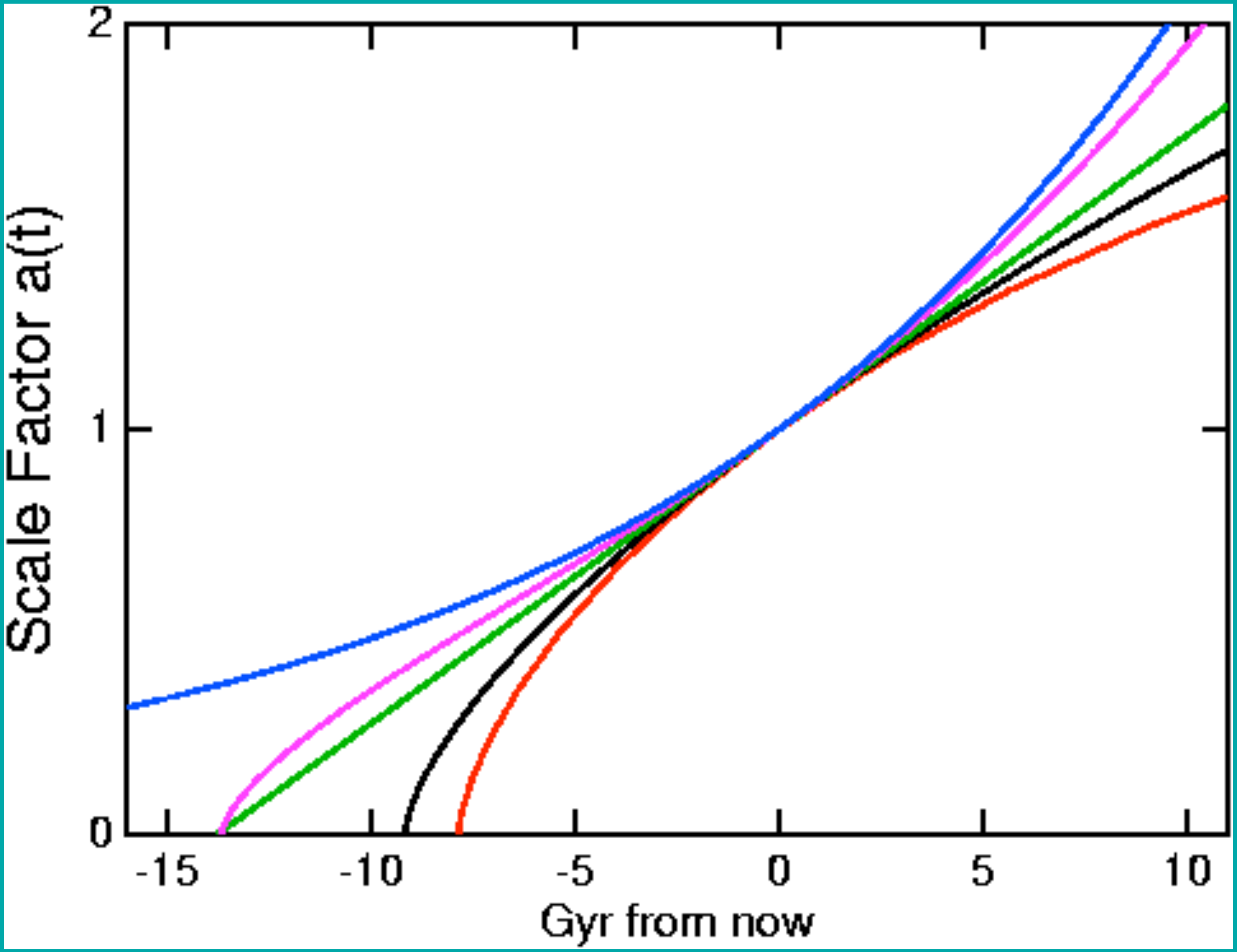


Hubble time = 14 billion years.

Age of universe = 13.7 billion years.

The Destiny of the Universe

The universe is expanding: that is, the scale factor a(t) is **increasing** with time.



Naïve question:

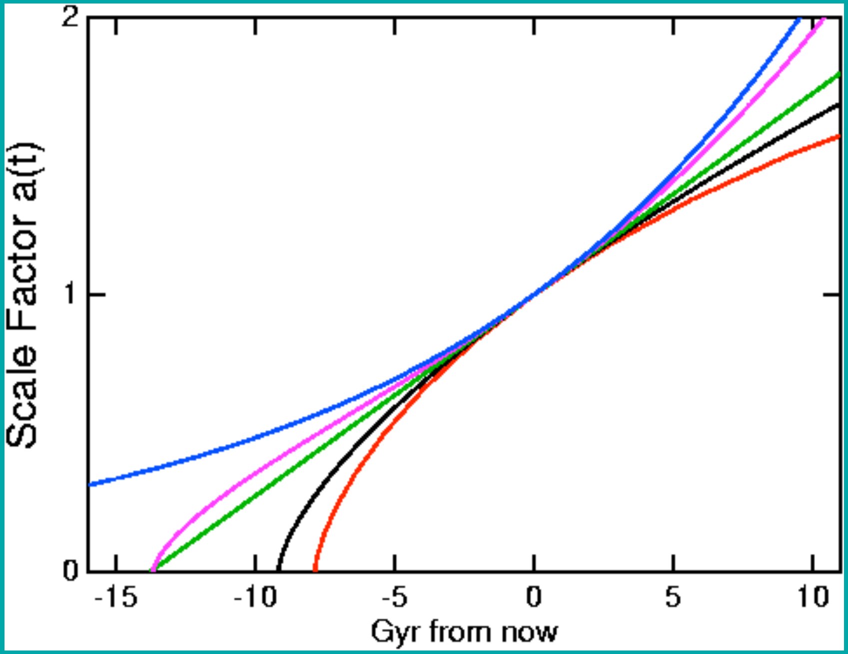
Why is the universe expanding?

Naïve answer:

The universe is expanding today because it was expanding yesterday.

(Objects in motion tend to remain in motion at constant velocity.)

The universe was expanding yesterday because it was expanding the day before yesterday.



…And so forth, back to the Big Bang (the beginning of expansion).

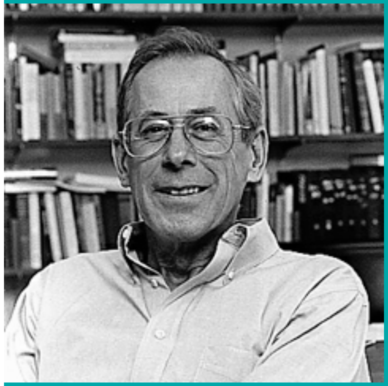
Naïve question:

Why did the universe **start** expanding?

(What put the “bang” in the Big Bang?)

Naïve answer:

We don’t know.



“The big bang theory describes how our universe is evolving, not how it began.”

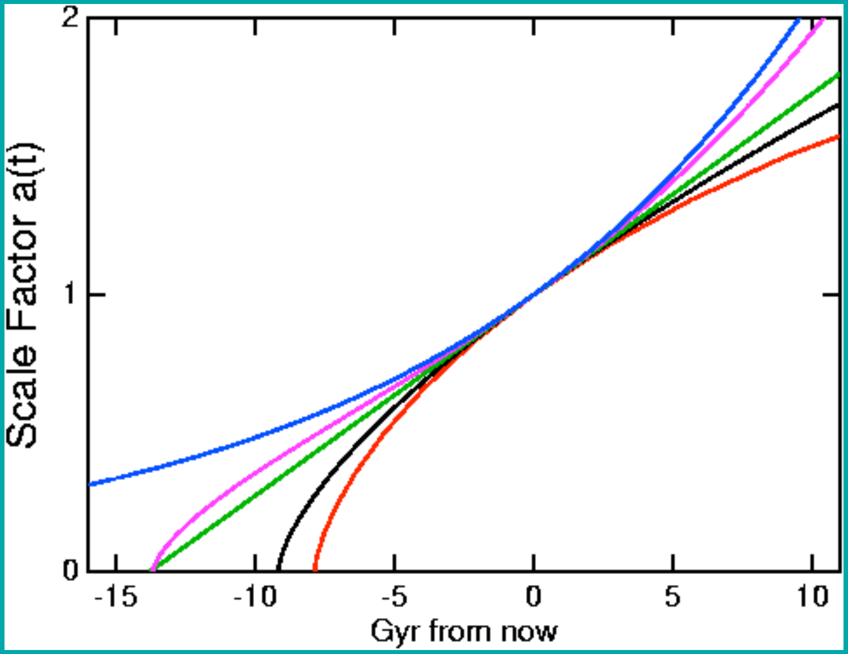
– Jim Peebles

The origin of the expansion
(in Newton’s terms, the force that
caused the initial acceleration)
was in the very early universe.

To describe the very early universe, we need a good theory of “quantum gravity”.

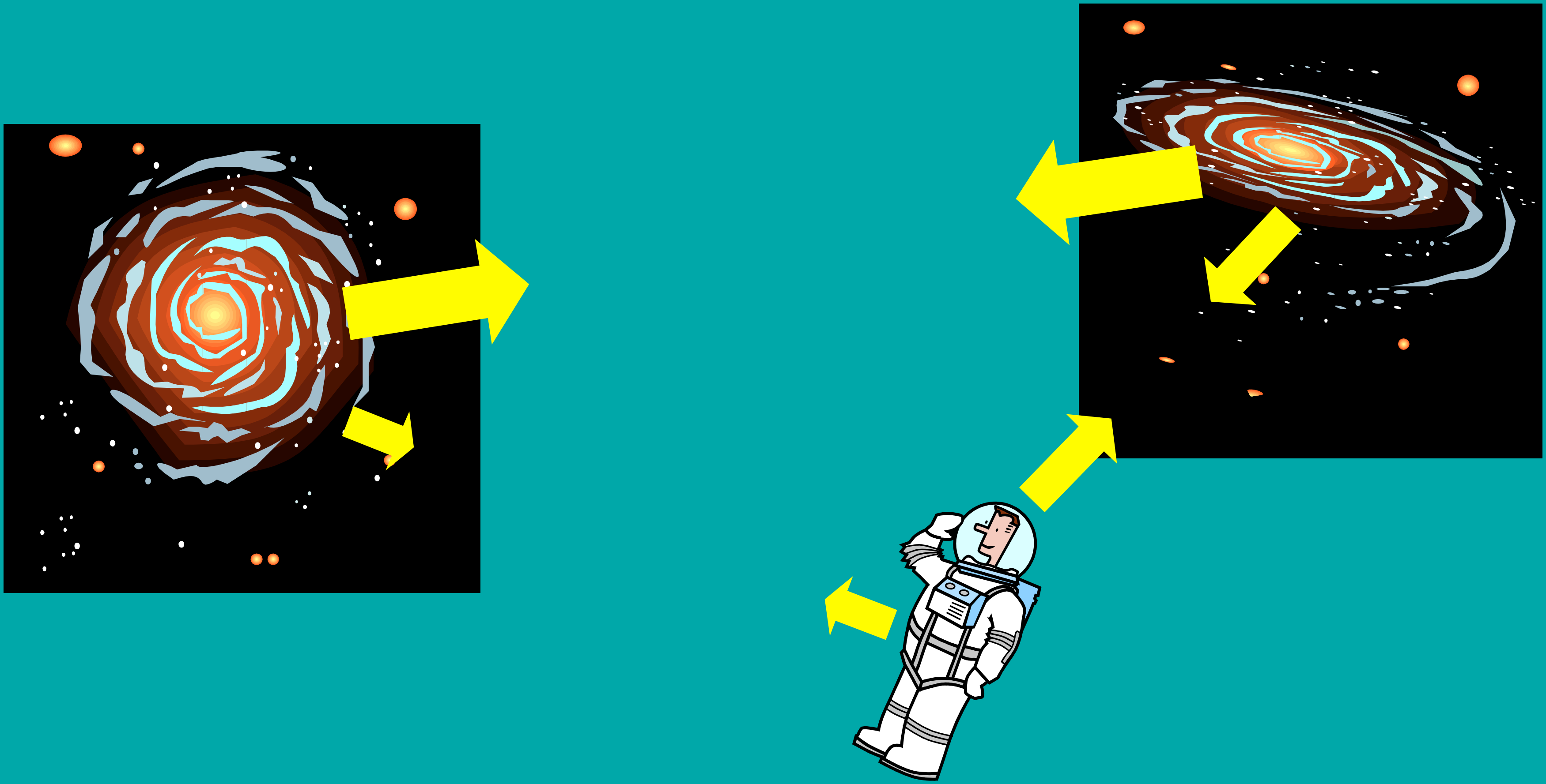
We haven’t got one.

Another naïve question: Will the universe expand forever?

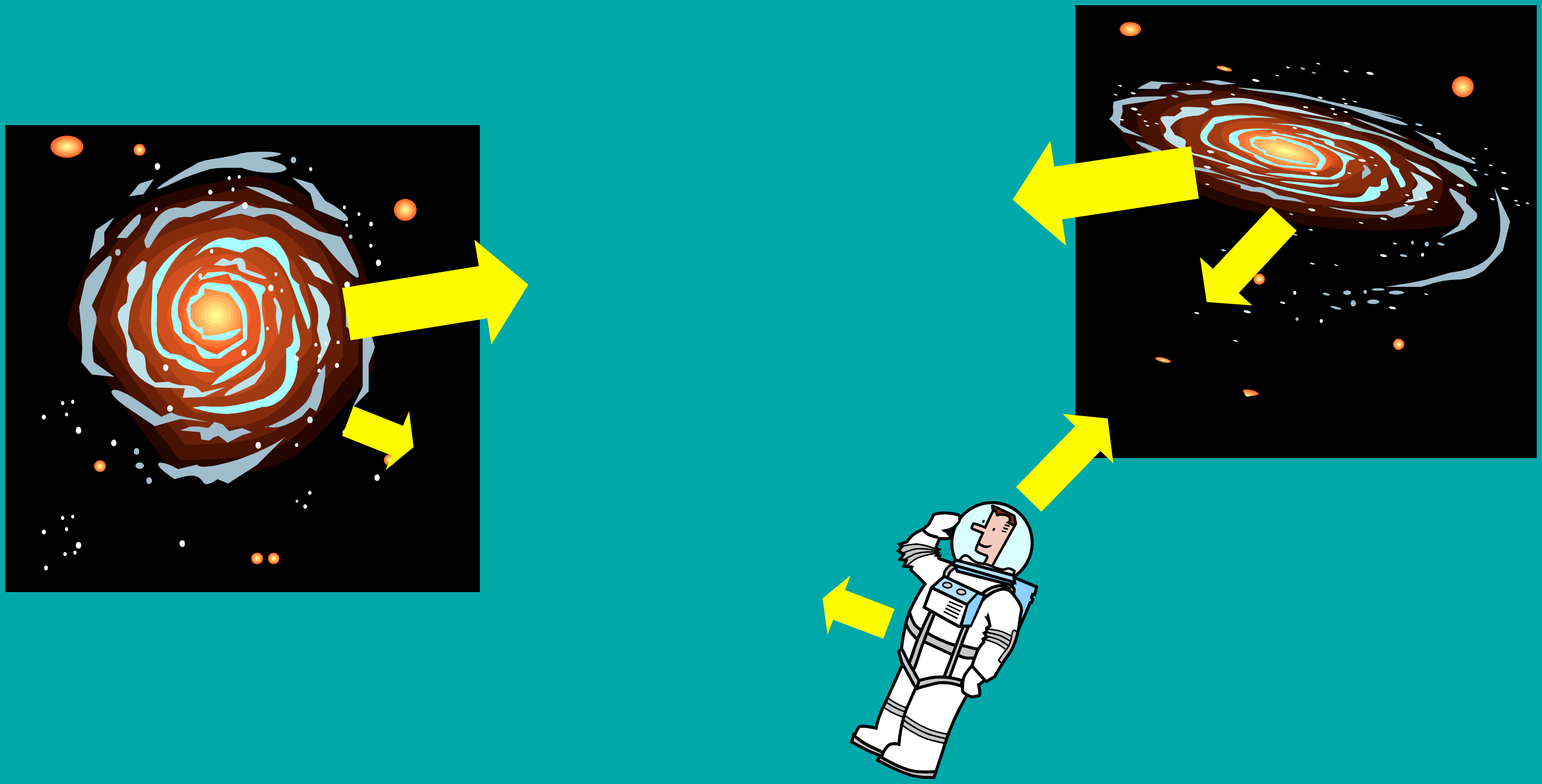


(A force can make motion speed up or slow down.)

What would Newton say? The universe is full of massive objects attracting each other through gravity.



Gravitational attraction slows the expansion. Can the expansion ever be brought to a halt by gravity?



Start with a related Newtonian problem:

A boy standing on the Earth throws an apple upward: initially, the distance between apple & Earth is **increasing**.

Is the attractive force between apple & Earth enough to stop the apple from rising?

What goes up must come down.

…unless it’s traveling faster than the **escape velocity**.

Small initial speed:
short distance upward.

Larger initial speed:
long distance upward.

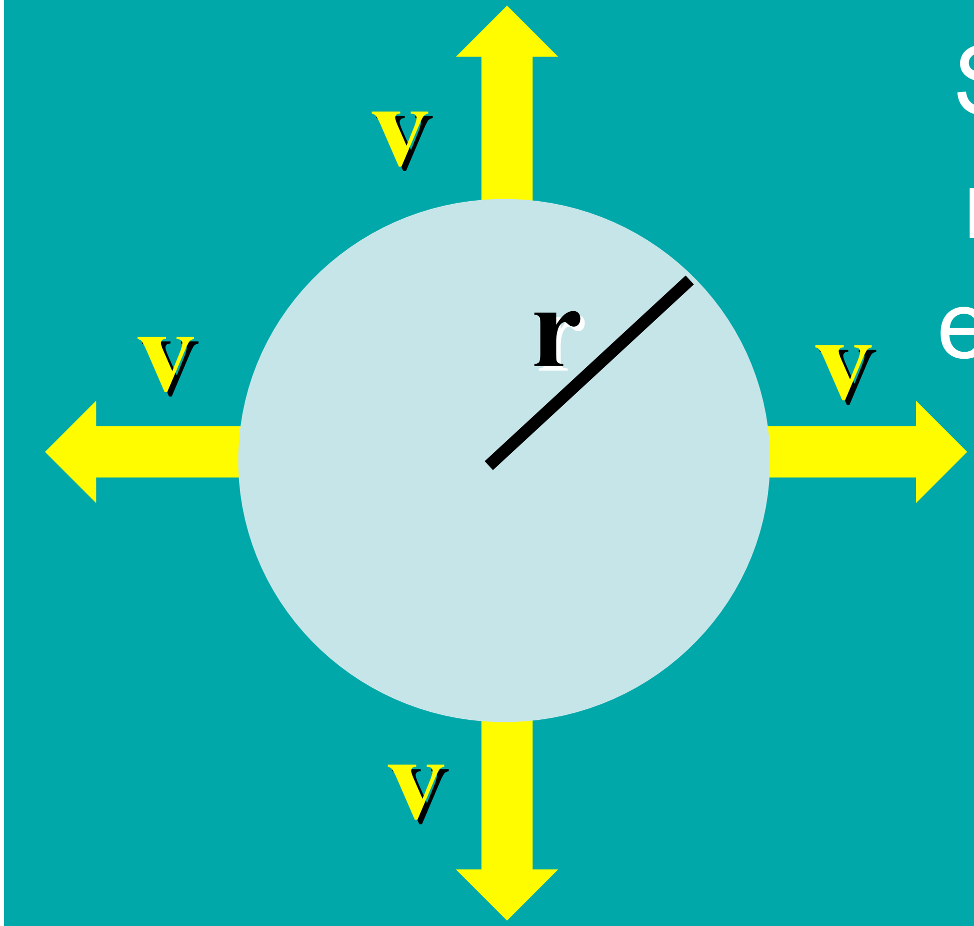
Speed > escape velocity:
to infinity!

Escape velocity from a planet (or star) depends on its **mass (M)** & **radius (r)**.

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

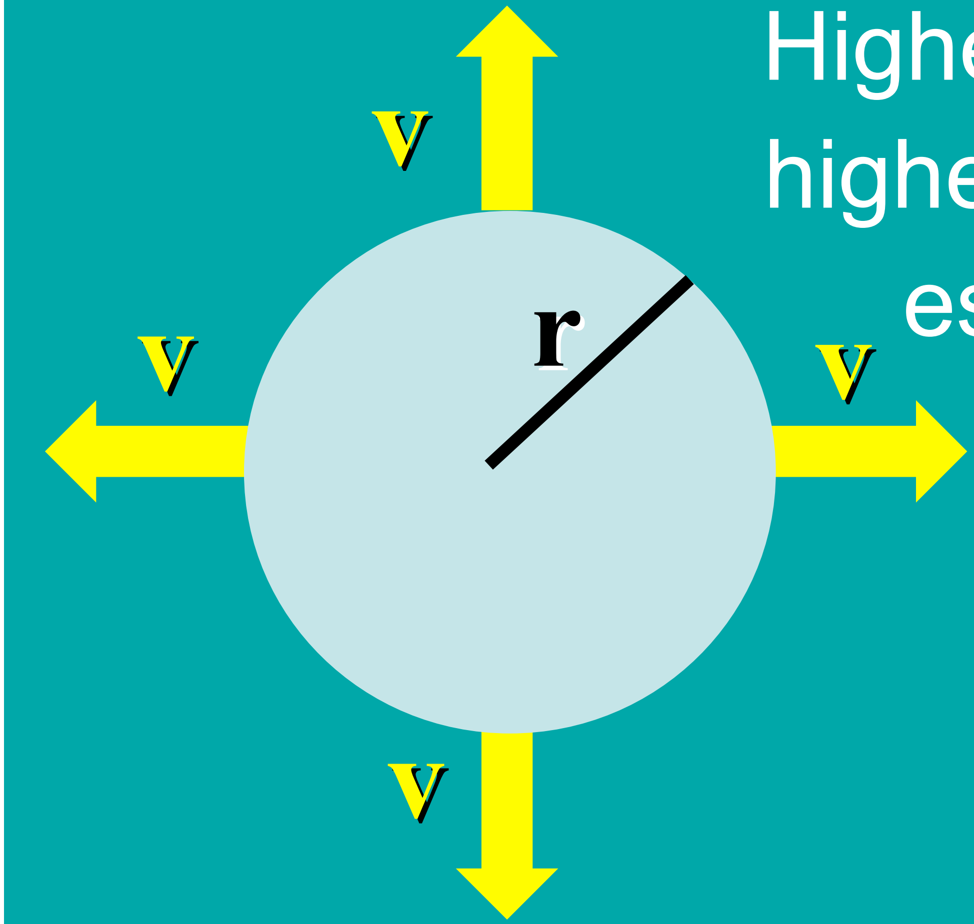
Escape velocity from **Earth**:
11 km/sec = 25,000 mph

Escape velocity from **Sun**:
620 km/sec = 1,400,000 mph



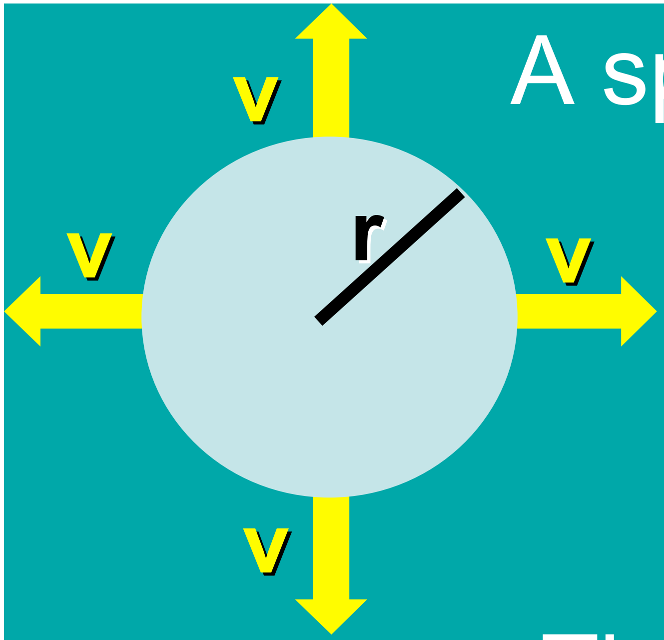
Suppose a sphere of matter (radius = r) is expanding outward at a speed v.

If expansion speed is greater than escape speed ($v > v_{esc}$), the sphere will expand forever.



Higher density ρ leads to a higher mass M, & a higher escape velocity v_{esc}.

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$



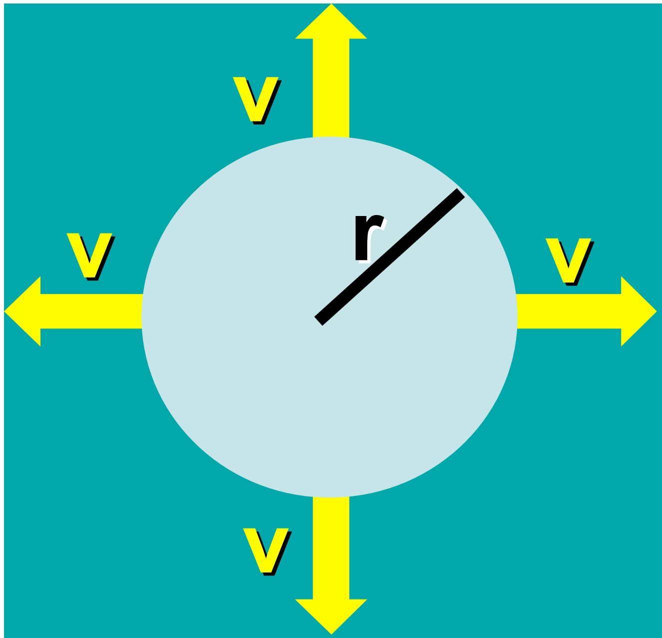
A sphere is expanding at exactly its escape velocity when

$$v = \sqrt{\frac{8\pi}{3} G r^2 \rho}$$

That is, when its density is

$$\rho = \frac{3}{8\pi G} \frac{v^2}{r^2}$$

This is the critical density for a sphere of radius r expanding at speed v.



$$\rho_{\text{crit}} = \frac{3}{8\pi G}\frac{v^2}{r^2}$$

Suppose our sphere of matter is part of the expanding universe, so that $v = H_0 r$.

$$\rho_{\text{crit}} = \frac{3}{8\pi G}\frac{(H_0\cancel{r})^2}{\cancel{r}^2}$$

$$\rho_{\mathrm{crit}} = \frac{3\,H_0^{\;2}}{8\pi\,G}$$

We found this result using
old-fashioned **Newtonian** physics.

However, it's the same as the critical
density required to make a flat universe,
according to Einstein!!

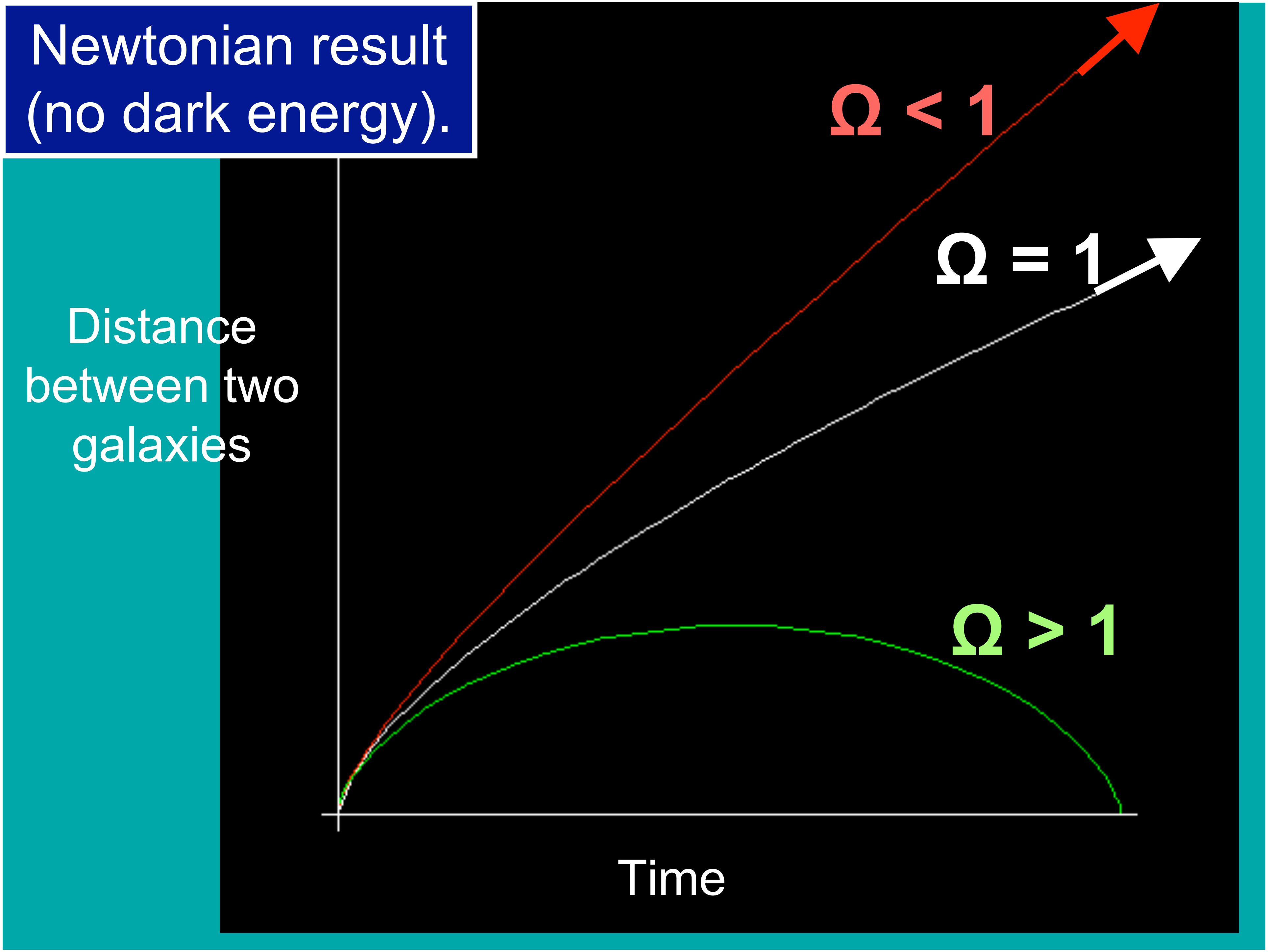
Newton says:

Destiny of the universe depends on the ratio of its **density** to the **critical density**.

$$\Omega = \frac{\rho}{\rho_{\mathrm{crit}}}$$

Omega (Ω) is also called the "**density parameter**".

Newtonian result (no dark energy).



Newtonian result
(no dark energy).

$\Omega > 1$ (density greater than critical):

The Big Crunch

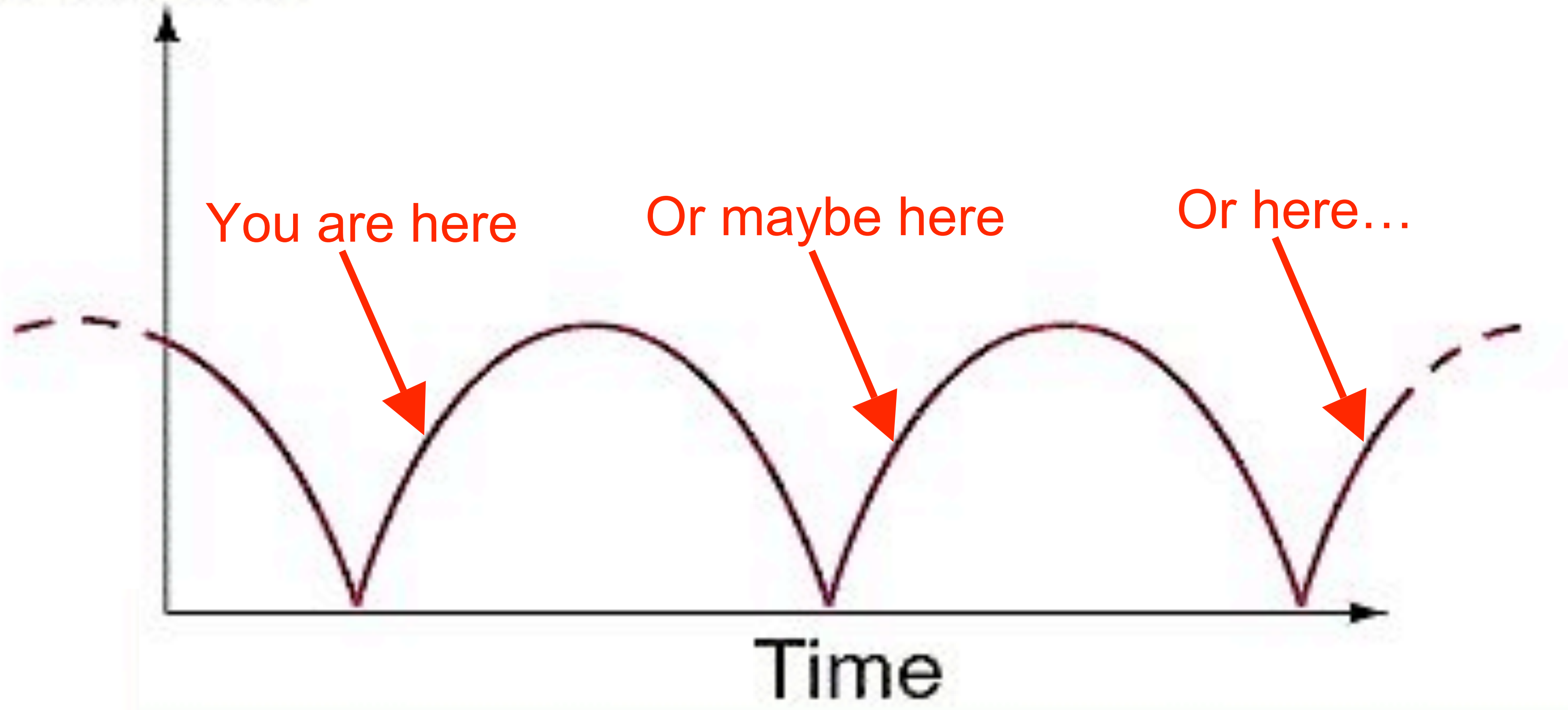
(recollapse, becoming hotter)

$\Omega \leq 1$ (density less than or equal to critical):

The Big Chill

(perpetual expansion, becoming cooler)

Amusing speculation: perhaps a Big Crunch would lead to a Big Bounce.



Einstein says:

Curvature of the universe depends on the ratio of its **density** to the **critical density**.

Now the density ρ includes dark energy and photons as well as matter.

Relativistic result (with dark energy).

Ω>1 (density greater than critical):

Positive curvature

Ω<1 (density less than critical):

Negative curvature

Ω=1 (density equal to critical):

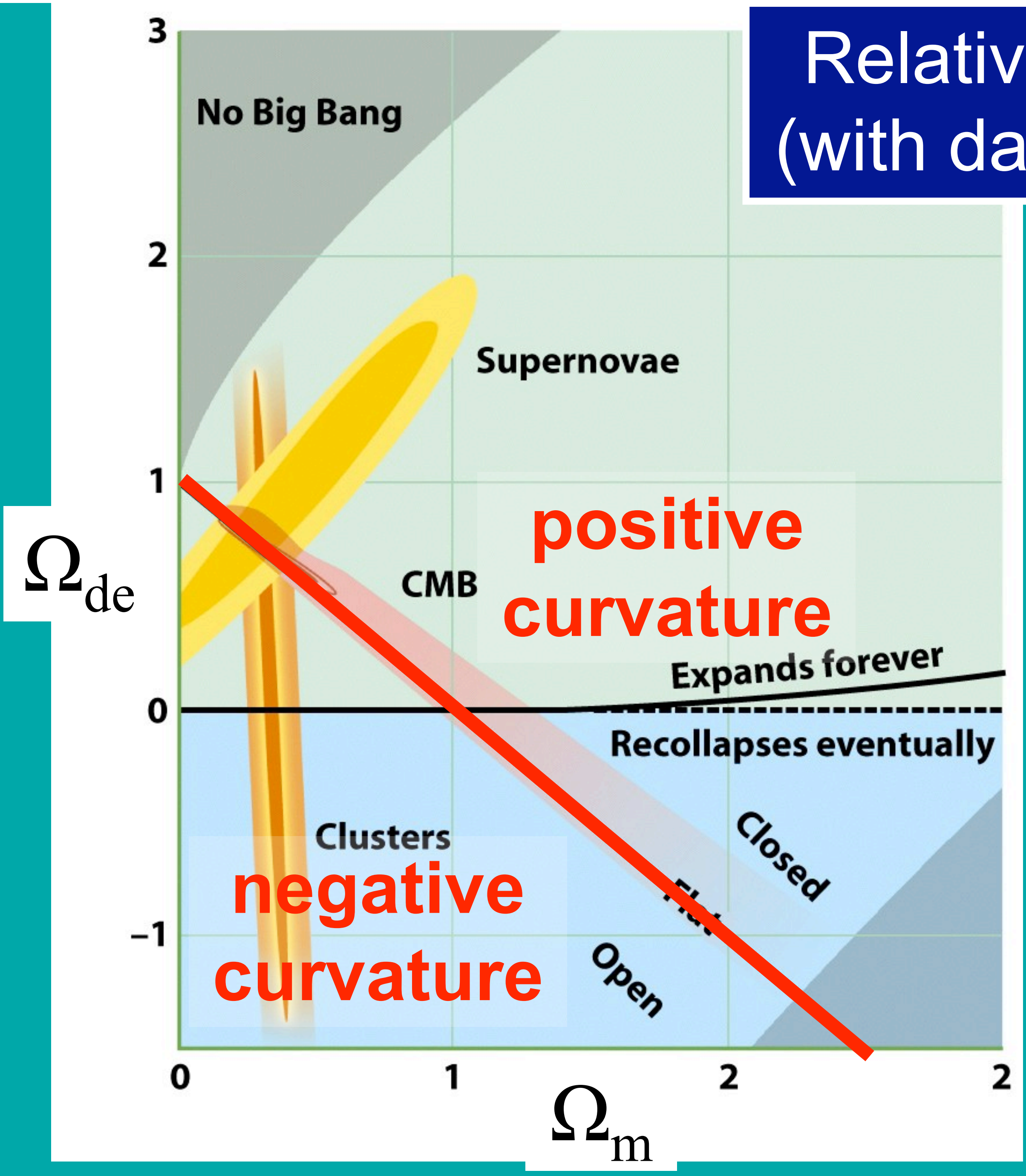
Flat

Einstein says:

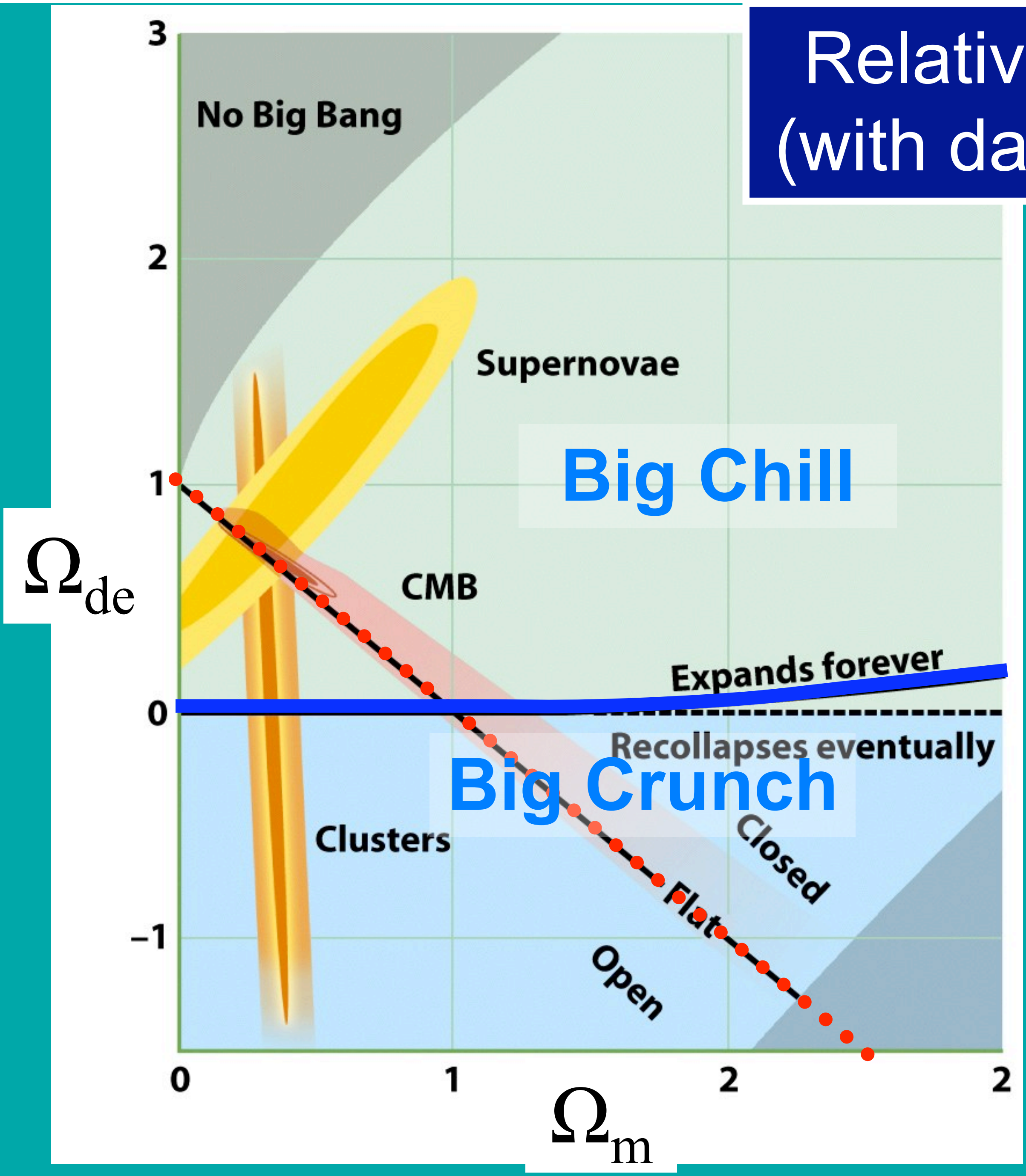
Destiny of the universe depends on the amounts of matter & dark energy today.

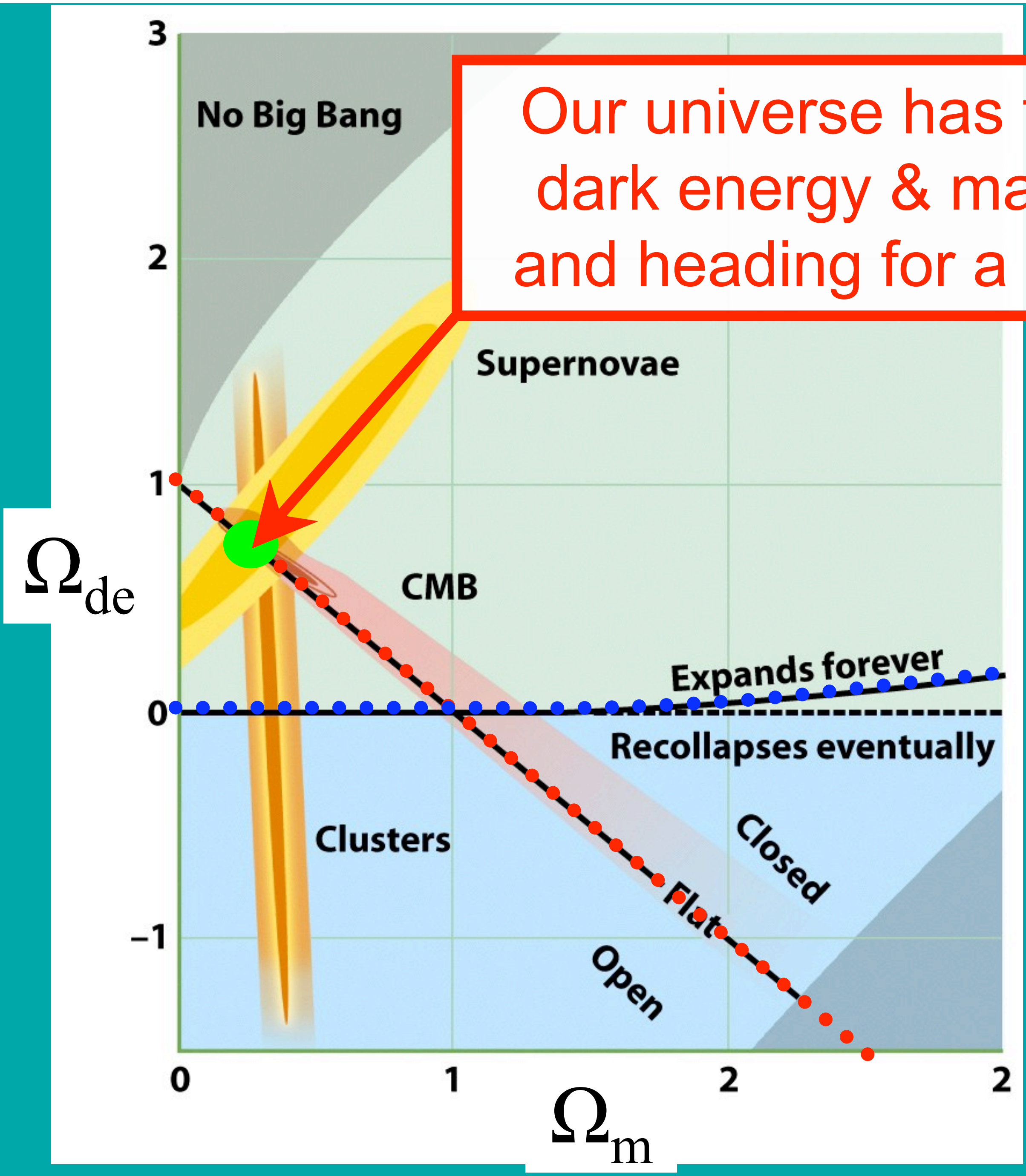
Today, $\Omega_m = 0.27$, $\Omega_{de} = 0.73$

Relativistic result (with dark energy).



Relativistic result (with dark energy).





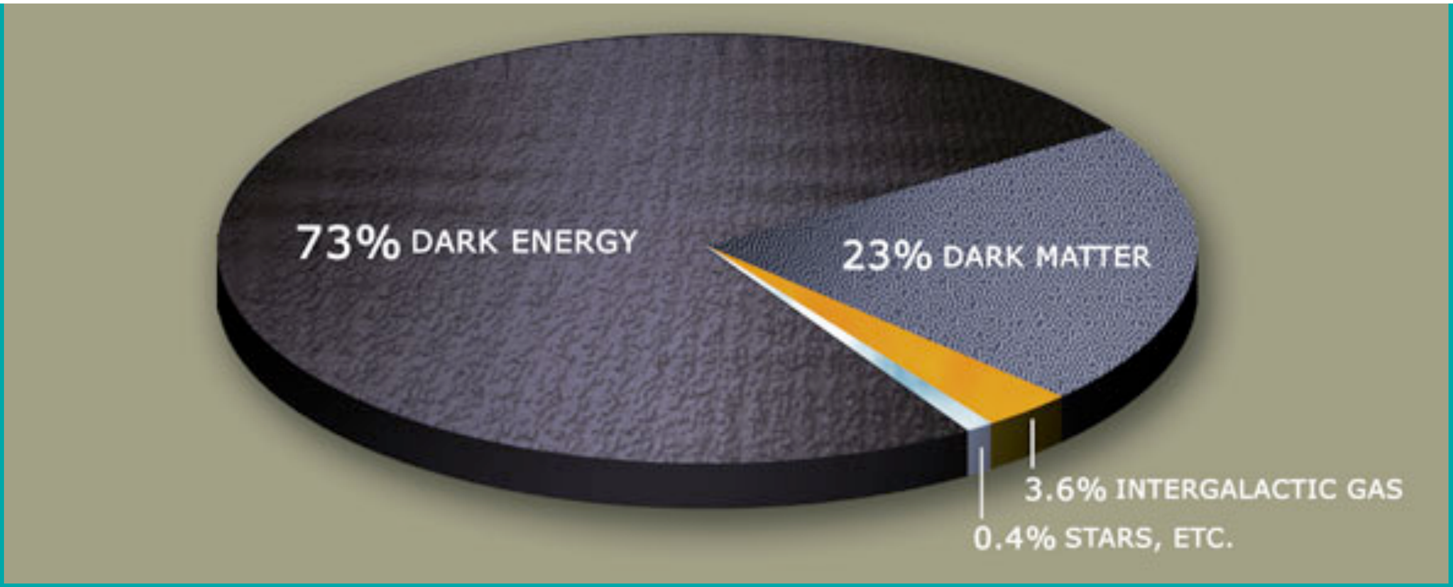
Our universe has this much dark energy & matter: **flat**, and heading for a **Big Chill**.

Why is the Universe Lumpy?

The average density of the universe is 10^{-26} kg/m^3.

However, most of the universe is slightly **less dense** than average (voids).

Some of the universe is **much denser** than average (stars, white dwarfs, black holes…)

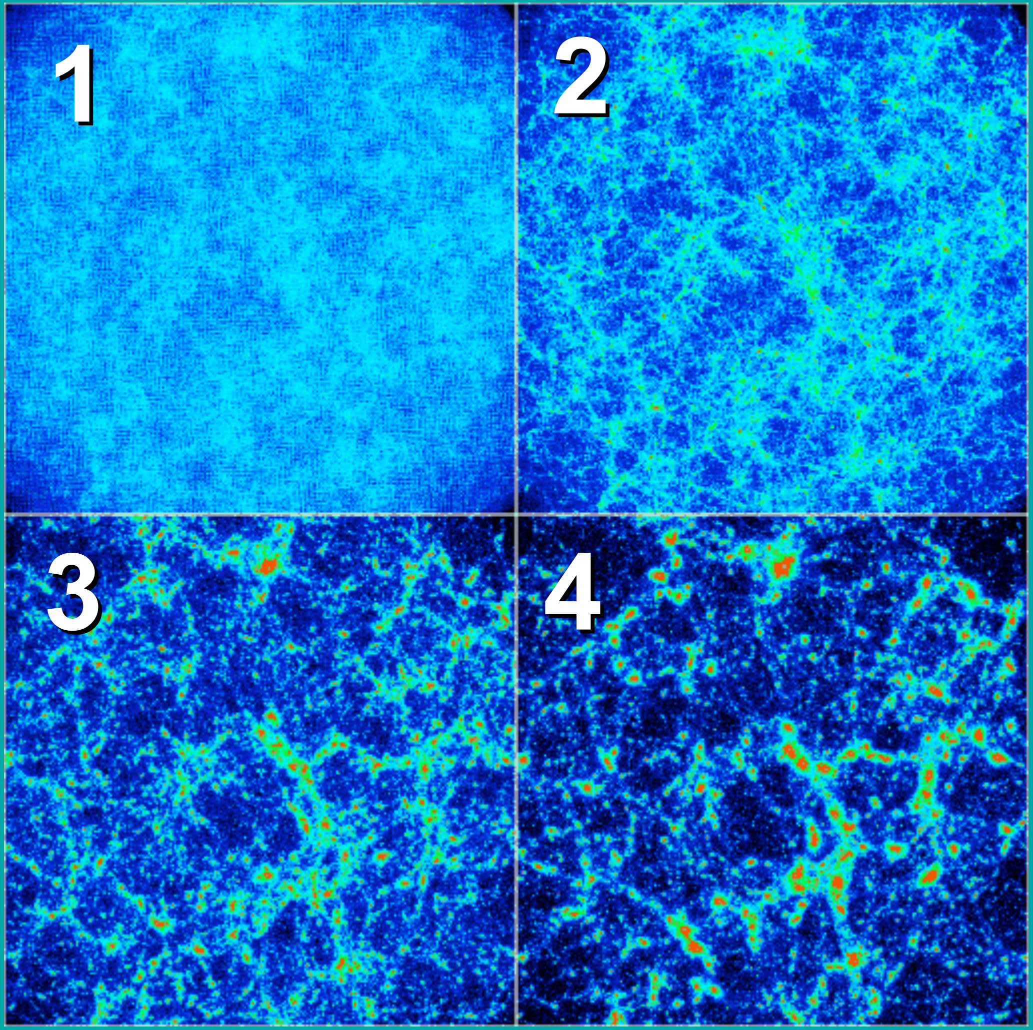


Dark energy: apparently uniform density, with no lumps.

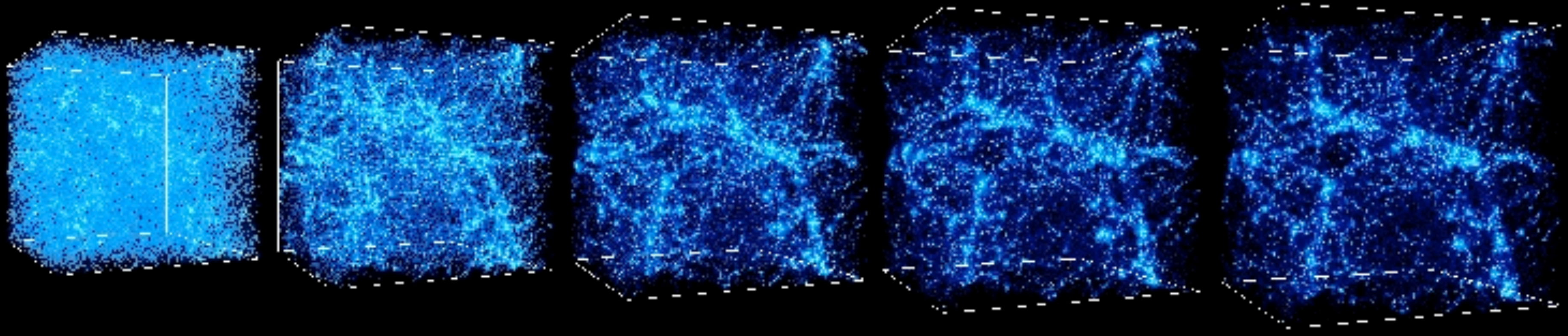
Dark matter: large lumps, about 1 megaparsec across.

Ordinary matter (protons, neutrons, electrons): small, but very dense, lumps.

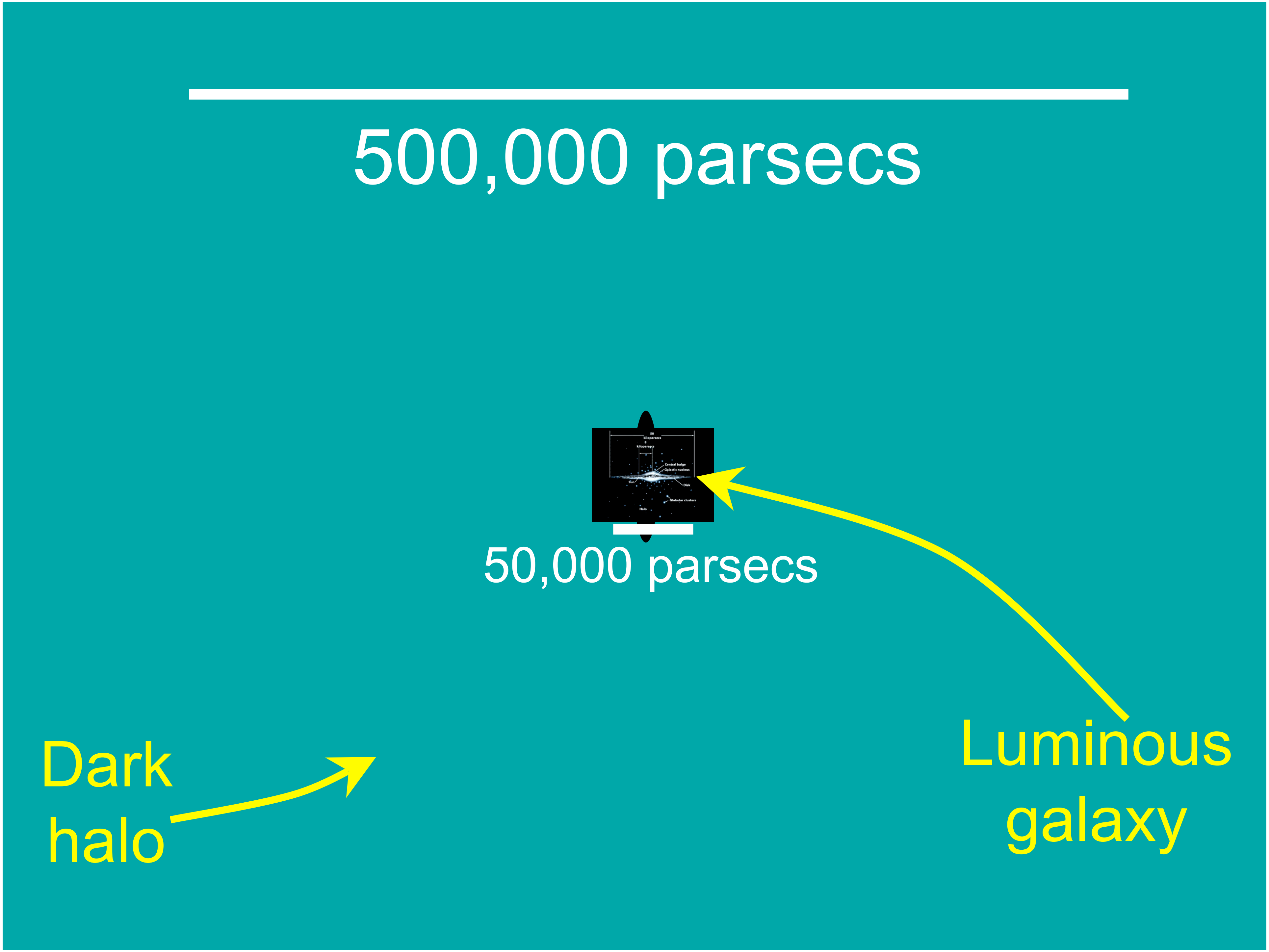
As we have seen, **gravity** tends to increase the lumpiness of matter.



The dense regions present when the universe became transparent have evolved tbecome clusters & superclusters today.



However, gravity alone can’t account for the **extreme** lumpiness of ordinary matter.



500,000 parsecs

50,000 parsecs

Dark halo

Luminous galaxy

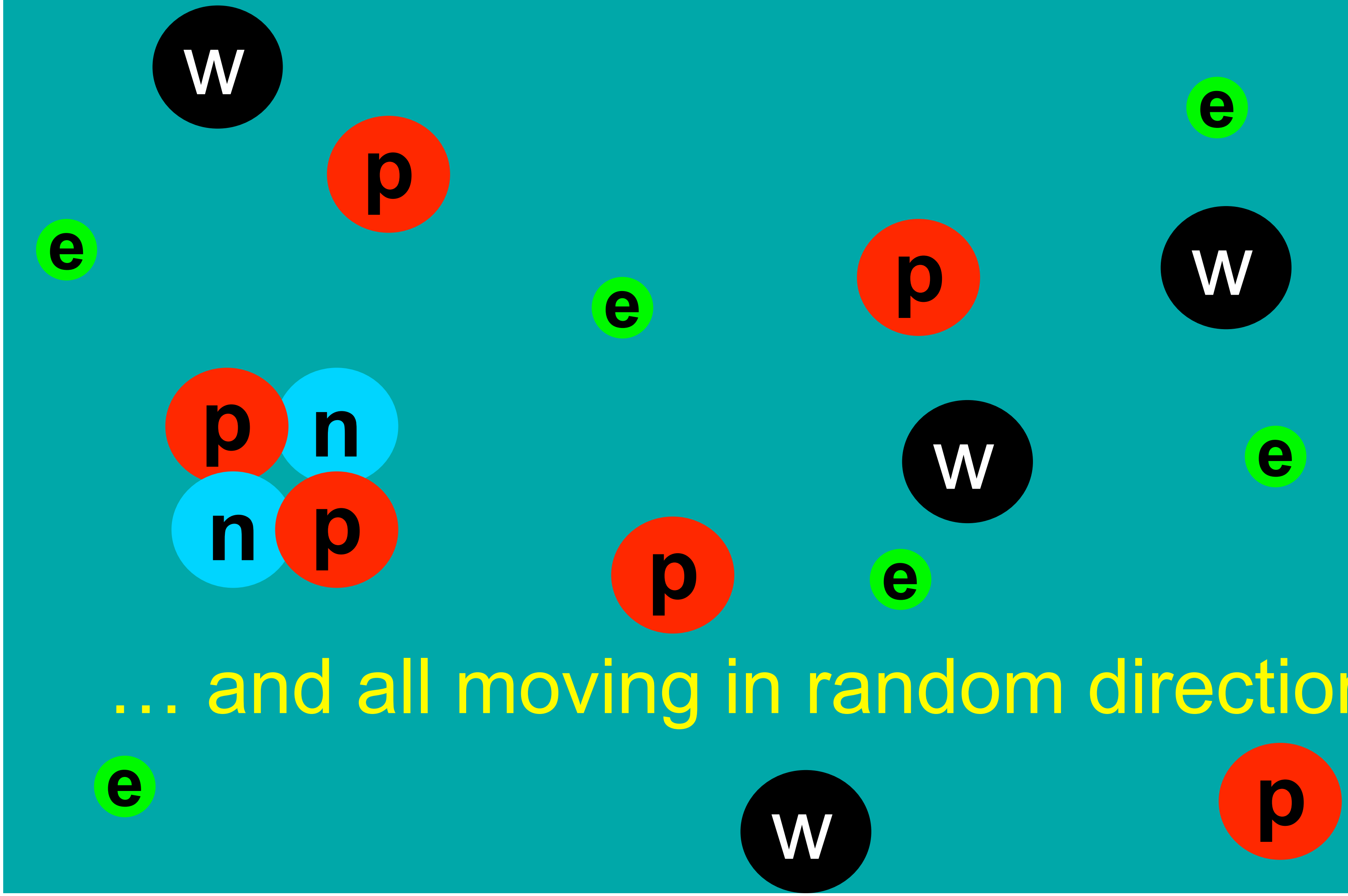
Luminous part of a galaxy
(electrons, protons, & neutrons)
is much smaller than the **dark** part
(Weakly Interacting Massive Particles).

What’s special about
electrons, protons, & neutrons
that concentrates them at the
center of dark halos?

“Tootsie pop” hypothesis: central luminous galaxy forms 1st, then is “dipped” in dark matter.

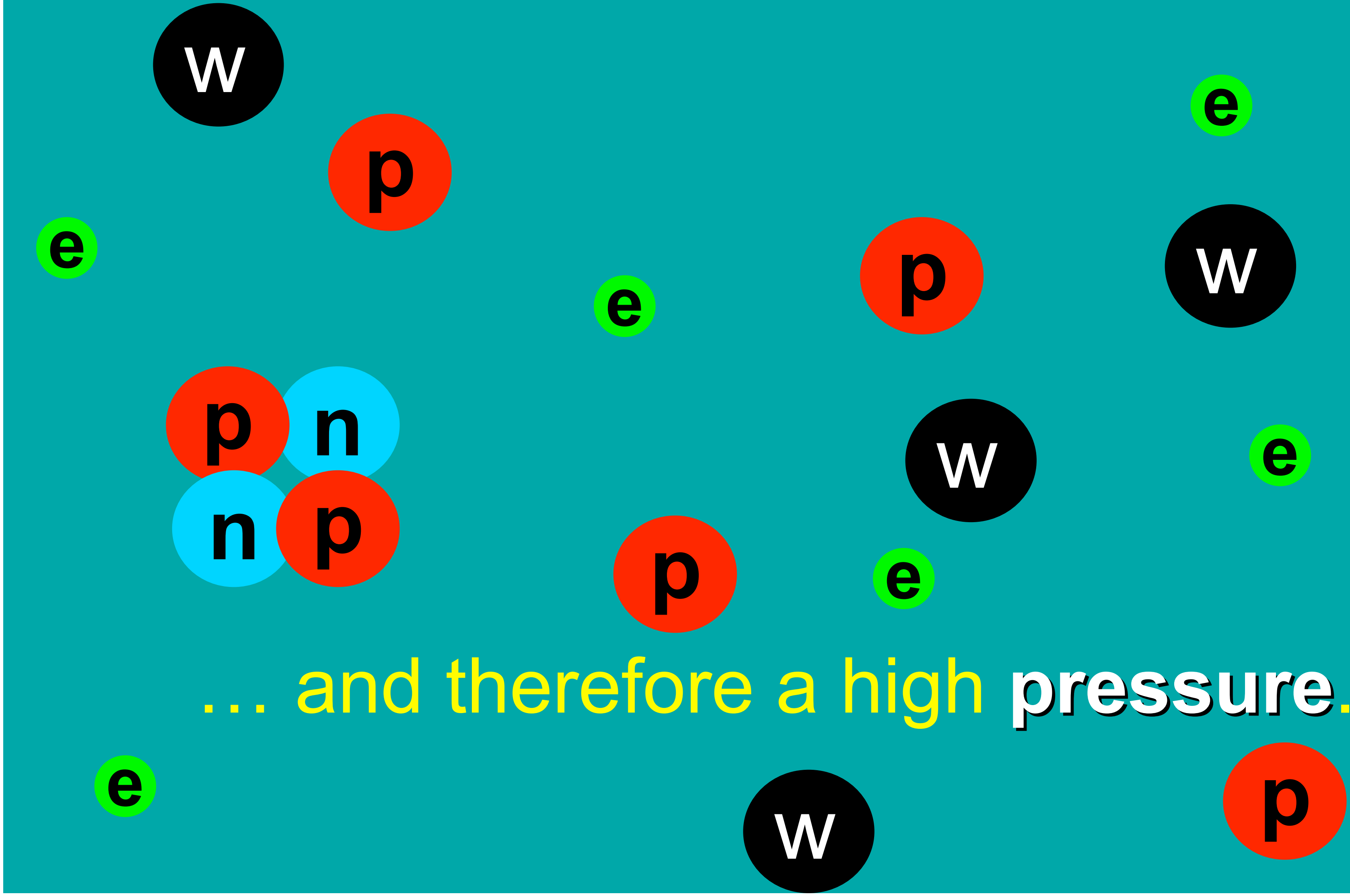
“Twinkie” hypothesis: outer dark halo forms 1st, then luminous galaxy is “injected”.

Consider a gas of electrons, protons, helium nuclei, and WIMPs all mixed together…



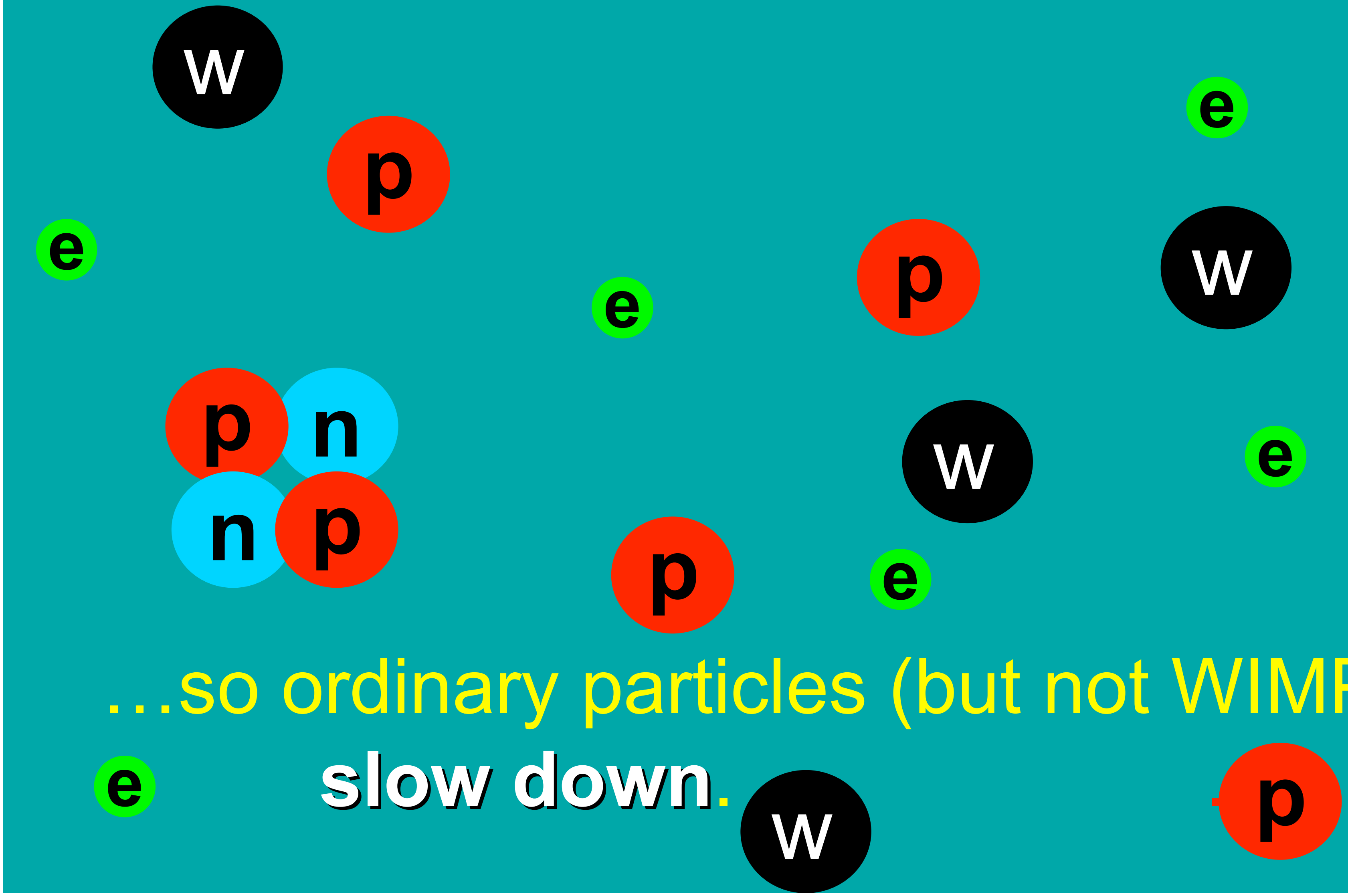
… and all moving in random directions.

Initially, the particles move rapidly.
They have a high **temperature**…



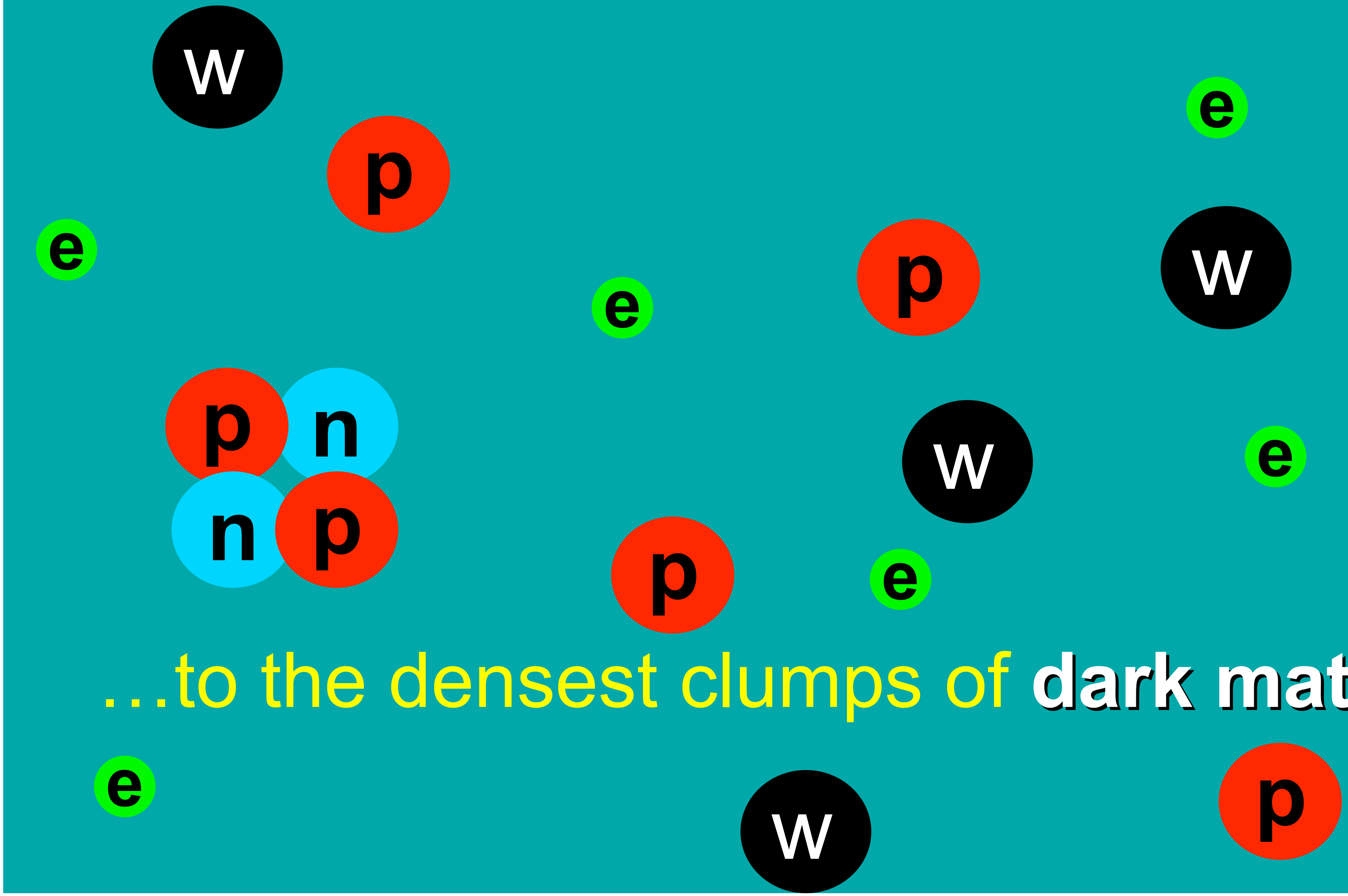
… and therefore a high **pressure**.

However, the ordinary particles emit photons, which carry away energy…

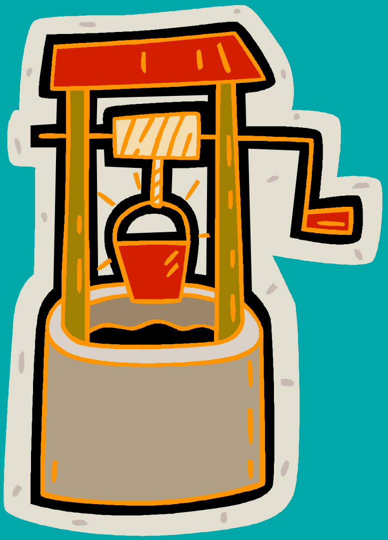


…so ordinary particles (but not WIMPs) **slow down**.

Ordinary particles, no longer supported by pressure, flow where gravity takes them…



…to the densest clumps of **dark matter**



Astronomy jargon:
“falling down the gravity well.”

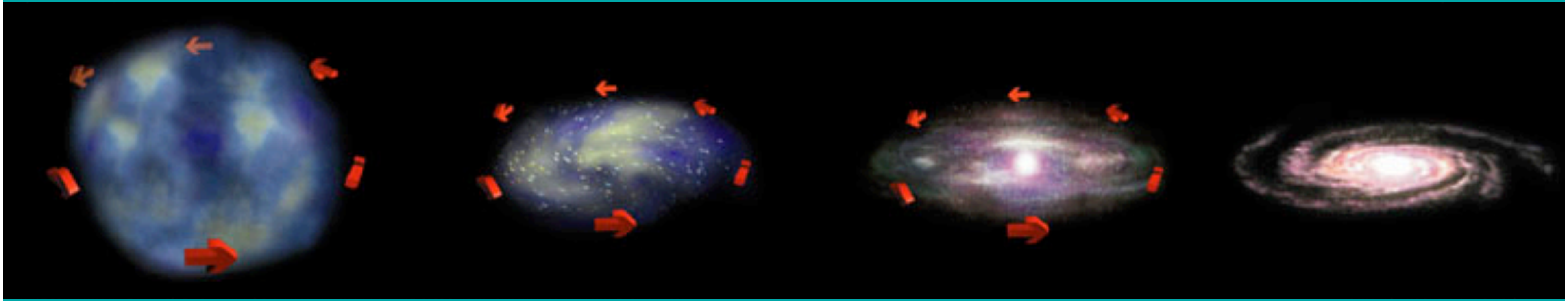
Since ordinary stuff,
made of electrons, protons, & neutrons,
can easily dump its excess energy,
it falls toward dense regions.

Modified Twinkie Hypothesis:

originally (dark matter) sponge cake & (ordinary matter) creme filling coexist.

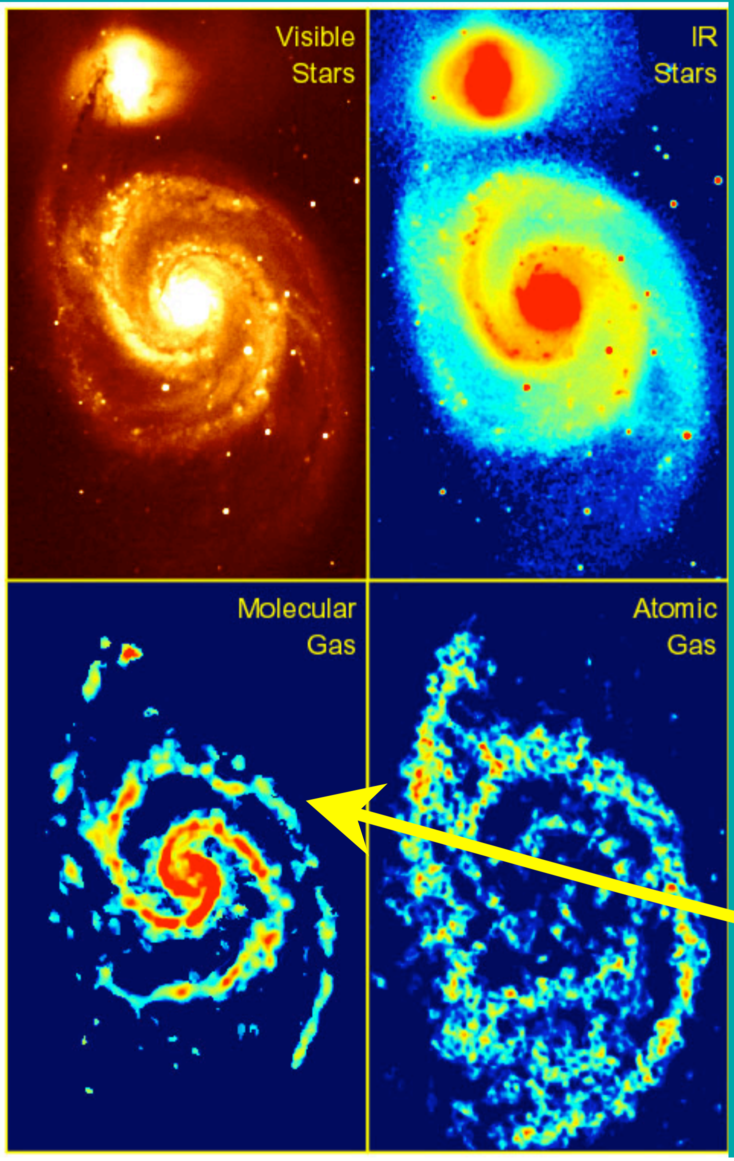
Gravity “injects” the ordinary matter to the center of the dark matter.

Galaxies form because ordinary matter cools down (by emitting photons) and falls to the center of dark halos.



Why do galaxies curdle into tiny stars, instead of remaining as homogenous gas clouds?

Look at where stars are forming **now**.

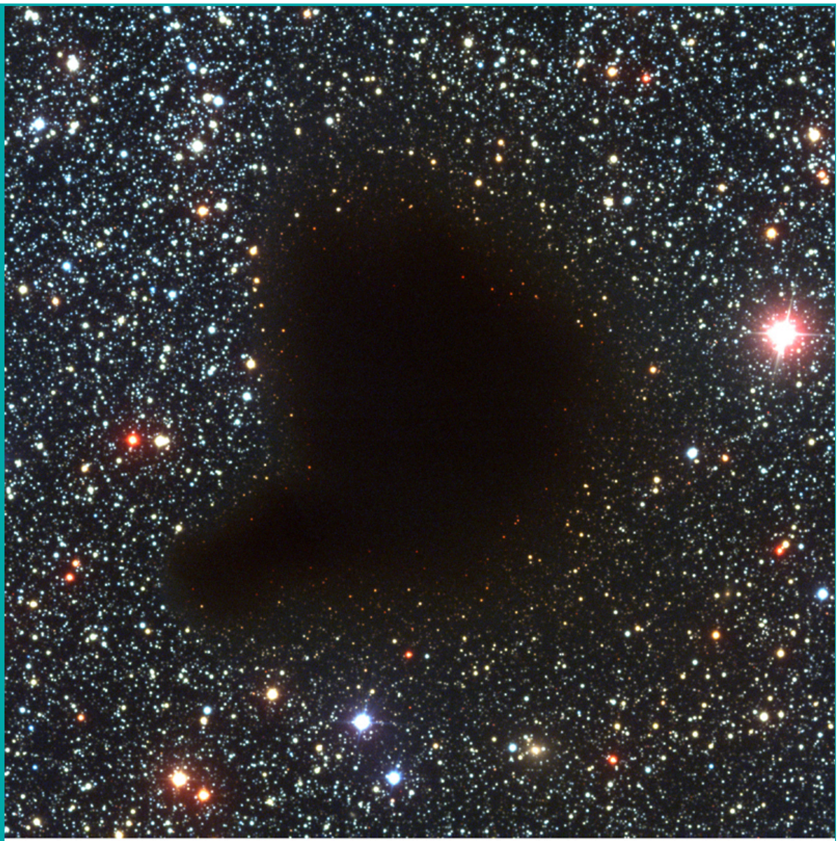


In the Whirlpool Galaxy, we see newly formed stars in dense, cold molecular clouds.

In regions where the gas is cooler and denser than elsewhere, hydrogen forms molecules (H_2).

These cool, dense regions are thus called “**molecular clouds**”.

Consider a small, dense molecular cloud.



ESO PR Photo 20a/99 (30 April 1999)

The "Black Cloud" B68
(VLT ANTU + FORS1)

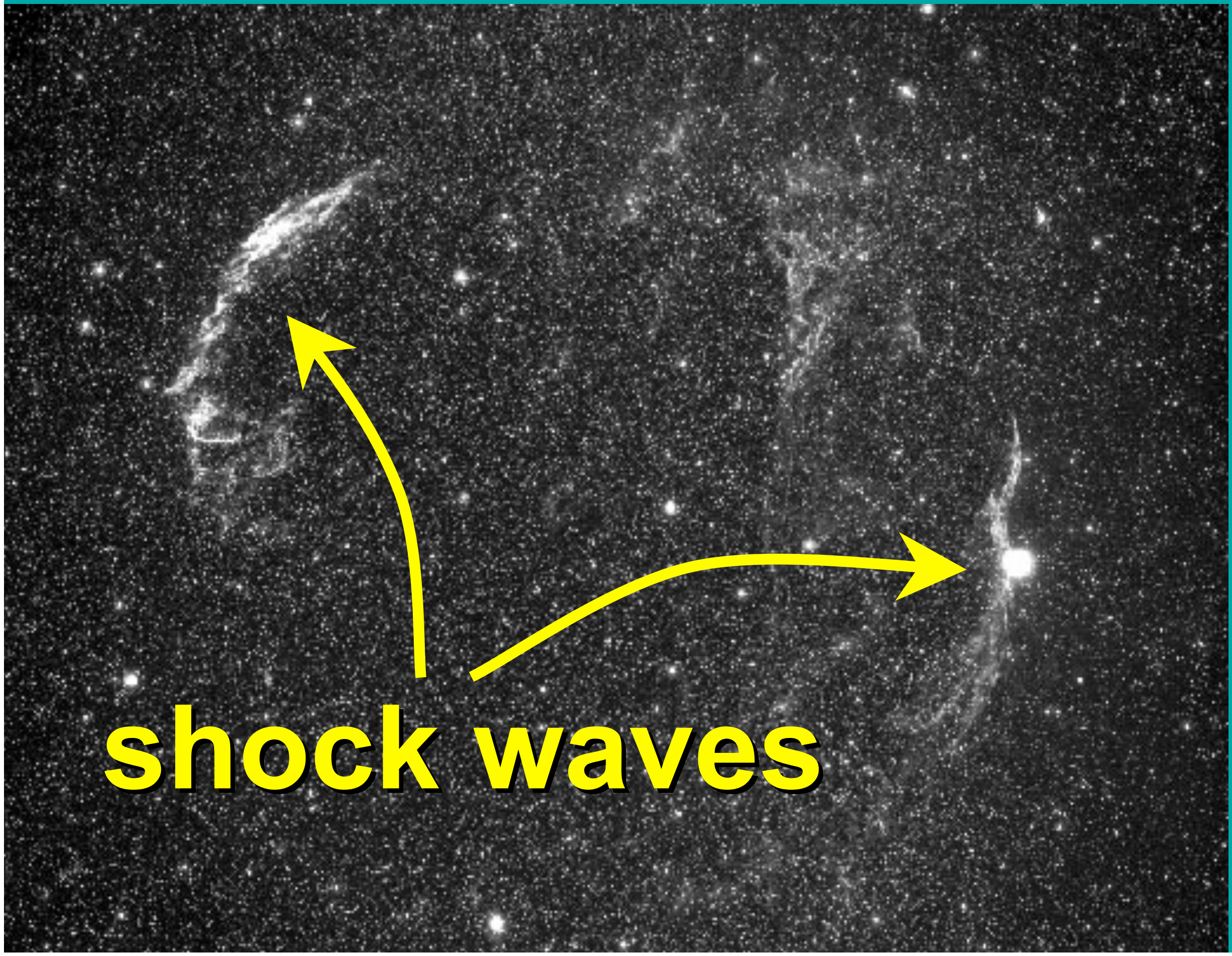
© European Southern Observatory

Mass = 1 M_{sun}

Radius = 0.1 pc = 4,000,000 R_{sun}

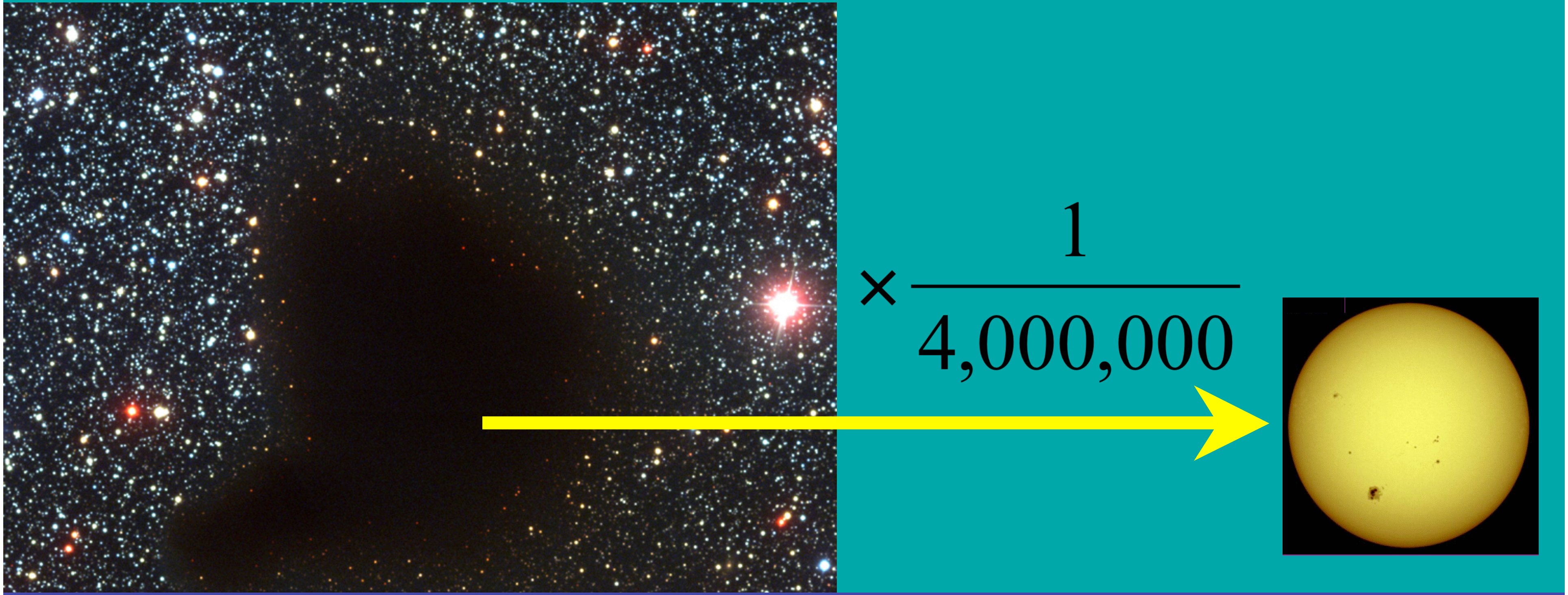
Temperature = 10 Kelvin = $T_{sun}/580$

Molecular clouds are usually stable; but if you hit them with a shock wave, they start to collapse gravitationally.



Once the collapse is triggered, it "snowballs".

Once gravity has reduced the radius of the cloud by a factor of 4,000,000, it’s the size of a star.



$$\times \frac{1}{4,000,000}$$

Why doesn’t the molecular cloud collapse all the way to a black hole?

Escape speed from
molecular cloud ≈ 0.3 km/sec

Escape speed from
star ≈ 600 km/sec

Escape speed from
black hole = 300,000 km/sec

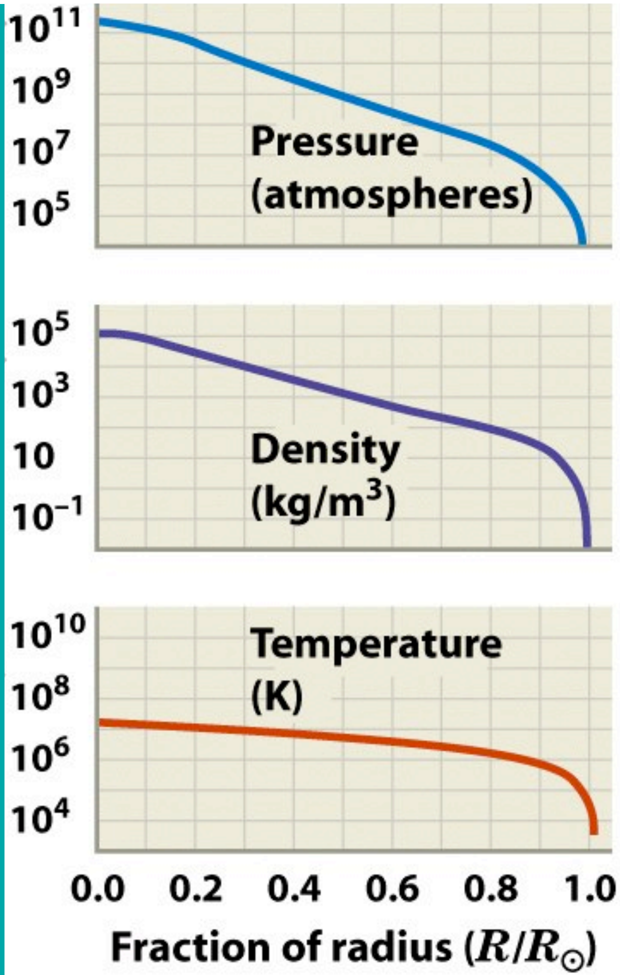
As the gas of the molecular cloud is compressed, it becomes denser.

As the gas is compressed,
it also becomes hotter.

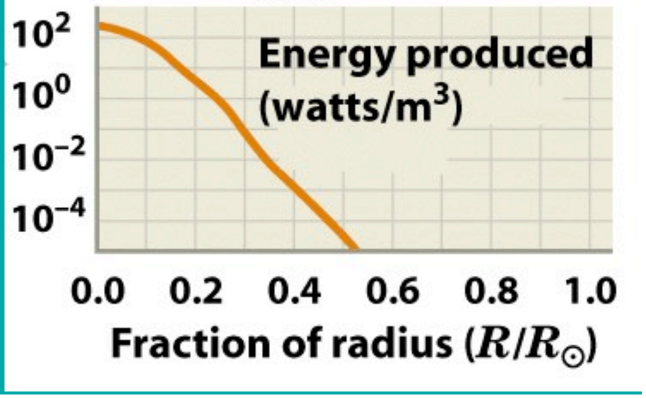
When the gas temperature is high enough (T ≈ 10 million Kelvin), nuclear fusion begins!

Nuclear fusion keeps the central **temperature** and **pressure** of the star at a constant level.

The star is static (not contracting or expanding) because it's in **hydrostatic equilibrium**.



(b) Energy generation

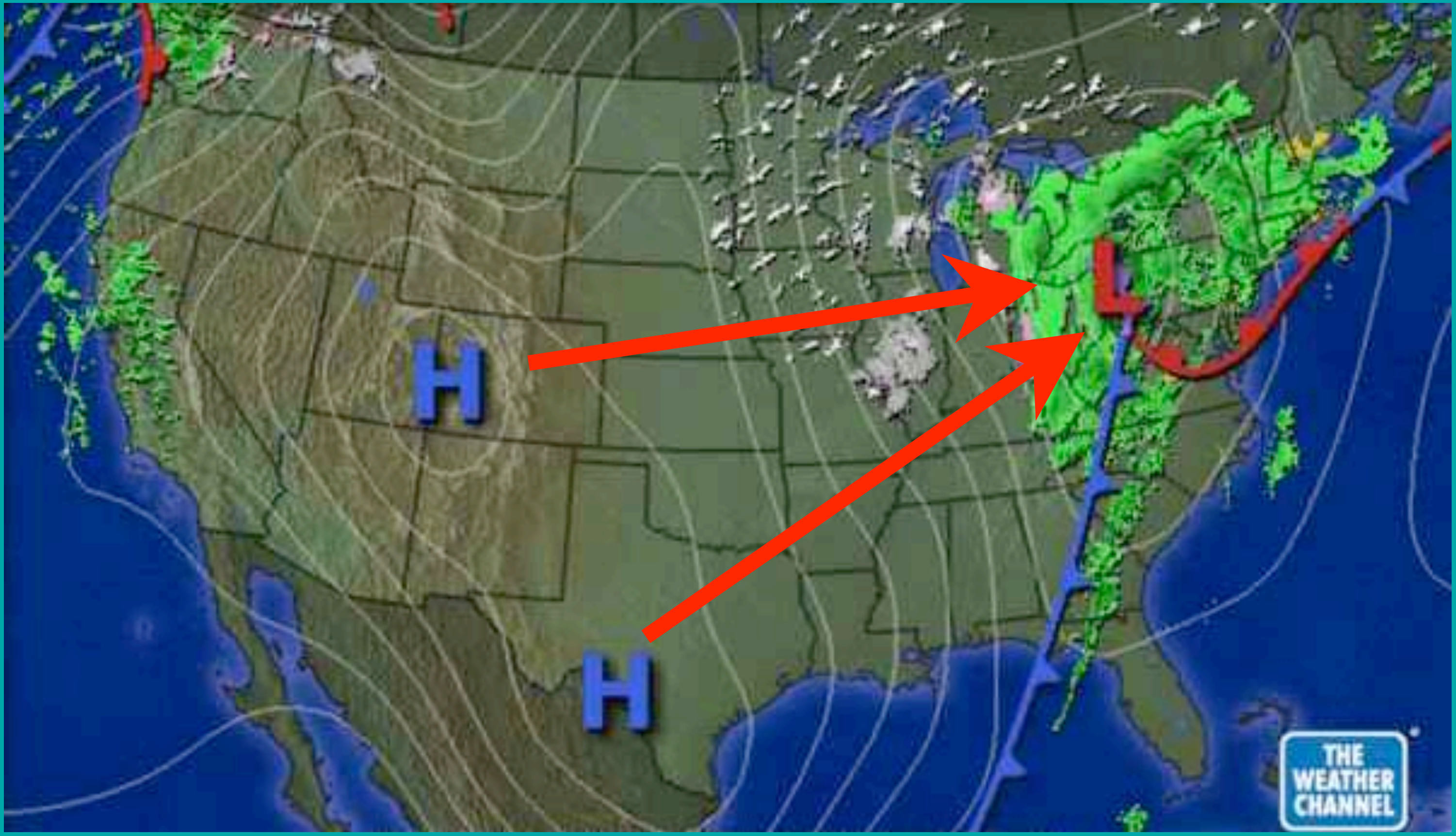


Hydrostatic equilibrium = a balance between gravity and pressure.

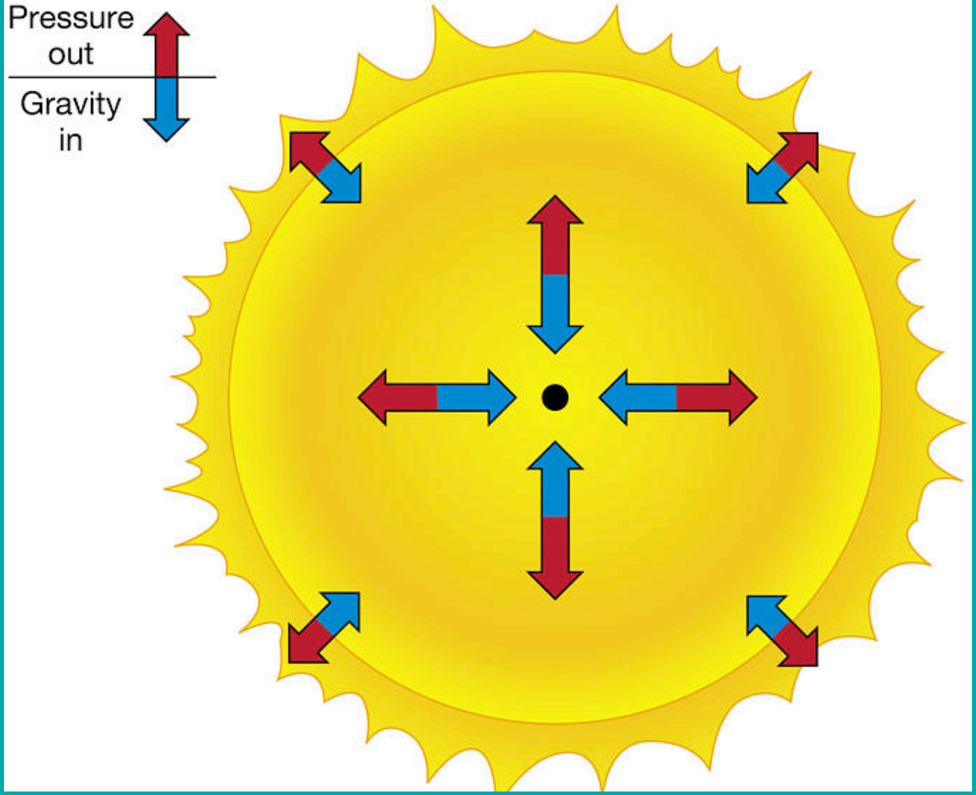
Pressure increases as you dive deeper into the ocean:

pressure increases as you dive deeper into the Sun.

Gas flows from regions of high pressure to regions of low pressure.

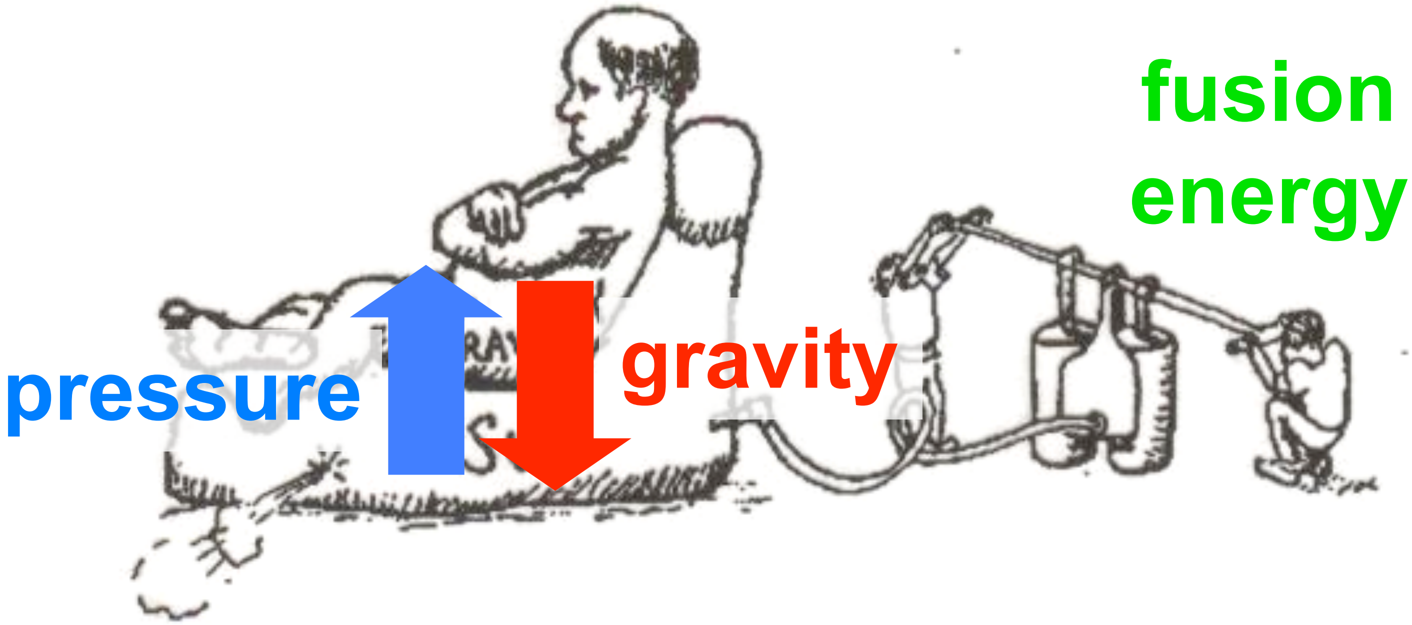


For gas in the Sun, pressure creates a net outward force, gravity creates a inward force.

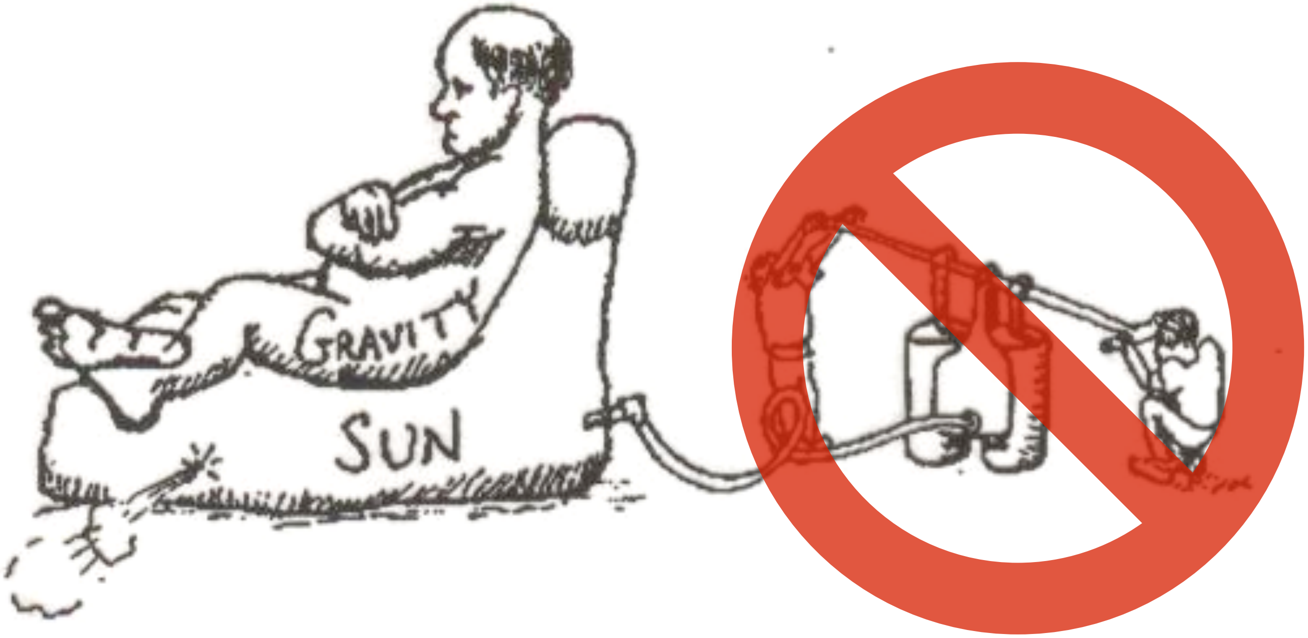


The Sun is in **hydrostatic equilibrium**.

The Sun is like a fat guy on an inflatable chair.



What happens when nuclear fusion ends inside a star?



Pressure drops: gravity compresses star to a denser object.

Small stars → white dwarf
(very dense)
Larger stars → neutron star
(very, very dense)
Largest stars → black hole
(ultimate in density)