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Study on the Effect of Object to Camera Distance on Polynomial Expansion Coefficients in Barrel Distortion Correction

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Abstract

Videoendoscopy is becoming increasingly popular in surgical procedures. Wide-angle lenses are commonly employed in such applications for enhanced viewing capability. However, images captured with these lenses suffer from barrel distortion. 2D distortion correction for images captured with wide-angle lenses has been widely investigated. In clinical applications, it is necessary to incorporate correction techniques that are independent of the distance from the object to the camera lens. In this paper, we prove that for a wide-angle camera lens with fixed focal length, the distortion correction coefficients remain the same for distances within the minimum and maximum range (depth of field). Experiments have also been performed to verify this.

Keywords: barrel distortion, distortion correction, expansion coefficients

1. Introduction

Minimally invasive therapy (MIT) is becoming increasingly popular in surgical procedures as it minimizes the destruction of healthy organs and tissues through the use of natural or artificial orifices of the body. Videoendoscopy has become one of the most commonly accepted forms of diagnostic and therapeutic procedures with the advent of miniature CCD camera and associated microelectronics [1][2]. The electronic endoscope has an excellent advantage over the conventional endoscope using fiber image guide as it facilitates observation, documentation, and electrical manipulation of the internal structure images of the gastrointestinal tract. In these endoscopes, cameras with wide viewing angle lens (fish-eye lens) are used to enhance the imaging capability by capturing a large field in a single image (i.e., large areas of the mucosa can be visualized rapidly [3]). However, images formed with these lenses suffer from spatial distortion, referred to in optics as "barrel" distortion due to the wide-angle nature of the endoscope's objective lens. Barrel distortion introduces nonlinear changes in the image, due to which, image areas near the distortion center are compressed less, while areas farther from the center are compressed more. Because of this, the outer areas of the images look significantly smaller that their actual size. This inhomogeneous image compression introduces significant errors in the results obtained during feature extraction. Continuous estimation of quantitative parameters, such as area and perimeter, is of considerable importance while performing clinical endoscopy. Unless the distortion is corrected, estimation errors could be very large [4]-[6]. Moreover, the distortion causes complications while using token matching techniques for pattern recognition. Distortion correction is also a prerequisite for the camera calibration to obtain extrinsic and intrinsic camera parameters [7][8].

Several researchers have presented various mathematical models of the image distortion and techniques to find the model parameters to complete the distortion-correction procedure. Simith et. al. [9] gave a formulation in which distortion was assumed to be purely radial, and orthogonal Chebyshev polynomials were used to determine the model parameters. Hideaki et. al. [10] presented a different method for estimating the model parameters, in which a moment matrix was obtained from a set of image points, and distorted grid lines in the image were straightened on the basis of the smallest characteristic root of the moment matrix. Vijayan et. al. [11] proposed a new technique based on least square estimation to obtain the coefficients of the correction polynomial. Furthermore, a method for accurate determination of the critical points on the calibration grid, based on a dual-step approach is also presented. The proposed technique is independent of the orientation of the calibration chart, hence precise placement of the chart and the placement errors in distortion-correction formulation are avoided. The proposed distortion correction technique is much faster than the existing techniques and gives sufficiently accurate results, which makes the concept of on-line calibration of the endoscopic camera feasible. This is desirable to enhance the accuracy of the quantitative results obtained from endoscopic images by machine analysis.

However, all the methods mentioned above have been presented for 2D distortion correction, whereby the distance from the object to camera lens is always constant. For practical videoendoscopy, it is imperative to explore the effect of varying this distance on the expansion coefficients of the 2D distortion correction algorithms. In this paper, we prove that for a fixed focal length wide-angle camera, the set of expansion coefficients obtained at a certain distance for 2D distortion correction has no relation with the distance from the object to camera lens. Simulations have also been performed to verify this.

2. 2-D distortion correction theoretical model

Vijayan et al. [11] proposed a new technique based on least square estimation to correct the non-linear distortion. The distortion correction algorithm assumes that the distortion is radial about the distortion center of the image captured by the wide-angle camera lens. Although nonlinear magnification of the distorted endoscopic image in two dimensions is needed to correct the barrel distortion, the assumption precludes the loss of generality, as a typical endoscope lens is circularly symmetric within narrow precision limit [9]. This assumption simplifies the model by converting a 2-D distortion problem into a one-dimensional (1-D) problem. Let the distorted and corrected (or undistorted) image spaces be represented by (U', V') and (U, V), respectively, and the distortion center and the corrected center by (u_c', v_c') and (u_c, v_c). The distortion center (u_c', v_c') is a point in the distorted image space such that the straight lines in the object space passing through it remains straight in the image space. The corrected center (u_c, v_c) is a point in the corrected image space about which the expansion of distorted image gives a final corrected image. In the distorted image space, magnitude ρ' of a vector $\boldsymbol{P'}$ from the distortion center to any pixel location (u', v') and the angle θ' made by this vector from the horizontal U'-axis are given by

$$\rho' = \sqrt{(u'-u_c')^2 + (v'-v_c')^2} \quad \theta' = \arctan\left(\frac{v'-v_c'}{u'-u_c'}\right) \tag{1}$$

Let the same pixel be assigned to a new location (u,v) in the corrected image space and the magnitude ρ and argument θ of the corresponding vector $\boldsymbol{P}$ drawn from the corrected center to the new pixel location are

$$\rho = \sqrt{(u-u_c)^2 + (v-v_c)^2} \quad \theta = \arctan\left(\frac{v-v_c}{u-u_c}\right) \tag{2}$$

The objective of the mathematical model is to obtain a relation between the vectors $\boldsymbol{P'}$ and $\boldsymbol{P}$. An expansion polynomial of degree N is defined to relate the magnitudes of the two vectors in distorted and corrected images as

$$\rho = \sum_{n=1}^{N} a_n \rho'^n \tag{3}$$

where a_n's are the expansion coefficients. As the distortion has been assumed to be purely radial, there will be no change in the arguments of the corresponding vectors $\boldsymbol{P'}$ and $\boldsymbol{P}$, i.e., $\theta' = \theta$. After obtaining the magnitude of the new vector, the new pixel location in the corrected image space can be calculated as

$$u = u_c + \rho\cos\theta', \quad v = v_c + \rho\sin\theta' \tag{4}$$

To map each pixel from the distorted image space onto the corrected image space, there are $N + 4$ unknowns, viz., N expansion coefficients (a_n's), distortion center (u_c', v_c'), and corrected center (u_c, v_c). The proposed distortion correction method involves two steps listed as follows:

1) Estimation of Distortion and Corrected Center

A reasonably correct estimation of the distortion center is essential for effective determination of the expansion coefficients. The distortion center is a fixed point for a particular camera and, once calculated, can be used for all the images obtained from that camera. The distortion center is computed based on the adjacent rows and columns of opposite curvatures. Please see Fig. 1.

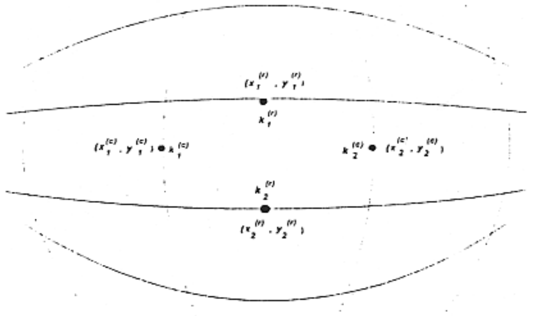

Fig. 1. Best-fit polynomial curves of adjacent rows and columns

Their curvatures are defined as $k_1^{(r)}$, $k_2^{(r)}$, $k_1^{(c)}$ and $k_2^{(c)}$, respectively. $(x_1^{(r)}, y_1^{(r)})$, $(x_2^{(r)}, y_2^{(r)})$, $(x_1^{(c)}, y_1^{(c)})$, $(x_2^{(c)}, y_2^{(c)})$ is the stationary point on the curve at which the above curvature is computed, respectively. The distortion center

$(u_c^{'}, v_c^{'})$ is estimated by interpolating the four curvatures $k_1^{(r)}$, $k_2^{(r)}$, $k_1^{(c)}$, and $k_2^{(c)}$ as

$$u_c^{'} = \frac{|k_1^{(c)}| x_2^{(c)} + |k_2^{(c)}| x_1^{(c)}}{|k_1^{(c)}| + |k_2^{(c)}|}$$

$$v_c^{'} = \frac{|k_1^{(r)}| y_2^{(r)} + |k_2^{(r)}| y_1^{(r)}}{|k_1^{(r)}| + |k_2^{(r)}|} \qquad (5)$$

The corrected image center is found based on the criterion that in the corrected image, pixel distances between the dot centers should be the same for all the grid lines in the horizontal and the vertical directions. The corrected image center is estimated by applying the expansion polynomial to this pixel location in the distorted image, which is obtained by iteratively minimizing the variation in distances between the test dot centers in the corrected image.

2) *Estimation of Expansion Coefficients*
The expansion coefficients are estimated on the basis of the degree of straightness of the points, which lie on a straight line before imaging. These are estimated in the distorted image space by straightening the grid lines of a distorted grid image. The least squares estimation and a line search approach of global convergence for the iterative procedure are used to obtain the optimum expansion coefficients a_n's as

$$a_n(\Delta+1) = a_n(\Delta) + \alpha_n^{\beta} E(\Delta) \frac{1}{\left(\frac{\partial E(\Delta)}{\partial a_n}\right)}, \quad for\ n = 1, \ldots N \qquad (6)$$

where α is the convergence rate parameter, β is the expansion index, $E(\Delta)$ is the total error for the whole grid image and $\partial E/\partial a_n$ is the error gradient. Here, α is chosen to ensure that for every $(\Delta+1)$th iteration, $E(\Delta+1)<E(\Delta)$.

3. Effect of object to camera distance on polynomial expansion coefficients

In this section, we will prove that for a wide angle camera with fixed focal length (i.e. keeping the aperture constant), and image plane size, the expansion coefficients a_n's obtained at a certain distance is applicable for correcting distorted images captured at varying distance.

Fig. 2 describes a simplified relation between the image plane, lens and object plane. Assuming they are in parallel with each other, O represents the lens and $d1$, $d2$ are two different distances from the lens to the object plane, whereby distances $d1$, $d2$ are not nearer or beyond the depth of field. OO' and the size of image plane is constant for a particular wide-angle camera with a fixed focal length. Assuming a set of polynomial expansion coefficients a_n's have been obtained for distance $d1$. We will now try to prove that the same set of expansion coefficients a_n's can be applied to any distance range.

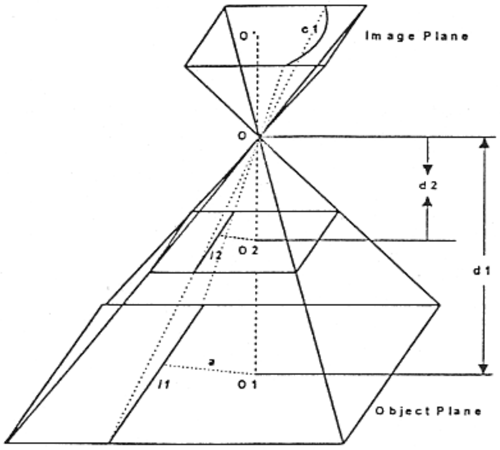

Fig. 2. Basic relations of imaging optics

Proof: Without loss of generality, the 2-D object plane can be regarded as a composition of infinite continuous parallel lines. Assuming that at distance $d1$, the 2-D object plane can be fully captured onto the image plane. This means that each parallel line in the object plane corresponds to a curve in the image plane. Let's assume that the size of the 2-D object plane is $S1$ and $l1$ is an arbitrary parallel line in the plane at distance $d1$. Also, $c1$ is the corresponding curve in the image plane of the line $l1$ and a, the distance from $O1$ to the line $l1$. When the object plane goes nearer to the lens at distance $d2$, only one part of the 2-D object plane $(\frac{d2}{d1})^2 \cdot S1$ can be captured onto the image plane. For line $l1$, there must exist a corresponding line $l2$ in this nearer object plane with a distance $\frac{d2}{d1} \cdot a$ from $O2$, which is reflected onto the same curve $c1$ as the line $l1$. From the known condition that a_n's can be used to straighten the curve $c1$, the same applies to correcting the distorted image captured at distance $d2$. The similar proof can be done for further distance ranging from $d1$ to the maximum distance. Hence, we can conclude that the distance of object to camera has no effect on polynomial expansion coefficients in barrel distortion correction for a wide-angle camera with a fixed focal length.

This can also be understood easily by the following explanations: For a fixed focal length wide-angle camera, the size of the image plane is always kept the same. No matter how the distance ranges from the object to the camera, the curvature and the location of each pixel in the distorted image array are kept the same throughout in the image plane. The only difference is within the range of the depth of field, when the object is nearer, less information will be captured in the image plane, but the image is bigger. If the object is further, more information will be captured in the image plane, but the image is smaller.

4. Experimental results

To verify the above theory, an experimental grid containing a rectangular array of dots was used. A camera sensor with a resolution of 480x640 pixels and a wide-angle lens of 158 degree was used to capture the images. The camera was oriented perpendicular to the grid surface at a distance of 8.6 cm. Fig. 3(a), clearly shows the barrel distortion effects of the wide-angle camera. We have known that the order of expansion coefficients N taken as 4 can obtain good distortion correction effect [11]. They are computed as follows:

$$a_1 = 1.00985160541818$$
$$a_2 = 0.00001011098960$$
$$a_3 = 0.00000002092793$$
$$a_4 = 0.00000001718244$$

The distorted image center is [259, 334]. The corrected image size is 1062×1416 and corrected image center is [595, 768]. These parameters can be directly used for the distortion correction of the images. The corrected image is shown in Fig. 3(b). Since magnification is controlled in order not to lose the information around the center of distortion, the marginal area is enlarged and the whole image becomes larger than the original one. From the comparison with the original image, the effect of the correction is obvious.

In the clinical case, imaging distance is variable. Therefore, it is necessary to investigate if the parameters obtained above are applicable to images recorded at different imaging distances. Fig. 4 and Fig. 5 show the images recorded at the distance 4 cm and 12 cm from the wide-angle camera and the corrected effects respectively. We can see that the same set of expansion coefficients lends well for these different distances.

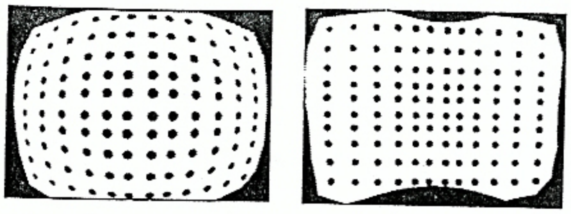

Fig.3. (a) Distorted image at distance of 8.6cm
(b) Corrected image corresponding to (a)

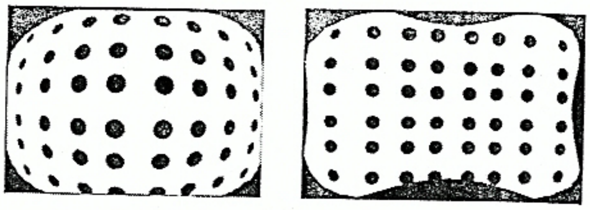

Fig. 4. (a) Distorted image at distance of 4cm
(b) Corrected image corresponding to (a)

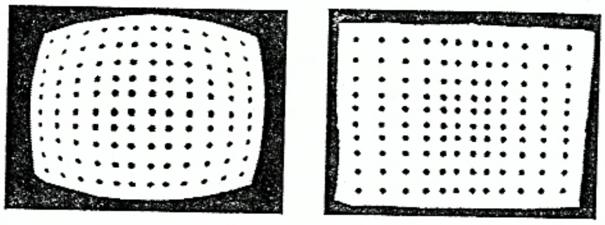

Fig. 5. (a) Distorted image at distance of 12cm
(b) Corrected image corresponding to (a)

5. Conclusion

In this paper, we study the relationship between the polynomial expansion coefficients and the distance from the object to the camera lens in barrel distortion correction. We found that for wide–angle camera lens with fixed focal lengths, whereby the size of the sensor plane is always constant, the curvature of each pixel in the image array will remain the same irregardless of the distance of the objective image to the camera lens. Hence, the expansion coefficients obtained at the certain distance can be used to correct the images captured at any distance within the depth of field range. This provides the impetus for employing existing distortion correction methods to clinical applications.

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