INTERFERENCE AVOIDANCE FOR WIRELESS SYSTEMS

BY DIMITRIE C. POPESCU

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ABSTRACT OF THE DISSERTATION

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Dissertation Director: Professor Christopher Rose

The study of interference avoidance for wireless communications is motivated by recent developments in telecommunications industry which demands new solutions for personal communications. Radios are becoming more sophisticated, and one can presently think of adjusting transmission/reception methods to suit the environment in which the communication system is operating. As a main application of interference avoidance techniques we see communication in unlicensed bands, where independent systems that interfere with each other will have to coexist without central control or coordination.

Our purpose is to provide insight into a class of distributed iterative algorithms that produce optimal codeword ensembles which maximize sum capacity in a multiuser system and minimize interference. Also known as *interference avoidance algorithms* they have been introduced in the context of chip-based CDMA systems but have been subsequently framed in a general signal space formulation so as to make them applicable to a wide variety of communication scenarios.

After an introduction to basic interference avoidance (IA) concepts and a review of previous work on IA, we describe how IA can be applied to codeword optimization in the uplink of a CDMA system in which the channel between each user and the basestation is assumed to be known. Each user sends frames containing multiple symbols (at least as many as there are signal space dimensions) and each symbol is assigned a signature waveform. Under the assumption that the frame duration is long enough, a common set of signal space basis functions consisting of real sinusoids is shared by all users at the base and an equivalent multiaccess vector channel is obtained. Application of interference avoidance methods is shown to produce optimal signature waveform ensembles which maximize sum capacity. Furthermore, loose assumptions on the channels seen by each user cause natural segregation of users and potentially large reductions in receiver complexity.

Fading channels, which are characteristic to wireless communications, are considered next, and answers to two main questions are sought: 1) When can a channel be considered quasistatic from the perspective of interference avoidance algorithms? 2) For rapidly varying channels which do not allow interference avoidance to be applied for each channel realization, does application of interference avoidance to the average channel result in capacity gains?

Application of interference avoidance to general multiaccess vector channels is also presented. It is shown that application of greedy interference avoidance increases sum capacity for multiaccess vector channels. Application of the eigen-algorithm is generalized for multiaccess vector channels and it is shown that sequential application of the eigen-algorithm for interference avoidance by all users in a multiuser system is equivalent to iterative water filling and always yields codeword ensembles that maximize the sum capacity of the multiaccess vector channel.

Using this general result, application of interference avoidance to multiuser systems with multiple inputs and multiple outputs (MIMO), and asynchronous CDMA systems is also presented. Multiuser MIMO systems are associated with the uplink of a wireless system in which users and the basestation have multiple antennas, while for asynchronous systems symbol intervals corresponding to different users are not necessarily synchronized at the receiver. In both cases a multiaccess vector channel model is derived for which application of interference avoidance becomes straightforward.

Empirical studies on various issues of the eigen-algorithm for interference avoidance are also performed. Since power is a main factor in determining interference, theoretical considerations leading to an algorithm that combines interference avoidance with a power control mechanism are presented. Transients of the eigen-algorithm are analyzed to show that no user can get poorer performance (in terms of signal-to-interference plus noise ratio) than in the begining of the algorithm. Codeword representation with finite precision and complexity issues are also explored.

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Dedication

To the memory of those Romanian people who gave their lives in December 1989, without whose sacrifice this work would not have been possible.

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Chapter 1

Introduction

The emergence of software radios is changing the way modern communication systems are designed by providing transceivers which can vary their output waveforms as well as their demodulation methods [1, 42, 68, 69]. The versatile transmitters and receivers allow a new class of wireless communication systems [63], where pairs of receivers and transmitters adaptively change modulation/demodulation methods in response to interference conditions to achieve better performance. For such a communication system, modulation and processing methods are adjusted to changing patterns of interference through *interference avoidance methods* [62, 63, 75]. The idea behind interference avoidance is that via feedback from the receiver, the transmitting radio is instructed to vary its waveform (or signature) in response to interference conditions. In an ensemble of users, this scenario can lead to optimized use of the shared medium. Even more, this optimum can be reached in a distributed fashion, through individual greedy optimization by each user.

We see the main application of interference avoidance to communication in the unlicensed bands which was enabled[1] by the release of 300 MHz of spectrum in the 5 GHz range for the Unlicensed National Information Infrastructure (U-NII). The FCC decision [12] is motivated by the belief that the creation of the U-NII band will stimulate the development of unlicensed communication systems which will provide more efficient and less expensive solutions for local access applications. However, in unlicensed bands one might expect to find an abundance of mutually interfering independent systems and no central control for efficient coordination. Thus, application of interference avoidance might be a useful control mechanism in such environments.

However, before applying the interference avoidance precept to arbitrary systems, interference avoidance must be understood in relation to the peculiarities of wireless channels. Therefore this

[1]In January 1997 by the Federal Commision for Communications [12].

thesis provides insight into interference avoidance algorithms by investigating their applications to non-ideal channels (e.g. dispersive, fading) and to general multiaccess vector channels. Application to multiple-input multiple-output (MIMO) systems and systems with asynchronous users as well as practical issues related to interference avoidance algorithms are also investigated.

1.1 Interference Avoidance Algorithms

Interference avoidance was introduced [75, 76] in the context of chip-based CDMA systems, but was subsequently developed [63] in a general signal space formulation which makes it applicable to a wide variety of communication scenarios.

Consider the uplink of a synchronous CDMA communication system with M users having signature waveforms $\{S_i(t)\}_{i=1}^M$ of finite duration T and equal received power at a common receiver (basestation). We will assume unit received power for each user. The received signal is

$$R(t) = \sum_{i=1}^{M} b_i S_i(t) + n(t) \tag{1.1}$$

where b_i is the information symbol sent by user i with signature $S_i(t)$, and $n(t)$ is an additive white Gaussian noise (AWGN) process. We assume that all signals are representable in an arbitrary N-dimensional signal space. Hence, each user's signature waveform $S_i(t)$ is equivalent to an N-dimensional vector $\mathbf{s}_i$ and the white noise process $n(t)$ is equivalent to a white noise vector $\mathbf{n}$. The equivalent received signal vector $\mathbf{r}$ at the basestation is then

$$\mathbf{r} = \sum_{i=1}^{M} b_i \mathbf{s}_i + \mathbf{n} \tag{1.2}$$

By defining the $N \times M$ matrix having as columns the user codewords $\mathbf{s}_i$

$$\mathbf{S} = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1 & \mathbf{s}_2 & \dots & \mathbf{s}_M \\ | & | & & | \end{bmatrix} \tag{1.3}$$

the received signal can be rewritten in vector-matrix form as

$$\mathbf{r} = \mathbf{S}\mathbf{b} + \mathbf{n} \tag{1.4}$$

with $\mathbf{b} = [b_1 \dots b_M]^\top$ containing the symbols sent by users.

Assuming simple matched filters at the receiver for all users, the signal-to-interference plus noise-ratio (SINR) for user k is

$$\gamma_k = \frac{(\mathbf{s}_k^\top \mathbf{s}_k)^2}{\sum_{j=1,j\neq k}^{M} (\mathbf{s}_k^\top \mathbf{s}_j)^2 + E[(\mathbf{s}_k^\top \mathbf{n})^2]} \tag{1.5}$$

Let us also define the correlation matrix of the interference seen by user k

$$\mathbf{R}_k = \sum_{i=1,i\neq k}^{M} \mathbf{s}_i \mathbf{s}_i^\top = \mathbf{S}\mathbf{S}^\top - \mathbf{s}_k \mathbf{s}_k^\top \tag{1.6}$$

and denote by σ^2 the spectral height of the white noise $\mathbf{n}$. Then, equation (1.5) can be rewritten as

$$\gamma_k = \frac{1}{\mathbf{s}_k^\top (\mathbf{R}_k + \sigma^2 \mathbf{I}) \mathbf{s}_k} \tag{1.7}$$

The idea behind interference avoidance algorithms is to use the SINR as a metric, and maximize it through adaptation of user codewords. This is also equivalent to minimizing the inverse SINR, defined as

$$\beta_k = \frac{1}{\gamma_k} = \mathbf{s}_k^\top (\mathbf{R}_k + \sigma^2 \mathbf{I}) \mathbf{s}_k \tag{1.8}$$

Note that for unit power codewords, equation (1.8) represents the Rayleigh quotient for matrix $\mathbf{R}_k + \sigma^2 \mathbf{I}$ and recall from linear algebra [72, p. 348] that this is minimized by the eigenvector corresponding to the minimum eigenvalue of the given matrix. Thus, the SINR for user k can be maximized by replacing the codeword $\mathbf{s}_k$ with the minimum eigenvector of the autocorrelation matrix $\mathbf{R}_k$ of the interference seen by user k. Sequential application by all users of this greedy procedure defines the minimum eigenvector algorithm for interference avoidance, or the *eigen-algorithm* [63] formally stated below:

The Eigen-Algorithm

1. Start with a randomly chosen codeword ensemble specified by the user codewords $\{\mathbf{s}_i\}_{i=1}^{M}$, given as columns of the codeword matrix $\mathbf{S}$ in equation (1.3)

2. for each user $k = 1, \ldots, M$

   (a) Compute the N-dimensional autocorrelation matrix $\mathbf{R}_k$ of the interference coming from other users

(b) Determine the minimum eigenvalue $\lambda_N^{(k)}$ of $\mathbf{R}_k$ and its associated unit eigenvector $\mathbf{v}_N^{(k)}$.

(c) If user k's codeword $\mathbf{s}_k$ is not already a suitable eigenvector of $\mathbf{R}_k$, replace it by $\mathbf{v}_N^{(k)}$.

3. Repeat step 2 for each user until a fixed point is reached, where all user codewords are minimum eigenvectors of coresponding autocorrelation matrices.

It so happens that at each step the eigen-algorithm reduces the total squared correlation (TSC) [63] defined as

$$\text{TSC} = \text{Trace}\left[(\mathbf{S}\mathbf{S}^\top)^2\right] = \sum_{i=1}^{M}\sum_{j=1}^{M}(\mathbf{s}_i^\top \mathbf{s}_j)^2 \tag{1.9}$$

which is a measure of the total interference in the system and is lower bounded by [88]

$$\text{TSC} \geq \frac{M^2}{N} \tag{1.10}$$

We note that codeword ensembles for which the bound in equation (1.10) is met with equality are also known as Welch Bound Equality (WBE) sets, and they satisfy the equality

$$\mathbf{S}\mathbf{S}^\top = \frac{M}{N}\mathbf{I}_N, \qquad \text{if } M > N \tag{1.11}$$

Using the TSC one can define a general class of algorithms for interference avoidance by changing a given user's codeword with a new one which does not increase the TSC. More specifically, we note from equation (1.6) that

$$\mathbf{S}\mathbf{S}^\top = \mathbf{R}_k + \mathbf{s}_k\mathbf{s}_k^\top \tag{1.12}$$

and when user k replaces its codeword with a new codeword $\mathbf{x}$ the difference in TSC can be written as

$$\Delta = \text{Trace}\left[(\mathbf{R}_k + \mathbf{s}_k\mathbf{s}_k^\top)^2\right] - \text{Trace}\left[(\mathbf{R}_k + \mathbf{x}\mathbf{x}^\top)^2\right] \tag{1.13}$$

This can be further rewritten after cancelling similar terms and replacing the traces by the corresponding quadratic forms as

$$\Delta = 2(\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k - \mathbf{x}^\top \mathbf{R}_k \mathbf{x}) \tag{1.14}$$

The requirement that $\Delta \geq 0$ yields the condition

$$\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \geq \mathbf{x}^\top \mathbf{R}_k \mathbf{x} \tag{1.15}$$

which defines a general class of algorithms in which a given user does not increase the TSC when replacing its codeword $\mathbf{s}_k$ by $\mathbf{x}$. The only assumption made is that other users' codewords do not change during the replacement. Algorithms for codeword adaptation based on (1.15) replace each user's codeword with a new one satisfying this requirement. Replacement is done sequentially, until all users have updated their codewords, at which point the cycle may be repeated until the TSC stops changing within some tolerance. The eigen-algorithm and the MMSE algorithm [63, 75, 78] are just particular cases of this class of algorithms for which convergence to absolute minimum TSC in equation (1.10) has been proven [3, 62]. Thus, codeword ensembles obtained by application of the eigen-algorithm or the MMSE algorithm form WBE sets. Other algorithms that yield WBE sequences have also been established and analyzed in the literature [14, 40, 84, 85].

One of the advantages of using WBE sequences as codewords for users in a CDMA system is that matched filters are *optimal linear receivers* and minimize the mean squared error (MSE) for each user. To see this, let us consider the detection scheme for user k with the received signal vector in equation (1.2) given by

$$\hat{b}_k = \mathbf{c}_k^\top \mathbf{r} \tag{1.16}$$

where $\mathbf{c}_k$ is the N-dimensional, unit norm, receiver filter corresponding to user k. The MSE is given by [37]

$$\text{MSE}_k = E[(\mathbf{c}_k^\top \mathbf{r} - b_k)^2] = \mathbf{c}_k^\top \mathbf{R}_k \mathbf{c}_k + (\mathbf{c}_k^\top \mathbf{s}_k - 1)^2 \tag{1.17}$$

and we first note that when a matched filter $\mathbf{c}_k = \mathbf{s}_k$ is used then the second term in equation (1.17) is zero. Furthermore, in the case of WBE sequences each codeword $\mathbf{s}_k$ is a minimum eigenvector of its corresponding interference-plus-noise covariance matrix $\mathbf{R}_k$. Therefore, the first term in equation (1.17), which corresponds to the Rayleigh quotient of matrix $\mathbf{R}_k$, will achieve its minimum value thus implying that the MSE is minimized. Hence, the matched filter and MMSE receiver filter are identical in this case, which is consistent with the similar results established in [84, 85]. We also note that the outputs of these matched filters provide sufficient statistics for joint processing of the users, as is performed by a multiuser receiver and required to achieve maximum sum capacity.

Let us also note that, when the number of users M is less than or equal to the dimension of the signal space N, both the eigen-algorithm and the MMSE algorithm produce a set of orthonormal codewords $\{\mathbf{s}_k\}_{k=1}^M$, which in this case satisfies

$$\mathbf{S}^\top \mathbf{S} = \mathbf{I}_M, \qquad \text{if } M \leq N \tag{1.18}$$

For the MMSE algorithm this may take several iterations, while the eigen-algorithm only needs a single pass through all codewords since each user chooses as new codeword an eigenvector which is orthogonal to the previously chosen codewords. Since all users are orthogonal, this case is trivial in the sense that for any given user interference from other users is zero, and the SINR is equal to the signal-to-noise ratio (SNR). More interesting is the case of *overloaded systems* having at least as many users as signal space dimensions. Therefore, we will assume that $M \geq N$ in our presentation throughout the thesis.

1.2 Capacity and WBE Sequences

In a white noise background, when the received powers of all users are the same, $p_k = p$, $\forall k = 1, \ldots, M$ the sum capacity is given by [80]

$$C_s = \frac{1}{2} \log \left[\det \left(\mathbf{I}_N + \frac{p}{\sigma} \mathbf{S}\mathbf{S}^\top \right) \right] = \frac{1}{2} \log \left[\det \left(\mathbf{I}_M + \frac{p}{\sigma} \mathbf{S}^\top \mathbf{S} \right) \right] \tag{1.19}$$

and is maximized [64] when user codewords are chosen such that they are either orthogonal when there are fewer users than signal space dimensions (that is $M \leq N$), or they form WBE sequences when the system is overloaded (that is $M > N$).

Furthermore, in [85] the user capacity of a CDMA system is defined as the maximum number of admissible users. For a signal space of dimension N and a desired common target SIR γ, M users are said to be admissible if there exist powers $p_i > 0$ and codewords ensembles $\{\mathbf{s}_k\}_{k=1}^M$ such that each user has an SIR at least as large as γ. It is shown in [85] that user capacity is the same when either matched filters or MMSE receivers are used, and is maximized if codewords are either orthonormal when there are fewer users than signal space dimensions, or they form WBE sequence sets when the system is overloaded.

This implies that codeword ensembles yielded by interference avoidance algorithms (the MMSE algorithm or the eigen-algorithm), which satisfy either equation (1.18) or equation (1.11), maximize both sum capacity and user capacity.

1.3 The Eigen-Algorithm in Colored Noise

In the presence of colored background noise with covariance matrix $E[\mathbf{n}\mathbf{n}^\top] = \mathbf{W}$ the SINR expression in equation (1.5) becomes

$$\gamma_k = \frac{1}{\mathbf{s}_k^\top(\mathbf{S}\mathbf{S}^\top - \mathbf{s}_k\mathbf{s}_k^\top + \mathbf{W})\mathbf{s}_k} \tag{1.20}$$

and we define in this case the correlation matrix of the interference plus noise seen by user k

$$\mathbf{R}_k = \mathbf{S}\mathbf{S}^\top - \mathbf{s}_k\mathbf{s}_k^\top + \mathbf{W} \tag{1.21}$$

Application of the eigen-algorithm remains unchanged and the current codeword $\mathbf{s}_k$ of user k is still replaced by the minimum eigenvector of matrix $\mathbf{R}_k$ in order to maximize user k SINR. In this case the eigen-algorithm yields codeword ensembles that minimize the trace of the square of the received signal covariance matrix $\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top]$, that is

$$\text{Trace}\left[\mathbf{R}^2\right] = \text{Trace}\left[(\mathbf{S}\mathbf{S}^\top + \mathbf{W})^2\right] \tag{1.22}$$

is minimized by the algorithm. By analogy to the white noise case, one could think of the resulting codeword ensembles as WBE sequences "tuned" to the particular noise stucture specified by covariance matrix $\mathbf{W}$.

The eigen-algorithm can be put into a similar information theoretic context in this case as well, by observing the equivalence between minimization of Trace $\left[\mathbf{R}^2\right]$ and maximization of the sum capacity [61–63]. In a colored noise background sum capacity is given by [61, 62]

$$C_s = \frac{1}{2}\log(\det\mathbf{R}) - \frac{1}{2}\log(\det\mathbf{W}) = \frac{1}{2}\log\left[\det(\mathbf{S}\mathbf{S}^\top + \mathbf{W})\right] - \frac{1}{2}\log(\det\mathbf{W}) \tag{1.23}$$

and we note that both Trace $\left[\mathbf{R}^2\right]$ and sum capacity depend on the eigenvalues of the received signal covariance matrix $\mathbf{R}$. While Trace $\left[\mathbf{R}^2\right]$ is a convex function in these eigenvalues, sum capacity is a concave function, and it can be shown [61, 62] that both are optimized when identical bounds on these eigenvalues are met with equality. More precisely, let us define the eigenvalues of the received signal covariance matrix $\mathbf{R}$ as $\mu_i,\ i = 1, \ldots, N$ and those of the noise covariance matrix $\mathbf{W}$ as $\sigma_i,\ i = 1, \ldots, N$. In terms of these eigenvalues we have

$$C_s = \frac{1}{2}\sum_{i=1}^{N}\log\mu_i - \frac{1}{2}\sum_{i=1}^{N}\log\sigma_i \tag{1.24}$$

$$\text{Trace}\left[\mathbf{R}^2\right] = \text{Trace}\left[(\mathbf{SS}^\top + \mathbf{W})^2\right] = \sum_{i=1}^{N} \mu_i^2 \tag{1.25}$$

For fixed energy codewords the sum of eigenvalues μ_i is constant and equal to

$$\sum_{i=1}^{N} \mu_i = \text{Trace}\left[\mathbf{SS}^\top + \mathbf{W}\right] = \text{const} \tag{1.26}$$

By using constrained optimization methods as in [61, 62] or results from majorization theory [38] as in [84, 87] one can show that optimal points that maximize sum capacity in equation (1.24) and minimize Trace $\left[\mathbf{R}^2\right]$ in equation (1.25), subject to the constraint in equation (1.26) are identical. Therefore, minimization of Trace $\left[\mathbf{R}^2\right]$ is completely equivalent to maximization of sum capacity C_s.

Hence, in a colored noise background the eigen-algorithm performs *aggregate water filling* over that portion of the signal space with least interference-plus-noise energy and maximize sum capacity. That is, the sum of rates of all users at which reliable communication can take place is maximized. We note that signal space "water filling" and the implied maximization of sum capacity are *emergent* properties of interference avoidance algorithms. That is, individual users do not attempt maximization of sum capacity via an individual or ensemble water filling scheme, but rather, they greedily maximize the SINR of their own codeword. In fact, individual water filling schemes over the whole signal space are impossible in this framework since each user's transmit covariance matrix $\mathbf{X}_\ell = \mathbf{s}_\ell \mathbf{s}_\ell^\top$ is of rank one an cannot possibly span an N-dimensional signal space. We also note that, by water filling those dimensions of the signal space with minimum background noise energy and avoiding "noisy" dimensions with energy above a certain threshold, the eigen-algorithm leads to a uniform maximum SINR for all users [63].

1.4 Thesis Road Map

Chapter 2 describes how interference avoidance can be applied to codeword optimization in the uplink of a CDMA system. We assume that the channel between each user and the basestation is known and that the symbol transmission interval (frame) is long enough that a common set of signal space basis functions (sinusoids) is shared by all users at the base. Each user sends multiple symbols (at least as many as there are signal space dimensions) and each symbol is assigned a signature waveform. Application of interference avoidance methods is shown to produce signature ensembles which achieve sum capacity. Such optimal signature waveforms provide to each user a uniform SINR for all its symbols and the optimal linear detector for each symbol is a matched filter. Such uniform identical receiver structures are often good candidates for integration. Furthermore, when multiple users are present, loose assumptions on the channels seen by each user cause natural segregation and potentially large reductions in receiver complexity.

In Chapter 3 application of interference avoidance is considered for multiple access fading channels and answers to the following questions are sought: 1) When can a channel be considered quasistatic from the perspective of interference avoidance algorithms? 2) For rapidly varying channels which do not allow interference avoidance to be applied for each channel realization, does application of interference avoidance to the average channel result in significant capacity gains?

Application of interference avoidance methods to multiaccess vector channels is presented in Chapter 4. Using a general signal space approach it is shown that application of greedy interference avoidance monotonically increases sum capacity. Furthermore, it is shown that sequential application of the eigen-algorithm for interference avoidance by all users in a multiuser system is equivalent to an *iterative water filling* procedure and always yields codeword ensembles that maximize sum capacity of the multiaccess vector channel. Application of interference avoidance to general dispersive channel models is also presented there as a particular case.

Application of interference avoidance to multiuser systems with multiple inputs and multiple outputs (MIMO) is presented in Chapter 5. Such systems are associated with the uplink of a wireless system in which users and the basestation have multiple antennas. Our approach is based on application of interference avoidance to general multiaccess vector channels for the particular multiaccess vector channel corresponding to a multiuser MIMO system. The use of an arbitrary signal space makes the approach general and applicable to any MIMO system models regardless of the choice of signal space basis functions. Information is transmitted over the MIMO channel using a multicode CDMA approach in which the sequence of information symbols that make up the frame to be sent by a given user over the MIMO channel is "spread" over the available dimensions using a precoding matrix. Optimal precoding matrices that maximize sum capacity are then obtained by application of interference avoidance methods. Numerical simulations have also been performed and the signal-to-noise ratio distribution for receiver antennas and complementary cumulative distribution functions for sum capacity with optimal precoding matrices are also presented.

Application of interference avoidance to asynchronous multiuser systems is presented in Chapter 6. These are systems for which symbol intervals corresponding to different users are not necessarily synchronized at the receiver and they are modeled as frame-synchronous with symbols sent in parallel using distinct signature waveforms of extended duration. This approach relaxes the synchronization requirements at the common receiver. A vector multiple access channel model is derived for which application of interference avoidance becomes straightforward.

Chapter 7 presents some empirical studies on various issues of the eigen-algorithm for interference avoidance. We start by presenting theoretical considerations leading to an algorithm that combines interference avoidance with a power control mechanism. Then, the transient part of the eigen-algorithm is analyzed and empirical evidence is presented which shows that if the user with the worst SINR had an acceptable connection with the basestation at the begining of the eigen-algorithm, then no one's connection will be any worse than this as the algorithm proceeds.

Next, the problem of codeword representation is analyzed. As opposed to current CDMA systems where uniform-amplitude codeword chips are used, interference avoidance employs real-valued "chips"– real-valued coefficients for a set of orthonormal basis functions of the signal space used by the transmitter and receiver. Thus, compact representation of codewords is extremely important since for practical implementation one must also consider the feedback channel between the receiver which calculates the codeword adjustments and the transmitter which uses them. An investigation of how codeword quantization affects the performance of interference avoidance algorithms is presented. Results indicate that using $4-5$ bits per chip for codeword representation is sufficient to maintain performance close to optimal values.

Complexity issues are also explored in Chapter 7. The operational complextity of the eigen-algorithm is evaluated and the reduction in receiver complexity under the implicit channelization produced by interference avoidance for dispersive channels is explored. Specifically, the properties of interference avoidance will naturally limit each user to fewer than the total number of signal dimensions, so each receiver need not necessarily span the whole signal space.

For each chapter, a detailed introduction, literature review about the specific problem, and a statement of the problem are presented in the begining in an attempt to make chapters more independently readable.

Most of the results in this thesis have been presented previously. The work on interference avoidance and dispersive channels in Chapter 2 was presented in part at the 37^{th} Annual Allerton Conference on Communication, Control, and Computing [46] and the 35^{th} Annual Asilomar Conference on Signals, Systems, and Computers [50]. Chapter 3 was presented at the 39^{th} Annual Allerton Conference on Communication, Control, and Computing [49]. Chapter 5 was presented in part at the 35^{th} Conference on Information Sciences and Systems [48]. Application of interference avoidance to general multiaccess vector channels in Chapter 4 was presented in part at the 2002 International Symposium on Information Theory - ISIT'02 [45]. The work on asynchronous systems in Chapter 6 was presented at the 36^{th} Conference on Information Sciences and Systems [51]. Codeword quantization in Chapter 7 was presented at the 2000 International Conference on Acoustics, Speech, and Signal Processing - ICASSP 2000 [47].

Chapter 2

Interference Avoidance and Dispersive Channels

In this chapter we extend application of interference avoidance methods to dispersive multiple access channels. Specifically, we consider optimization of uplink codewords for a CDMA system in which the dispersive channels between users and basestation are known. We begin in section 2.1 with a brief literature review to place this work in context with prior art. The problem of signature waveform optimization for uplink CDMA systems is stated in section 2.2. Section 2.3 contains background material on channel eigenfunctions and their use in converting a waveform channel into an equivalent vector channel. Although the results in this section are not new, they are important enough to be placed in a distinct section. The single-user case is presented in section 2.4 where it is shown that by using a multicarrier CDMA (MC-CDMA) modulation scheme the usual dispersive channel problem is equivalent to a multiuser detection problem. This equivalence allows direct application of interference avoidance techniques to our codeword optimization problem.

In section 2.5 we proceed to the multiple access channel problem corresponding to the uplink of a CDMA system. In this case multiple users are received at the basestation over non-identical channels. It is first shown that application of greedy interference avoidance does not decrease sum capacity. Then interference avoidance algorithms are presented and shown to converge to sum capacity optimal ensembles. We then focus on the case where the received signal covariance matrix is diagonal and explore some properties of these solutions. We find, under a loose set of assumptions on channel gain matrices, that users cannot overlap in more than one frequency and that if two or more users have energy over all frequencies, their channels must be identical. These results are similar to those found in [17, 44].

2.1 Related Work

It is well known that wireless channels can be dispersive and dispersion leads to intersymbol interference (ISI) where energy from symbol signals spills over into the observation intervals of subsequent symbols at the receiver. As noted in [32] there have been primarily two ways to combat ISI – equalization and coding – with large associated literatures.

In [32] and [35] an approach was proposed based on partitioning the communication channel into a set of parallel and independent subchannels each associated with its own carrier. Such channel partitioning is known in general as multicarrier modulation [6] and has been present in the literature for over 30 years. A signaling scheme that relates to multicarrier modulation as the communication interval is extended is also proposed in [30], where optimum signal sets for dispersive channels are derived in a framework based on channel eigenvectors.

A framework for using multicarrier modulation in frequency dispersive multiple access channels based on discrete multi-tone (DMT) is presented in [16], where a multiuser bit-loading algorithm is also proposed. The algorithm is a multiuser water filling scheme [11] over the DMT tones and maximizes the sum capacity of the multiaccess channel. The fact that a DMT scheme with appropriately loaded carriers is optimal with respect to maximizing sum capacity subject to a given power constraint has been proved to be optimal [44].

Recently, in the context of multiuser detection, methods for transmitter and receiver adaptation [60] have also been used for non-ideal channels [57, 58]. In this case transmitter/receiver adaptation compensates for the distortion introduced by the channel and avoids multiaccess interference. Transmitter and receiver adaptation methods have also been used in the context of multicode CDMA and MC-CDMA systems [36, 89].

Our approach to multiuser communication over dispersive channels is based on a form multicarrier modulation and transmitter/receiver adaptation through codeword optimization. The eigenfunctions of the channel autocorrelation [29] are used as modulation building blocks to form a convenient orthonormal basis for representing input and output signals of a dispersive channel and for converting it into a discrete vector channel. Assuming that symbol intervals $\mathcal{T}$ are long compared to the duration of channel impulse responses, then channel eigenfunctions are approximately sinusoids [29] and the same orthonormal basis can be used for all users *at the receiver*.[1]

Information symbols for a given user are sent in parallel as frames, using distinct signature waveforms of extended duration $\mathcal{T}$ for each symbol in the frame. This approach is different from prior work where symbol signatures were allowed to have length longer than the symbol interval [90]. Also, unlike [32, 35] there is no explicit precoding. Rather, the approach, which assigns multiple codes to a given user, is a form of CDMA in which each symbol in the given user's frame corresponds to a different virtual user. In the context of multicarrier modulation this implies that a MC-CDMA [92] modulation scheme is employed by each user. However, different from [92], we do not assume orthogonal codewords for different symbols/users.

In this context, our goal is to derive optimal ensembles of signature waveforms (codewords) that maximize the sum capacity of the multiaccess dispersive channel in the uplink of a CDMA system. We will apply interference avoidance methods to our signature waveform/codeword optimization problem since it has been shown that interference avoidance always produces sum capacity maximizing codeword ensembles [3,61,62]. Furthermore, the optimal linear receivers for such codewords are simple matched filters [84, 85].

2.2 Problem Statement

We consider the uplink of a synchronous CDMA system with L users communicating with a common basestation. Each user sends frames containing multiple symbols using a multicode CDMA approach in which each symbol in the frame is assigned a specific signature waveform as described schematically in Figure 2.1. Thus, the signals sent by users are written as

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t) \qquad \ell = 1, \ldots, L \tag{2.1}$$

where $b_m^{(\ell)}$, $m = 1, \ldots, M_\ell$, denote the symbols sent by user ℓ and $s_m^{(\ell)}(t)$ is the signature waveform assigned to convey symbol m of user ℓ assumed of finite duration $\mathcal{T}$. By loose analogy, one could think of each waveform $s_m^{(\ell)}(t)$ as a musical "chord" and each user sending a set of scaled chords.

[1]The sinusoidal basis functions of each user are advanced and retarded to that they coincide properly at the receiver.

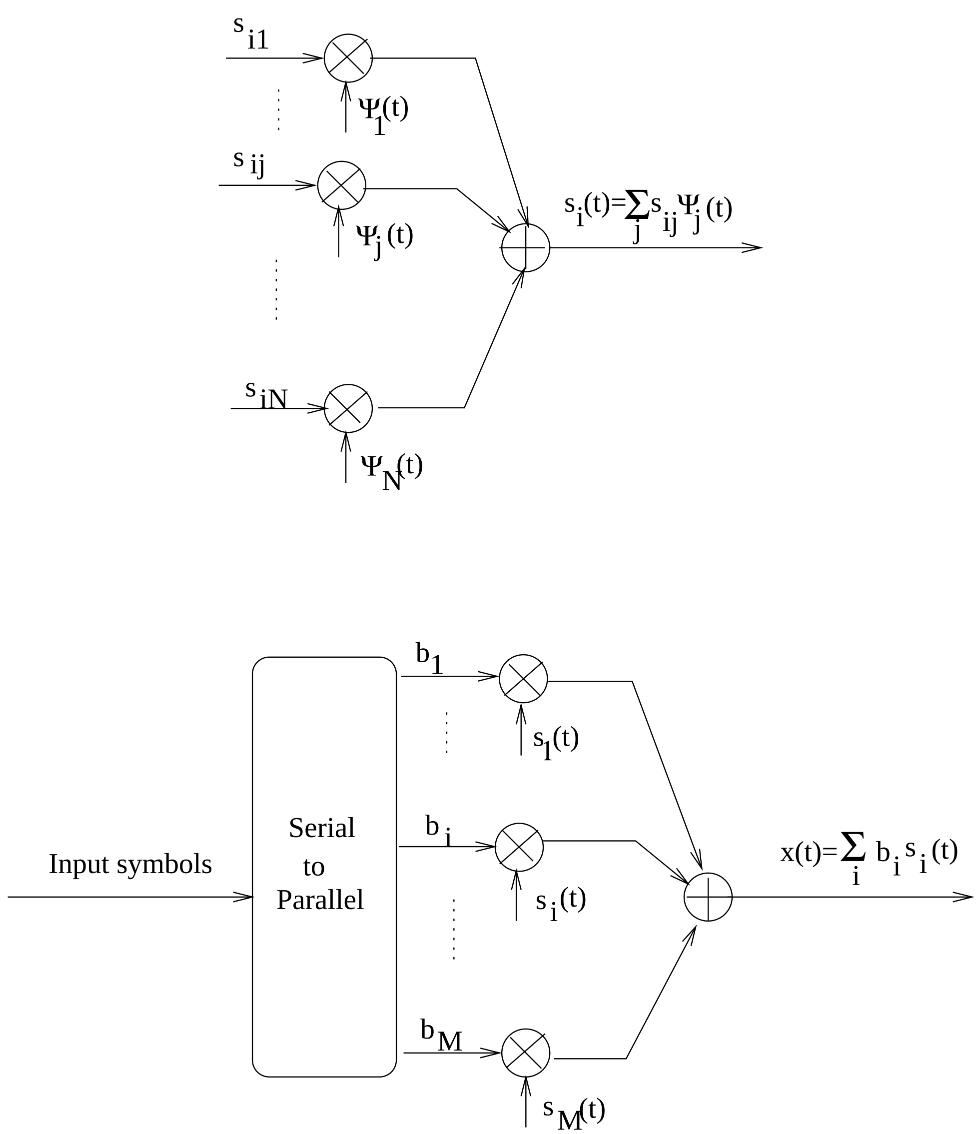

Figure 2.1: Multicode CDMA approach for sending frames of information. Each symbol b_i in the frame is assigned a signature waveform and the resulting signal $x(t)$ is a superposition of signature waveforms scaled by their corresponding information symbols. Signature waveforms $s_i(t)$ are expressed in terms of a set of basis functions for the signal space. Our problem will be to find optimal $\{s_{ij}\}$.

The channel between a given user ℓ and the basestation is assumed to be linear and time-invariant and is characterized by the causal impulse response $h_\ell(t)$ of duration T_ℓ. We assume that the frame duration is much larger than the duration of all channel impulse responses $\mathcal{T} \gg T_\ell$, $\forall \ell = 1, \ldots, L$. Thus, one can safely ignore ISI between successive frames of duration $\mathcal{T}$ by placing a relatively small "zero pad" between them to allow settling of channel responses. Where necessary we also assume cyclic extension of basis functions so that there are no gaps in the transmission interval.

The received signal at the basestation is a sum of signals transmitted by all users convolved with their corresponding impulse responses plus additive Gaussian noise $n(t)$

$$r(t) = \sum_{\ell=1}^{L} x_\ell(t) * h_\ell(t) + n(t) \tag{2.2}$$

Our goal is to derive optimal signature waveform ensembles $\{s_m^{(\ell)}(t)\}$, $\ell = 1, \ldots, L$, $m = 1, \ldots, M_\ell$ that maximize the sum capacity of the multiaccess channel. This is different from previous work which has concentrated mainly on optimal power allocation schemes. We will show that interference avoidance algorithms yield such optimal ensembles, and as a necessary *byproduct*, provides optimal power allocation across signal dimensions for users according to a water filling scheme.

In order to do this we will convert the continuous time (waveform) multiaccess channel in equation (2.2) into an equivalent vector multiaccess channel. We note that a vector channel is a natural representation of a waveform channel in a finite dimensional signal space [20, Ch. 8] implied by finite $\mathcal{T}$ and finite bandwidth constraints.

2.3 Channel Eigenfunctions and Equivalent Vector Channels

The material in this section is essentially review, but we feel it is important enough to be presented in a distinct section as we use a slightly different formulation than the complex representation of channels favored in the OFDM literature. We note that although it is theoretically possible to apply interference avoidance using complex channel models such application results in unrealizable signal sets (see Appendix 2.C for details). We therefore use a decomposition for which all channel gains are real non-negative numbers and all channel eigenvectors are real as well.

For a linear and time-invariant channel characterized by impulse response $h(t)$ a useful set of basis functions which allows convenient representation of channel inputs and outputs are the eigenfunctions of the channel impulse response *autocorrelation function* introduced in [29]. This set of functions $\{\Psi_n(t)\}$ forms an orthonormal basis for the signal space and can be used to represent channel inputs as vectors in a Euclidian space of the same dimension as the signal space. In addition, channel responses to these functions are also orthogonal, thus allowing convenient representation of channel outputs as scaled versions of the input vectors. For the sake of completeness we briefly show how these functions are derived. Note that complete details can be found in [29].

The requirement that channel responses to functions $\{\Psi_n(t)\}$ be orthogonal as well implies that on the interval $(0, \mathcal{T})$ where $\mathcal{T}$ is assumed much larger than the duration of $h(t)$ we have

$$\int_0^{\mathcal{T}} \left[\int_0^{\mathcal{T}} \Psi_i(\tau)h(t-\tau)d\tau\right]\left[\int_0^{\mathcal{T}} \Psi_j(\mu)h(t-\mu)d\mu\right] dt = \lambda_i\delta_{ij} \tag{2.3}$$

By defining the channel autocorrelation function as

$$R_h(\tau-\mu) = \int_0^{\mathcal{T}} h(t-\tau)h(t-\mu)dt \tag{2.4}$$

we obtain

$$\int_0^{\mathcal{T}}\int_0^{\mathcal{T}} \Psi_j(\mu)\Psi_i(\tau)R_h(\tau-\mu)d\tau d\mu = \lambda_i\delta_{ij} \tag{2.5}$$

which implies that the set $\{\Psi_n(t)\}$ must satisfy the integral equation

$$\int_0^{\mathcal{T}} \Psi_i(\mu)R_h(\tau-\mu)d\mu = \lambda_i\Psi_i(\tau) \tag{2.6}$$

Since $R_h(\cdot)$ is a correlation function, a complete set of orthonormal $\Psi_n(t)$ along with a corresponding set of *real non-negative* λ_n exists and can be used to represent any input in the mean square sense [79, p.181]. We note that this formulation is different from the usual OFDM treatment which allows complex channel eigenvalues. Regardless, these $\{\Psi_n(t)\}$ eigenfunctions can therefore be used to represent any input in the mean square sense as

$$x(t) = \text{l.i.m}_{N\to\infty}\sum_{n=1}^{N} x_n\Psi_n(t) \tag{2.7}$$

Similarly, any channel output can be written as a linear superposition of the set $\{\tilde{\Psi}_n(t)\}$ defined as

$$\tilde{\Psi}_n(t) = \lambda_n^{-1/2}\int_0^{\mathcal{T}} \Psi_n(\tau)h(t-\tau)d\tau \tag{2.8}$$

for non-zero λ_i. We note that each of the $\Psi_n(t)$ and $\tilde{\Psi}_n(t)$ is assumed to have unit energy

$$\int_0^{\mathcal{T}} \Psi_n^2(t)dt = \int_0^{\mathcal{T}} \tilde{\Psi}_n^2(t)dt = 1 \tag{2.9}$$

Thus, the channel output corresponding to input $x(t)$ is written as

$$y(t) = x(t) * h(t) = \int_0^{\mathcal{T}} \left(\sum_n x_n \Psi_n(\tau) \right) h(t-\tau)d\tau = \sum_n x_n \lambda_n^{1/2} \tilde{\Psi}_n(t) \tag{2.10}$$

For finite time and bandwidth constraints the signal space is "essentially" finite dimensional [34] and, as it is also pointed out in [79, p. 193], the eigenvalues λ_n rapidly approach zero for $n > 2B\mathcal{T}$ where B is the allowable bandwidth. Thus, we assume that there are only N non-negligible eigenvalues λ_n. This implies that the useful input signal space – that portion of the signal space whose component energies are not strongly absorbed by the channel – is of dimension N. Therefore, the input signal can be written in vector form as

$$x(t) = \mathbf{\Psi}(t)^\top \mathbf{x} \tag{2.11}$$

where $\mathbf{x}$ is the N-dimensional vector of coefficients x_n corresponding to the N eigenfunctions $\Psi_n(t)$ with non-negligible eigenvalues arranged in the vector of functions $\mathbf{\Psi}(t)$. Likewise, the channel output can be written as

$$y(t) = \tilde{\mathbf{\Psi}}(t)^\top \mathbf{\Lambda}^{1/2} \mathbf{x} \tag{2.12}$$

where $\mathbf{\Lambda}$ is the $N \times N$ diagonal matrix of the real non-negative significant eigenvalues.

It is worth noting here that in what follows we will always assume that channel gain matrices $\mathbf{\Lambda}$ are invertible. Almost paradoxically, this implies no loss of generality since we show in section 2.4 that the optimum solution using a non-invertible gain matrix $\mathbf{\Lambda}^*$ (with diagonal elements λ_i^* and at least one of them non-zero) is identical to that using an invertible matrix $\mathbf{\Lambda}$ with diagonal elements λ_i such that

$$\lambda_i = \begin{cases} \lambda_i^* & \text{for} \quad \lambda_i^* \neq 0 \\ \epsilon & \text{for} \quad \lambda_i^* = 0 \end{cases} \tag{2.13}$$

where $\epsilon > 0$ is a sufficiently small number.

By performing the outlined eigen-decomposition, the waveform channel with input $x(t)$ and output $y(t)$ described by the impulse response $h(t)$ can be represented by the diagram in Figure 2.2. Note that Figure 2.2 can be put in the simple form of Figure 2.3 which shows the output of the

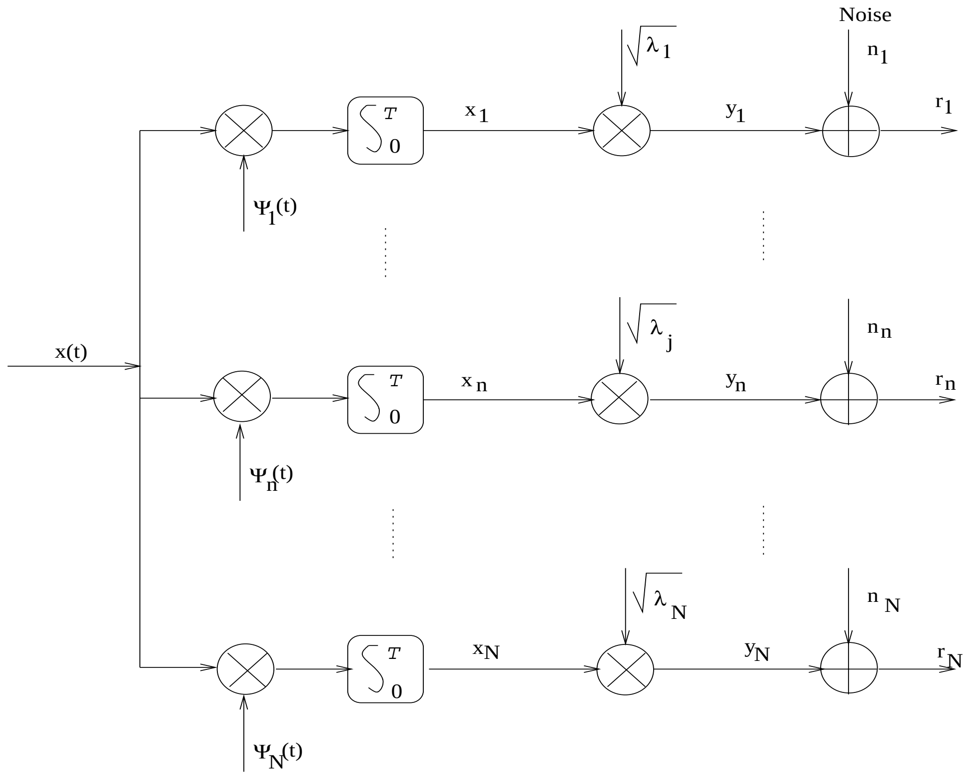

Figure 2.2: Dispersive channel with impulse response $h(t)$ represented as an N-dimensional vector channel characterized by different gains λ_n corresponding to different dimensions (eigenfunctions $\Phi_n(t)$).

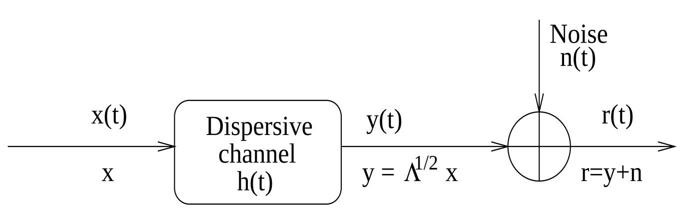

Figure 2.3: Equivalent vector representation of the channel given in Figure 2.2.

dispersive channel as a scaled version of the input corrupted by the corresponding noise vector $\mathbf{n}$.

At this point a final remark about channel eigenfunctions is necessary. In the multiuser case where different users are received at the basestation through different channels with different impulse responses the set of channel eigenfunctions may be different for distinct users. However, since it is assumed that the frame duration $\mathcal{T}$ is large relative the durations of all the channel impulse responses, the channel eigenfunctions will all be approximately sinusoidal "tonebursts"[2]. Substitution of sinusoids for channel eigenfunctions has been studied in detail in [29] and appears practical for a wide range of channels. Also, similar to [29] we choose to use separate dimensions for sine and cosine of the same frequency rather than complex exponentials. The use of complex channel models in conjunction with interference avoidance methods is presented in Appendix 2.C. However, we again mention that such use can result in unrealizable signals $s_k(t)$.

2.4 The Single User Case: Multicarrier CDMA and Interference Avoidance

2.4.1 Multicarrier CDMA

In the case of a single user, the transmitted signal consists of a superposition of signature waveforms scaled by the appropriate information symbols in the frame (see Figure 2.1)

$$x(t) = \sum_{k=1}^{M} b_k s_k(t) \tag{2.14}$$

where $s_k(t)$ is the signature waveform assigned to convey symbol b_k. Unlike [32, 35] there is no explicit precoding. As already mentioned, the method is a form of CDMA in which each symbol corresponds to an independent "virtual user". The collection of these M virtual users comprises the actual user's signal. Each of the waveforms $s_k(t)$ can be represented in terms of the channel eigenfunctions described in the previous section

$$s_k(t) = \sum_{j=1}^{N} s_{kj} \Psi_j(t) = \mathbf{\Psi}(t)^\top \mathbf{s}_k \tag{2.15}$$

which implies that the input signal can be written as

$$x(t) = \sum_{k=1}^{M} b_k \sum_{j=1}^{N} s_{kj} \Psi_j(t) = \mathbf{\Psi}(t)^\top \mathbf{S}\mathbf{b} \tag{2.16}$$

---

[2]Special thanks are due to Prof. M. Honig for this perspective.

where $\mathbf{b} = [b_1 \ldots b_M]^\top$ is the vector containing the frame to be sent and $\mathbf{S}$ is the $N \times M$ matrix ($\in \mathbb{R}^{N\times M}$) with columns $\mathbf{s}_k$ being the coordinate vectors (codewords) of each waveform $\{s_k(t)\}_{k=1}^{M}$ with respect to the basis defined by the channel eigenfunctions. As previously argued, we assume that channel eigenfunctions are approximately sinusoidal and find that this decomposition implies a frequency representation of the signature waveforms as superposition of sinusoids specified by the corresponding codewords leading to a MC-CDMA [92] scheme.

The received signal is corrupted by additive zero mean white Gaussian noise as in

$$r(t) = y(t) + n(t) \tag{2.17}$$

which when projected onto the channel eigenfunctions yields

$$\mathbf{r} = \mathbf{\Lambda}^{1/2}\mathbf{S}\mathbf{b} + \mathbf{n} \tag{2.18}$$

with $\mathbf{n} = [n_1 \ldots n_N]^\top$ a vector containing the projections of the noise process $n(t)$ onto the basis functions – $n_j = \int_0^{\mathcal{T}} n(t)\tilde{\Psi}_j(t)dt$. Since $n(t)$ is a white Gaussian process, the projections onto orthogonal functions are mutually independent, zero-mean Gaussian random variables with variances $E[n_j^2] = N_0$. Now, the problem of optimizing the signature waveforms $s_k(t)$ is completely equivalent to optimizing the codeword matrix $\mathbf{S}$ in equation (2.18).

Since we assume $\mathbf{\Lambda}$ invertible, we can rewrite equation (2.18) to obtain

$$\tilde{\mathbf{r}} = \mathbf{\Lambda}^{-1/2}\mathbf{r} = \mathbf{S}\mathbf{b} + \tilde{\mathbf{n}} \tag{2.19}$$

in which $\tilde{\mathbf{n}} = \mathbf{\Lambda}^{-1/2}\mathbf{n}$ is a new vector of noise components, still uncorrelated, but no longer white – the variance of each component is now N_0/λ_i.

The equivalent problem defined in equation (2.19) is a typical multiuser detection problem [83, ch. 4] where an ensemble of user transmissions $b_k\mathbf{s}_k$ must be jointly decoded subject to a constraint on transmitted energy for each codeword. One could turn to multiuser detection theory at this point and seek to design the best receivers (decision feedback, MMSE etc.) for a given ensemble of codewords. However, our goal here is to find codeword ensembles which maximize sum capacity.

2.4.2 Interference Avoidance

To generate such optimal signal sets, interference avoidance algorithms will be used. We use the greedy *eigen-algorithm* for interference avoidance in the current context and describe properties of the fixed point codeword ensemble. For the single-user case the eigen-algorithm is formally stated next:

The Single-User Eigen-Algorithm for Dispersive Channels

1. Start with an initial set of signatures waveforms $\{s_k(t)\}_{m=1}^{M}$, with corresponding codewords $\{\mathbf{s}_k\}_{m=1}^{M}$ with respect to the orthonormal basis defined by the channel eigenfunctions.

2. For each symbol m

   (a) Compute the N-dimensional autocorrelation matrix of the interference generated by the other $M-1$ independent symbols and noise $\mathbf{R}_m = \mathbf{S}\mathbf{S}^\top - \mathbf{s}_m\mathbf{s}_m^\top + N_0\mathbf{\Lambda}^{-1}$

   (b) Find the minimum eigenvalue μ_m^* of $\mathbf{R}_m$ and its associated eigenvector $\mathbf{x}_m^*$

   (c) Replace $\mathbf{s}_m$ by $\mathbf{x}_m^*$

3. Repeat step 2 until a fixed point is reached where all codewords are minimum eigenvectors of $\mathbf{R}_m$.

4. If the fixed point is suboptimal, employ escape methods [62] to continue iterations

As already been mentioned, this procedure monotonically decreases the trace of the square of the received signal covariance matrix which is equal in this case with

$$\text{Trace}\left[\tilde{\mathbf{R}}^2\right] = \text{Trace}\left[(\mathbf{S}\mathbf{S}^\top + N_0\mathbf{\Lambda}^{-1})^2\right] \tag{2.20}$$

Furthermore, the following theorem shows that the codeword ensemble which minimizes Trace $\left[\tilde{\mathbf{R}}^2\right]$ corresponds to a "water filling" solution which implies that it also maximizes channel capacity.

Theorem 2.1 : *The codeword set that minimizes Trace* $\left[\tilde{\mathbf{R}}^2\right]$ *subject to the total power constraint Trace* $\left[\mathbf{S}\mathbf{S}^\top\right] = M$ *water fills those dimensions of the inverted channel signal space with minimum noise energy*[3]*.*

Proof: In order to find the minimum Trace $\left[\tilde{\mathbf{R}}^2\right]$, let us write the matrix $\mathbf{S}\mathbf{S}^\top + N_0\Lambda^{-1}$ in terms of canonical eigenvectors $\mathbf{e}_i = [0,\ldots,0,1,0,\ldots,0]^\top$ (zero except for 1 in the i^{th} position) of $\mathbb{R}^N$.

[3]Recall that by definition of the equivalent inverted channel problem the actual noise energy in a given signal space dimension is scaled by the channel eigenvalue corresponding to that dimension.

Then we have

$$\mathbf{S}\mathbf{S}^\top + N_0\boldsymbol{\Lambda}^{-1} = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{M} s_{ik}s_{jk}\mathbf{e}_i\mathbf{e}_j^\top + \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{N_0}{\lambda_j}\mathbf{e}_i\mathbf{e}_j^\top = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)\mathbf{e}_i\mathbf{e}_j^\top \quad (2.21)$$

We can then write

$$\begin{aligned}(\mathbf{S}\mathbf{S}^\top + N_0\boldsymbol{\Lambda}^{-1})^2 &= \left[\sum_{i=1}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)\mathbf{e}_i\mathbf{e}_j^\top\right]\left[\sum_{m=1}^{N}\sum_{n=1}^{N}\left(\sum_{\ell=1}^{M} s_{m\ell}s_{n\ell} + \frac{N_0}{\lambda_\ell}\right)\mathbf{e}_m\mathbf{e}_n^\top\right] \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{n=1}^{N}\sum_{k=1}^{M}\left(s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)\sum_{\ell=1}^{M}\left(s_{j\ell}s_{n\ell} + \frac{N_0}{\lambda_j}\right)\mathbf{e}_i\mathbf{e}_n^\top\end{aligned} \quad (2.22)$$

Since Trace $\left[\mathbf{e}_i\mathbf{e}_n^\top\right] = \mathbf{e}_n^\top\mathbf{e}_i = \delta_{in}$, we then have

$$\text{Trace}\left[(\mathbf{S}\mathbf{S}^\top + N_0\Lambda^{-1})^2\right] = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)^2 \quad (2.23)$$

By defining the total signal energy projection along the i^{th} eigenfunction as $P_i = \sum_{k=1}^{M} s_{ik}^2$ we can rewrite (2.23) as

$$\text{Trace}\left[(\mathbf{S}\mathbf{S}^\top + N_0\boldsymbol{\Lambda}^{-1})^2\right] = \sum_{i=1, i\neq j}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)^2 + \sum_{j=1}^{N}\left(P_j + \frac{N_0}{\lambda_j}\right)^2 \quad (2.24)$$

Since all terms in the summations in i and j are non-negative we can lower bound the Trace $\left[\tilde{\mathbf{R}}^2\right]$ by

$$\text{Trace}\left[(\mathbf{S}\mathbf{S}^\top + N_0\boldsymbol{\Lambda}^{-1})^2\right] \geq \sum_{j=1}^{N}\left(P_j + \frac{N_0}{\lambda_j}\right)^2 \quad (2.25)$$

where the right hand side term is a convex function in P_j. Minimization of this term with the requirements that $P_j \geq 0,\ \forall j$ and $\sum_{j=1}^{N} P_j = \text{Trace}\left[\mathbf{S}\mathbf{S}^\top\right] = M$ (total power sent – M symbols each with unit energy) leads to the "water filling" distribution of powers [15, 20]

$$P_i = \left(c^* - \frac{N_0}{\lambda_i}\right)^+, \qquad \text{where } (x)^+ = \begin{cases} x \text{ if } x \geq 0 \\ 0 \text{ if } x < 0 \end{cases} \quad (2.26)$$

where c^* is the water level mark and can be calculated from the total power constraint.

We then recall [72] that if the eigenvalues of a matrix $\mathbf{A}$ are μ_i then the eigenvalues of $\mathbf{A}^2$ are μ_i^2, and since the trace is the sum of eigenvalues we have

$$\text{Trace}\left[(\mathbf{S}\mathbf{S}^\top + N_0\boldsymbol{\Lambda}^{-1})^2\right] = \sum_{i=1}^{N}\beta_i^2 \quad (2.27)$$

where the $\{\beta_i\}$ are the eigenvalues of $\mathbf{S}\mathbf{S}^\top + N_0\mathbf{\Lambda}^{-1}$. If these β_i are chosen to be the $P_i + N_0/\lambda_i$ where the P_i satisfy (2.26), then the bound of (2.25) is met with equality. ■

We note that existence of codeword ensembles with the above stated properties has been proved in [84, 85] and convergence of the single-user eigen-algorithm for dispersive channels to such codeword ensembles is guaranteed by its convergence to the minimum value of Trace $\left[\tilde{\mathbf{R}}^2\right]$ proved in [62].

So in summary, once the problem of signature waveform optimization for a single user is recast as a multiuser detection problem using the equivalent vector channel defined by channel eigenfunctions, codeword adaptation can be brought to bear and an optimal codeword found for each symbol to be transmitted by the user. The optimal codeword ensemble maximizes sum capacity, and assuming simple matched filters for each symbol's codeword, the symbols each attain the maximum achievable uniform SINR. We also note that matched filters are optimal linear receivers and minimize the MSE [84, 85] for each symbol in the frame, which is not true for general channel models with a non-diagonal structure.

Finally, we revisit the issue of channel invertibility. Water filling solutions dictate that if the noise energy in a dimension is large enough relative to other dimensions, then *no* signal energy can reside in that dimension. Thus, in the context of equation (2.19), the optimal codeword ensembles will be identical for non-invertible channels and their counterparts made invertible by replacing zero gain elements by sufficiently small but nonzero gains. A careful definition of "sufficiently small" can be stated as a theorem whose proof is a simple consequence of water filling.

Theorem 2.2 : *Following equation (2.18), define a non-invertible channel gain matrix* $\mathbf{\Lambda}^*$ *with* $k > 0$ *nonzero gains as*

$$\mathbf{\Lambda}^* = \begin{bmatrix} \lambda_1^* & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \lambda_k^* & 0 & \cdots & \cdots & \vdots \\ \vdots & \cdots & 0 & 0 & \ddots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \end{bmatrix} \tag{2.28}$$

and assume with no loss of generality that $\lambda_i \geq \lambda_{i+1}$. *Likewise define a noise covariance matrix*

$$E[\mathbf{n}\mathbf{n}^\top] = \mathbf{W} = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_N \end{bmatrix} \tag{2.29}$$

with $\sigma_i > 0$, $i = 1, \cdots, N$. *Finally assume unit energy symbols* b_i *so that the total transmitted signal energy over all dimensions is* $E = \text{Trace}\left[\mathbf{S}\mathbf{S}^\top\right]$. *If we construct the invertible matrix*

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1^* & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \ddots & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \lambda_k^* & 0 & \cdots & \cdots & \vdots \\ \vdots & \cdots & 0 & \epsilon & \ddots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & \epsilon \end{bmatrix} \tag{2.30}$$

where ϵ *is chosen such that*

$$\frac{\sigma_j}{\epsilon} < \frac{1}{k}\left[E + \sum_{i=1}^{k} \frac{\sigma_i}{\lambda_i}\right]$$

$j = k+1, \cdots, N$, *then the set of codeword ensembles which maximize sum capacity for the channel of equation (2.18) will be identical for* $\mathbf{\Lambda}^*$ *and* $\mathbf{\Lambda}$.

Proof: The theorem is a simple consequence of water filling and of the fact that interference avoidance provides a codeword ensemble which water fills the signal space. ■

Therefore, in what follows we assume all channels are invertible with no loss of generality.

2.5 The Multiuser Case

In this section we consider multiple users received at the basestation over distinct channels with different impulse responses $h_\ell(t)$. This corresponds to the uplink scenario in a CDMA system where we seek to derive optimal codeword ensembles that maximize sum capacity. We present and analyze two algorithms for interference avoidance: the multiuser version of the eigen-algorithm introduced in the previous section in which all codewords of a given user are modified sequentially, and a looser version based on random codeword updates in which codewords are updated randomly over all users (as opposed to updating a single user's codewords until convergence and then moving on). Both algorithms are based on greedy interference avoidance in which at each step a given codeword is replaced by the minimum eigenvector of the corresponding interference-plus-noise process.

As previously assumed in section 2.3, the channel eigenfunctions for all users are sinusoids. This implies that each user in the system employs a MC-CDMA modulation scheme to transmit frames. By further assuming that the corresponding channel eigenfunctions of all users are synchronized *at the receiver* we may then write the received vector at the basestation as

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{\Lambda}_\ell^{1/2} \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{2.31}$$

where $\mathbf{S}_\ell$ is the $N \times M_\ell$ matrix with columns $\mathbf{s}_m^{(\ell)}$ being the codewords for user ℓ and $\mathbf{b}_\ell$ the vector containing the symbols in the frame to be sent by user ℓ. From the perspective of user k equation (2.31) can be rewritten as

$$\mathbf{r} = \mathbf{\Lambda}_k^{1/2} \mathbf{S}_k \mathbf{b}_k + \sum_{\ell=1, \ell \neq k}^{L} \mathbf{\Lambda}_\ell^{1/2} \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{2.32}$$

Note that the first term in equation (2.32) is the desired signal corresponding to user k while the rest represents interference coming from other users and noise. Also note that all the $\mathbf{\Lambda}_\ell$ are invertible, although some of their elements may be of $O(\varepsilon)$. However, as pointed out in section 2.4 via Theorem 2.2, this does not restrict application of the interference avoidance algorithms since the dimensions corresponding to very small eigenvalues will be completely avoided.

We consider the more general case of colored noise with uncorrelated components which implies that the autocovariance matrix of the received signal is

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \sum_{\ell=1}^{L} \mathbf{\Lambda}_\ell^{1/2} \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{\Lambda}_\ell^{1/2} + \mathbf{W} \tag{2.33}$$

with $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top]$ a diagonal matrix with elements equal to $\sigma_i,\ i = 1 \ldots N$, representing the noise variances along each signal space dimension.

From the perspective of an individual user, our problem is again that of selecting input waveforms for its symbols in the presence of colored "noise" consisting of both additive Gaussian noise and interference from other users.

2.5.1 The Eigen-Algorithm in the Multiuser Case

Similar to equation (2.19) we define an equivalent problem for user k, pre-multiplying by the corresponding inverse channel eigenvalue matrix $\mathbf{\Lambda}_k^{-1/2}$ in equation (2.32) to obtain

$$\mathbf{r}_k = \mathbf{S}_k\mathbf{b}_k + \mathbf{\Lambda}_k^{-1/2}\left(\sum_{\ell\neq k}\Lambda_\ell^{1/2}\mathbf{S}_\ell\mathbf{b}_\ell + \mathbf{n}\right) \tag{2.34}$$

We then apply the eigen-algorithm for user k to the equivalent problem by modifying its codewords; the codeword corresponding to symbol m of user k being replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process under channel k inversion.

$$\mathbf{R}_m^{(k)} = \mathbf{S}_k\mathbf{S}_k^\top - \mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top} + \mathbf{\Lambda}_k^{-1/2}\left(\sum_{\ell\neq k}\mathbf{\Lambda}_\ell^{1/2}\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{\Lambda}_\ell^{1/2} + \mathbf{W}\right)\mathbf{\Lambda}_k^{-1/2} \tag{2.35}$$

A formal statement of the eigen-algorithm in the multiuser case is given below:

The Multiuser Eigen-Algorithm for Dispersive Channels

1. Start with a randomly chosen codeword ensemble specified by the user codeword matrices $\mathbf{S}_1, \ldots, \mathbf{S}_L$

2. For each user $k = 1 \ldots L$

   (a) Define the equivalent problem for user k as in equation (2.34)

   (b) adjust user k's codewords sequentially: the codeword corresponding to symbol m of user k is replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process in equation (2.35)

   (c) Repeat step (b) iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

   (d) If a suboptimal point is reached use escape methods [62] and repeat steps (b)-(c).

3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

We establish that fixed points of the eigen-algorithm in the multiuser case not only exist, but that the fixed points produced imply capacity maximizing codeword ensembles.

2.5.2 Greedy Interference Avoidance and Sum Capacity

We first show that application of greedy interference avoidance does not decrease sum capacity. Formally we have with received signal vector $\mathbf{r}$ and the transmitted symbol frames $\mathbf{b}_1, \ldots, \mathbf{b}_L$ in equation (2.31)

$$C_{\text{sum}} = \max I(\mathbf{r}; \mathbf{b}_1, \ldots, \mathbf{b}_L) \tag{2.36}$$

Expanding the mutual information in terms of entropy we get

$$I(\mathbf{r}; \mathbf{b}_1, \ldots, \mathbf{b}_L) = h(\mathbf{r}) - h(\mathbf{r}|\mathbf{b}_1, \ldots, \mathbf{b}_L) \tag{2.37}$$

Given the transmitted symbol frames, the second term in the right hand side is just the noise entropy which for Gaussian noise is

$$h(\mathbf{n}) = \frac{1}{2} \log \left[(2\pi e)^N \det \mathbf{W} \right] \tag{2.38}$$

and the first term is maximized when $\mathbf{r}$ is also Gaussian vector in which case the sum capacity expression becomes

$$C_{\text{sum}} = \frac{1}{2} \log \left[\det \left(\sum_{\ell=1}^{L} \mathbf{\Lambda}_\ell^{1/2} \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{\Lambda}_\ell^{1/2} + \mathbf{W} \right) \right] - \frac{1}{2} \log \left(\det \mathbf{W} \right) \tag{2.39}$$

In order to prove that sum capacity C_{sum} is not decreased by application of greedy interference avoidance we will use majorization theory [38] which provides a partial ordering relation on real vectors. The majorization relation between two N-dimensional vectors $\mathbf{a} = [a_1, \ldots, a_N]^\top$ and $\mathbf{b} = [b_1, \ldots, b_N]^\top$ with components in decreasing order, written as $\mathbf{a} \prec \mathbf{b}$ ($\mathbf{a}$ is majorized by $\mathbf{b}$), is formally defined by the sequence of inequalities

$$\begin{aligned} \sum_{i=1}^{n} a_i &\le \sum_{i=1}^{n} b_i, \quad n = 1, \ldots, N-1 \\ \sum_{i=1}^{N} a_i &= \sum_{i=1}^{N} b_i \end{aligned} \tag{2.40}$$

A comprehensive reference on majorization theory and inequalities is [38] which will be referred throughout this work. Majorization theory has been applied in relatively recent work on wireless communication systems to sum capacity problems [84, 85] as well as to signal design and power control for CDMA systems [22–24].

The following lemma is useful.

Lemma 2.1 : *Consider the matrix $\mathbf{V} = \mathbf{Q} + \mathbf{x}\mathbf{x}^\top$ (with $\|\mathbf{x}\| = 1$) for which we apply greedy interference avoidance, i.e. $\mathbf{x}$ is replaced by the minimum eigenvector of matrix $\mathbf{Q}$. Then, the eigenvalues of $\mathbf{V}$ after the replacement are majorized by the eigenvalues of $\mathbf{V}$ before the replacement.*

Proof: See Appendix 2.A. ■

Theorem 2.3 : *The sum capacity of the multiple access vector channel defined by equation (2.31) is not decreased by application of greedy interference avoidance for any given user.*

Proof: The sum capacity in equation (2.39) can be rewritten from the perspective of user k as

$$\begin{aligned} C_{\text{sum}} &= \frac{1}{2}\log\left\{\det\left[\mathbf{S}_k\mathbf{S}_k^\top + \mathbf{\Lambda}_k^{-1/2}\left(\sum_{\ell\neq k}\mathbf{\Lambda}_\ell^{1/2}\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{\Lambda}_\ell^{1/2} + \mathbf{W}\right)\mathbf{\Lambda}_k^{-1/2}\right]\right\} \\ &\quad + \frac{1}{2}\log(\det\mathbf{\Lambda}_k) - \frac{1}{2}\log(\det\mathbf{W}) \end{aligned} \tag{2.41}$$

We define the eigenvalues of

$$\mathbf{R}^{(k)} = \mathbf{S}_k\mathbf{S}_k^\top + \mathbf{\Lambda}_k^{-1/2}\left(\sum_{\ell\neq k}\mathbf{\Lambda}_\ell^{1/2}\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{\Lambda}_\ell^{1/2} + \mathbf{W}\right)\mathbf{\Lambda}_k^{-1/2} \tag{2.42}$$

as $\mu_1, \ldots, \mu_N$, and rewrite equation (2.41) as

$$C_{\text{sum}} = \frac{1}{2}\sum_{j=1}^{N}\log\mu_j + \frac{1}{2}\log(\det\mathbf{\Lambda}_i) - \frac{1}{2}\log(\det\mathbf{W}) \tag{2.43}$$

Now suppose user k applies greedy interference avoidance to the codeword corresponding to symbol m – that is user k replaces the m-th column of $\mathbf{S}_k$ with the minimum eigenvector of the corresponding autocorrelation matrix of the noise-plus-interference under channel inversion. We then have

$$\mathbf{R}_m^{(k)} = \mathbf{S}_k\mathbf{S}_k^\top - \mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top} + \mathbf{\Lambda}_k^{-1/2}\left(\sum_{\ell\neq k}\mathbf{\Lambda}_\ell^{1/2}\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{\Lambda}_\ell^{1/2} + \mathbf{W}\right)\Lambda_k^{-1/2} \tag{2.44}$$

Let us call this minimum eigenvector $\mathbf{x}_m^{(k)}$. We can rewrite the matrix $\mathbf{R}^{(k)}$ in terms of $\mathbf{R}_m^{(k)}$ as

$$\mathbf{R}^{(k)} = \mathbf{R}_m^{(k)} + \mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)\top} \tag{2.45}$$

and therefore, according to Lemma 2.1, the eigenvalues after the replacement are majorized by the eigenvalues before the replacement

$$\boldsymbol{\mu}' \prec \boldsymbol{\mu} \tag{2.46}$$

It is known from majorization theory [38] that for any Schur concave function $g(\cdot)$ equation (2.46) implies $g(\boldsymbol{\mu}') \geq g(\boldsymbol{\mu})$. Since the function $g(\boldsymbol{\mu}) = \sum_{j=1}^{N} \log \mu_j$ is Schur concave, and because the other two terms in equation (2.43) are constant before and after the replacement of codeword $\mathbf{s}_m^{(k)}$, the sum capacity of the channel is not decreased by application of greedy interference avoidance algorithm. ■

2.5.3 Fixed-Point Properties of the Eigen-Algorithm for Multiaccess Dispersive Channels

Since application of greedy interference avoidance at each step of the eigen-algorithm does not decrease sum capacity, and because sum capacity is upper bounded, we can conclude that the algorithm converges to some fixed point. We now consider the properties of these fixed points and will later show that they also imply a sum capacity maximizing codeword ensemble.

At a fixed point of the multiple user eigen-algorithm we can write

$$\mathbf{R}_m^{(k)} \mathbf{s}_m^{(k)} = \rho_k \mathbf{s}_m^{(k)}, \quad \forall\, m = 1, \dots, M_k,\ k = 1, \dots, L \tag{2.47}$$

which is also equivalent to

$$(\mathbf{R}_m^{(k)} + \mathbf{s}_m^{(k)} \mathbf{s}_m^{(k)\top}) \mathbf{s}_m^{(k)} = (\rho_k + 1) \mathbf{s}_m^{(k)}, \quad \forall\, m = 1, \dots, M_k,\ k = 1, \dots, L \tag{2.48}$$

or

$$\left[\mathbf{S}_k \mathbf{S}_k^\top + \boldsymbol{\Lambda}_k^{-1/2} \left(\sum_{\ell \neq k} \boldsymbol{\Lambda}_\ell^{1/2} \mathbf{S}_\ell \mathbf{S}_\ell^\top \boldsymbol{\Lambda}_\ell^{1/2} + \mathbf{W} \right) \boldsymbol{\Lambda}_k^{-1/2} \right] \mathbf{s}_m^{(k)} = (\rho_k + 1) \mathbf{s}_m^{(k)}, \qquad \begin{array}{c} m = 1, \dots, M_k \\ k = 1, \dots, L \end{array} \tag{2.49}$$

which can be made compact for a given user k as

$$\left[\mathbf{S}_k \mathbf{S}_k^\top + \boldsymbol{\Lambda}_k^{-1/2} \left(\sum_{\ell \neq k} \boldsymbol{\Lambda}_\ell^{1/2} \mathbf{S}_\ell \mathbf{S}_\ell^\top \boldsymbol{\Lambda}_\ell^{1/2} + \mathbf{W} \right) \boldsymbol{\Lambda}_k^{-1/2} \right] \mathbf{S}_k = (\rho_k + 1) \mathbf{S}_k, \quad \forall\, k = 1, \dots, L \tag{2.50}$$

We can also write equation (2.50) as

$$\boldsymbol{\Lambda}_k^{-1/2} \mathbf{R} \boldsymbol{\Lambda}_k^{-1/2} \mathbf{S}_k = \mu_k \mathbf{S}_k, \quad \forall\, k = 1, \dots, L \tag{2.51}$$

where $\mathbf{R}$ is the autocovariance of the received signal defined in equation (2.33). This property characterizes a fixed point of the multiple channel eigen-algorithm. Note that in equation (2.51) we have denoted $\mu_k = \rho_k + 1$, since it will turn out that this is exactly the water level under channel inversion for user k which appears in the following theorem (Theorem 2.4) and again later on. Also, it is easy to see from equation (2.42) that for any pair of users i and j matrices $\mathbf{R}^{(i)}$ and $\mathbf{R}^{(j)}$ are related to matrix $\mathbf{R}$ by

$$\mathbf{R} = \mathbf{\Lambda}_i^{1/2}\mathbf{R}^{(i)}\mathbf{\Lambda}_i^{1/2} = \mathbf{\Lambda}_j^{1/2}\mathbf{R}^{(j)}\mathbf{\Lambda}_j^{1/2} \quad \forall\, i, j = 1, \ldots, L \tag{2.52}$$

Next, we recall that application of the eigen-algorithm for a system with more codewords than signal space dimensions ($M_k \geq N$) in a colored noise background [62, 63] performs an *aggregate water filling* over that portion of the signal space with least interference-plus-noise energy, leading to an optimum SINR for all codewords. This formulation is identical to our inverted channel problem and therefore we restate the result here:

Theorem 2.4 : *For $M_k \geq N$, application of the eigen-algorithm by any user k water fills those dimensions of the user k inverted channel problem with minimum interference plus noise energy, and avoids dimensions with interference plus noise energy above a "water level" μ_k given by equation (2.51).*

Proof: The reader is referred to [62, 63] for the proof of this theorem. ■

We also note that as a consequence of Theorem 2.3, repeated application of interference avoidance implies convergence to some value of C_{sum} as given in equation (2.39). We do not yet claim that this is the maximum possible value, just that convergence to some value is certain. In fact, we will see in the following section that a looser application of interference avoidance whereby codewords are updated randomly over all users (as opposed to updating a single user's codewords until convergence and then moving on) is also guaranteed by Theorem 2.3 to converge to some value. With this in mind, Theorem 2.4 has the following corollary for the resultant fixed point.

Corollary 2.1 : *Application of the eigen-algorithm over multiaccess dispersive channels has a fixed point corresponding to water filling for all users in their respective inverted channel problems. Furthermore [62] guarantees that all symbols of a given user k will achieve the same signal-to-interference plus noise ratio SINR$= 1/\rho_k = 1/(\mu_k - 1)$.* ■

Figure 2.4 is a depiction of such a fixed-point in signal space for two distinct users.

Theorem 2.5 : *Sequential application of the eigen-algorithm for interference avoidance can have a fixed point for which the received signal autocovariance matrix* $\mathbf{R}$ *in equation (2.33) is diagonal*[4].

Proof: First, we note that the received signal covariance $\mathbf{R}$ obeys equation (2.52). Next, in order to obtain a diagonal $\mathbf{R}$ through interference avoidance, we start with all users silent and add them to the system one by one. In the presence of uncorrelated noise with diagonal autocorrelation matrix $\mathbf{W}$, the orthogonal basis used for generating codewords [63] for user 1 is exactly the canonical basis of $\mathbb{R}^N$ consisting of vectors $\mathbf{e}_i = [0, \ldots, 0, 1, 0, \ldots, 0]^\top$ (zero except for 1 in the i^{th} position). Since channel eigenvalue matrices are also diagonal, the resulting $\mathbf{R}^{(1)}$ will also be diagonal, which in turn implies that when the next user is added to the system, it will also see a diagonal noise-plus-interference autocorrelation matrix in its corresponding inverted channel problem, and so on, for remaining users added one by one into the system. After all users are added, we may begin again with an arbitrarily chosen user. Regardless, the noise/interference structure seen by that user is still diagonal and the chosen user will occupy a subspace spanned by canonical vectors. Thus, sequential application of the eigen-algorithm for interference avoidance will yield at equilibrium diagonal matrices $\mathbf{R}^{(k)}$, $k = 1, \ldots, L$. Using equation (2.52) to relate $\mathbf{R}^{(k)}$ and $\mathbf{R}$ we get the desired result – $\mathbf{R}$ is also diagonal. ■

In conclusion, there exists a fixed point of the multiuser eigen-algorithm for which the covariance matrix $\mathbf{R}$ of the received signal is diagonal with eigenvalues corresponding to a water filling distribution for all users in their respective inverted channel problems.

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[4]In all experiments using diagonal noise covariances $\mathbf{W}$, no matter which version of interference avoidance was used (codeword update ordering), a scaled identity received covariance structure was obtained. However, we could not prove in general that this was the only structure which could result from application of the algorithm. We therefore constructed a not too implausible scenario under which a diagonal received signal covariance will be obtained.

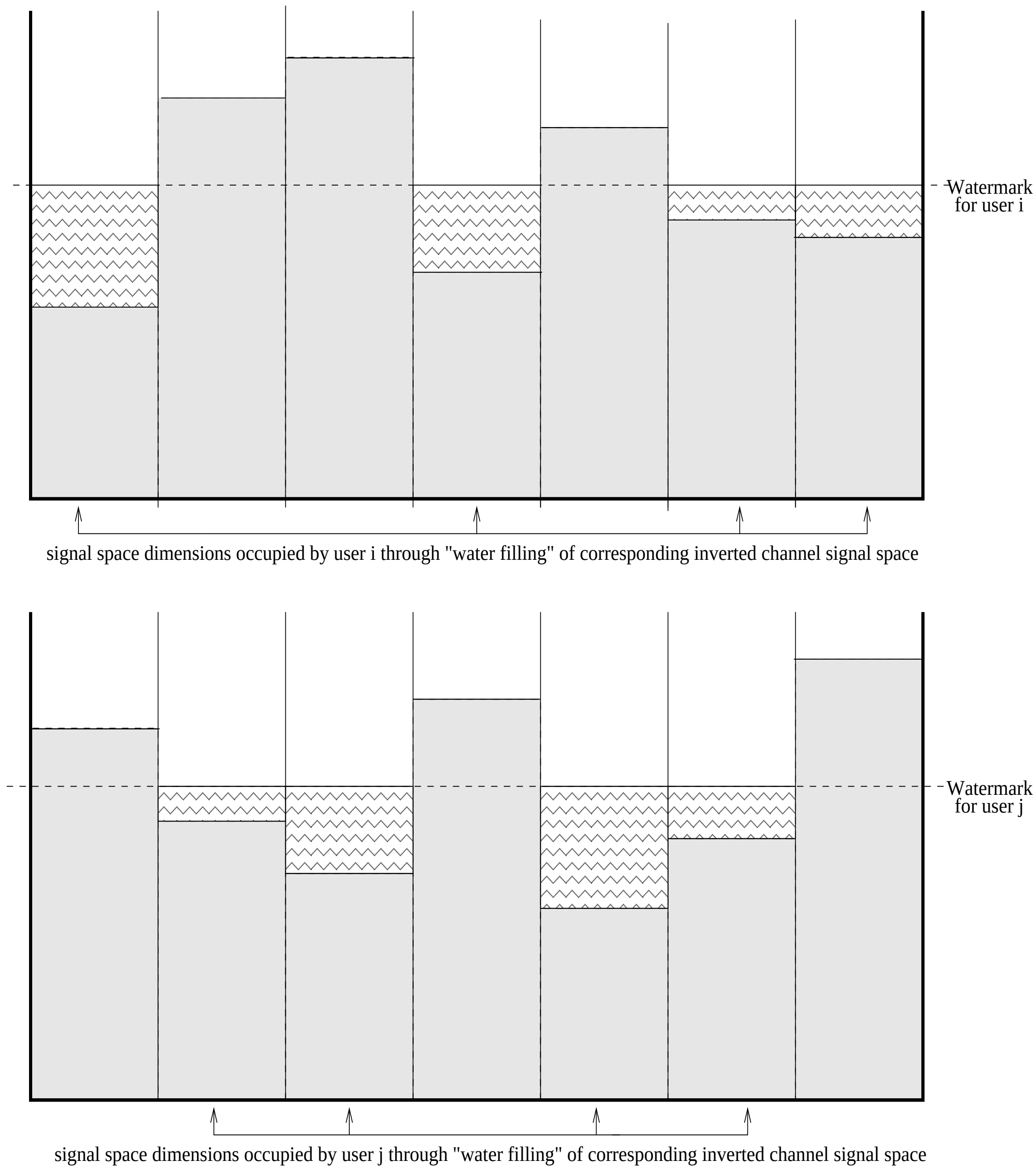

Figure 2.4: Example of "water filling" for two distinct users. Due to the different interference-plus-noise levels on different signal space dimensions, users i and j span different (possibly overlapping) subspaces.

2.5.4 Optimality of the Eigen-Algorithm Fixed Point: Water Filling the Inverted Channel

We now seek to show that the fixed point of the eigen-algorithm where each user has water filled its own inverted signal space corresponds to a signal constellation which maximizes sum capacity. To do so, we derive an expression for sum capacity in the context of our formulation and show that it is maximized by any signal set which satisfies the fixed point of the eigen-algorithm.

In order to maximize the sum capacity in equation (2.39) for the multiple access channel defined by equation (2.31), the eigenvalues of the received signal covariance matrix $\mathbf{R}$ must be solutions of the following optimization problem

$$\max_{\mathbf{s}_m^{(\ell)},\, m=1,\ldots,M_\ell,\, \ell=1,\ldots,L} C_{\text{sum}} \tag{2.53}$$

subject to some power constraint on the codewords. Since unit energy codewords have been assumed for all users, the power constraint is written as

$$\text{Trace}\left[\mathbf{S}_\ell \mathbf{S}_\ell^\top\right] = M_\ell, \quad \forall \ell = 1, \ldots, L \tag{2.54}$$

We note that the second term in equation (2.39) is fixed and therefore sum capacity is maximized when the first term is maximized. Let us denote the vector of eigenvalues of $\mathbf{R}$ in equation (2.33) by $\boldsymbol{\mu}$ in decreasing order $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_N$.

First, we show by using results from majorization theory [38] that sum capacity is upper bounded by the case corresponding to $\mathbf{R}$ being a diagonal matrix. We rewrite equation (2.39) in terms of eigenvalues of matrices involved as

$$C_{\text{sum}} = \frac{1}{2}\sum_{i=1}^{N} \log \mu_i - \frac{1}{2}\sum_{i=1}^{N} \log \sigma_i \tag{2.55}$$

and note again that the function $g(\boldsymbol{\mu}) = \sum_{i=1}^{N} \log \mu_i$ is Schur concave which implies that for any two vectors $\boldsymbol{\mu}$ and $\boldsymbol{\mu}'$ such that $\boldsymbol{\mu} \prec \boldsymbol{\mu}'$ we have $g(\boldsymbol{\mu}) \geq g(\boldsymbol{\mu}')$. The following majorization result (theorem 9.C.1. in [38]) which we state as a lemma is useful.

Lemma 2.2 : *Let* $\mathbf{H}$ *and* $\bar{\mathbf{H}}$ *be* $n \times n$ *symmetric matrices of the form*

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix}, \qquad \bar{\mathbf{H}} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{22} \end{bmatrix}$$

with matrices $\mathbf{H}_{11}$ *and* $\mathbf{H}_{22}$ *of dimensions* $l \times l$*, respectively* $m \times m$ $(l + m = n)$ *and other block matrices of appropriate dimensions (*$\mathbf{0}$ *denotes a matrix with all elements equal to zero). Then*

$$\bar{\boldsymbol{\lambda}} \prec \boldsymbol{\lambda}$$

that is, the eigenvalues $\boldsymbol{\lambda}$ *of* $\mathbf{H}$ *majorize the eigenvalues* $\bar{\boldsymbol{\lambda}}$ *of* $\bar{\mathbf{H}}$. ■

Since $|\bar{\mathbf{H}}| = |\mathbf{H}_{11}||\mathbf{H}_{22}|$, using Lemma 2.2 recursively one obtains the Hadamard inequality formulated in terms of majorization theory: *the eigenvalues of a symmetric matrix majorize the diagonal elements*[5]. From the sum capacity point of view this result has an important implication which we state as a lemma.

Lemma 2.3 : *For any non-diagonal matrix* $\mathbf{A}$ *with sum capacity* $C_{\mathbf{A}}$ *there exists a diagonal* $\mathbf{Q}$ *with Trace* $[\mathbf{A}] =$ *Trace* $[\mathbf{Q}]$ *such that the sum capacity* $C_{\mathbf{Q}} \geq C_{\mathbf{A}}$*. Thus, we may maximize sum capacity by searching only over diagonal* $\mathbf{Q}$. ■

Now consider that $\mathbf{W}$ and all the $\boldsymbol{\Lambda}_\ell$ are diagonal. So ignoring the question of existence for now, let us assume that at the optimum point all the products $\mathbf{S}_\ell \mathbf{S}_\ell^\top$, $\forall \ell = 1, \ldots, L$, will also be diagonal matrices with eigenvalues $\varrho_n^{(\ell)}$, $n = 1, \ldots, N$. Thus we can write them as

$$\mathbf{S}_\ell \mathbf{S}_\ell^\top = \begin{bmatrix} \varrho_1^{(\ell)} & & & & \\ & \ddots & & & \\ & & \varrho_n^{(\ell)} & & \\ & & & \ddots & \\ & & & & \varrho_N^{(\ell)} \end{bmatrix} \tag{2.56}$$

We can then write

$$\det \mathbf{R} = \prod_{n=1}^{N} \left(\sum_{\ell=1}^{L} \lambda_n^{(\ell)} \varrho_n^{(\ell)} + \sigma_n \right) \tag{2.57}$$

which implies that we can re-state the constrained optimization problem in equations (2.53)–(2.54) as

[5]The well-known Hadamard inequality states that the determinant of a positive semidefinite matrix is upper bounded by the product of its diagonal elements. This can be seen as a consequence of the majorization relation between eigenvalues and diagonal elements.

$$\text{maximize} \;\; \frac{1}{2}\sum_{n=1}^{N}\log\left(1+\frac{1}{\sigma_n}\sum_{\ell=1}^{L}\lambda_n^{(\ell)}\varrho_n^{(\ell)}\right) \tag{2.58}$$

subject to the constraints

$$\text{Trace}\left[\mathbf{S}_\ell\mathbf{S}_\ell^\top\right]=\sum_{n=1}^{N}\varrho_n^{(\ell)}=M_\ell, \qquad \forall\ell=1,\ldots,L \tag{2.59}$$

and

$$\varrho_n^{(\ell)}\geq 0, \qquad \forall\ell=1,\ldots,L,\; n=1,\ldots,N \tag{2.60}$$

Theorem 2.6 : *The eigenvalue distribution for the codeword ensemble that maximizes sum capacity corresponds to a water filling solution in the inverted channel signal space for each user.*

Proof: See Appendix 2.B. ■

We then note that the expression for capacity is concave in the $\varrho_n^{(i)}$. Thus, any solution which satisfies the extremal conditions of Theorem 2.6 must also be optimal. Since the water filling distribution of eigenvalues along with the diagonal structure implied by the codeword ensemble that maximizes sum capacity is identical with that attained by application of the multiuser eigen-algorithm (Theorems 2.4 and 2.5) we conclude that interference avoidance generates an optimal codeword ensemble which maximizes sum capacity of the multiaccess dispersive channel.

Finally, it is worth noting that the solution of the constrained optimization problem in equations (2.53)–(2.54) corresponds in fact to the optimal power distribution for all users along the signal space dimensions for the codeword ensemble that maximizes sum capacity of the given multiaccess dispersive channel. This is so because diagonal elements in $\mathbf{S}_\ell\mathbf{S}_\ell^\top$ represent the amount of power (energy) put by user ℓ into the corresponding signal space dimension. This comes in agreement with recent results [94] which show that a *simultaneous water filling* solution in which all $\mathbf{R}^{(k)}=\mathbf{\Lambda}_k^{-1/2}\mathbf{R}\mathbf{\Lambda}_k^{-1/2}$ are water filling solutions to the problem

$$\max_{\mathbf{S}_k\mathbf{S}_k^\top}\left[\det\mathbf{R}^{(k)}\right]=\max_{\mathbf{S}_k\mathbf{S}_k^\top}\left\{\det\left[\mathbf{S}_k\mathbf{S}_k^\top+\mathbf{\Lambda}_k^{-1/2}\left(\sum_{\ell\neq k}\mathbf{\Lambda}_\ell^{1/2}\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{\Lambda}_\ell^{1/2}+\mathbf{W}\right)\mathbf{\Lambda}_k^{-1/2}\right]\right\} \tag{2.61}$$

attains the largest possible sum capacity. Also, from the perspective of the algorithm defined in [94] the multiuser eigen-algorithm is an instance of *iterative water filling*.

2.6 An Alternative Algorithm for Interference Avoidance

We have seen in the previous section that the multiuser version of the eigen-algorithm converges to a fixed-point where the sum capacity of the multiaccess channel is maximized. The algorithm is based on updating all codewords of a given user in a sequence until convergence and then moving on to another user. We have found in practice, however, that regardless of the order in which codewords are replaced, sum capacity of the multiple access channel is always maximized. Although we have been unable to prove this result in general, we have identified an alternative interference avoidance algorithm for which we also prove convergence to the maximum sum capacity.

The Maximum Capacity Increase Algorithm for Interference Avoidance

1. Start with a randomly chosen codeword ensemble specified by the user codeword matrices $\mathbf{S}_1, \ldots, \mathbf{S}_L$
2. Define the equivalent problems for all users k as in equation (2.34)
3. Identify the codeword $\mathbf{s}_m^{(k)}$ whose replacement will maximally increase sum capacity. If no codeword will increase sum capacity, and suboptimal maxima escape methods [62] are ineffective for improvement, then stop. Otherwise,
   (a) adjust $\mathbf{s}_m^{(k)}$: replacement by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process in equation (2.35)
   (b) Return to step 2

We will prove in a moment that this procedure guarantees convergence of the codeword ensembles to sets which water fill their inverted channels. We note however, that we do *not* claim in any algorithm (the multiuser eigen-algorithm or the maximum capacity increase algorithm) that the *codewords* converge, but rather that they converge *in class* [62].

First we note that because at each step of the maximum capacity increase algorithm is a greedy interference avoidance procedure, it cannot decrease sum capacity (Theorem 2.3). Furthermore, the maximum capacity increase algorithm stops only if sum capacity cannot be increased. Thus, the sequence of sum capacity values along any update trajectory must be strictly increasing. This observation leads to the following theorem.

Theorem 2.7 : *In the limit, the maximum capacity increase algorithm for interference avoidance produces codewords* $\mathbf{s}_m^{(k)}$ *which are eigenvectors of* $\mathbf{R}^{(k)}$, $k = 1, 2, \ldots, L$, $m = 1, 2, \ldots, M_k$.

Proof: Let k_p be the user index and m_p the codeword index for that user chosen for update at algorithm step p. Let the N eigenvalues $\{\gamma_i\}$ of $\mathbf{R}_{m_p}^{(k_p)}$ be ordered from largest to smallest and let the associated eigenvectors be $\{\boldsymbol{\phi}_i\}$. Then, C_p, the sum capacity after each step p of the algorithm is

$$C_p = \frac{1}{2}\log\left|\mathbf{R}_{m_p}^{(k_p)} + \boldsymbol{\phi}_N\boldsymbol{\phi}_N^\top\right| - \frac{1}{2}\log|\mathbf{W}| + \frac{1}{2}\log\left|\boldsymbol{\Lambda}_{k_p}\right| \tag{2.62}$$

The change in sum capacity can then be written as

$$C_p - C_{p-1} = \Delta_p = \frac{1}{2}\log\left|\mathbf{R}_{m_p}^{(k_p)} + \boldsymbol{\phi}_N\boldsymbol{\phi}_N^\top\right| - \frac{1}{2}\log\left|\mathbf{R}_{m_p}^{(k_p)} + \mathbf{s}_{m_p}^{(k_p)}(\mathbf{s}_{m_p}^{(k_p)})^\top\right| > 0 \tag{2.63}$$

which we rewrite as

$$e^{-2\Delta_p} = \frac{\left|\mathbf{R}_{m_p}^{(k_p)} + \mathbf{s}_{m_p}^{(k_p)}(\mathbf{s}_{m_p}^{(k_p)})^\top\right|}{\left|\mathbf{R}_{m_p}^{(k_p)} + \boldsymbol{\phi}_N\boldsymbol{\phi}_N^\top\right|} \tag{2.64}$$

Then we note that for

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\phi}_1 & \boldsymbol{\phi}_2 & \cdots & \boldsymbol{\phi}_N \end{bmatrix} \tag{2.65}$$

and

$$\boldsymbol{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \cdots & \cdots & 0 \\ 0 & \gamma_2 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \gamma_N \end{bmatrix} \tag{2.66}$$

we have

$$\mathbf{R}_{m_p}^{(k_p)} = \boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\Phi}^\top \tag{2.67}$$

so that

$$\mathbf{R}_{m_p}^{(k_p)} + \boldsymbol{\phi}_N\boldsymbol{\phi}_N^\top = \boldsymbol{\Phi}\left(\boldsymbol{\Gamma} + \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & 0 & 0 \\ 0 & \cdots & \cdots & 1 \end{bmatrix}\right)\boldsymbol{\Phi}^\top \tag{2.68}$$

This allows us to rewrite equation (2.64) as

$$e^{-2\Delta_p} = \left| \begin{bmatrix} 1 & \cdots & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & 1 & 0 \\ 0 & \cdots & \cdots & \frac{\gamma_N}{1+\gamma_N} \end{bmatrix} + \mathbf{\Gamma}^{-1/2}\mathbf{\Phi}^\top \mathbf{s}_{m_p}^{(k_p)} (\mathbf{s}_{m_p}^{(k_p)})^\top \mathbf{\Phi}\mathbf{\Gamma}^{-1/2} \right| \tag{2.69}$$

Finally, we can rewrite $\mathbf{s}_{m_p}^{(k_p)}$ as

$$\mathbf{s}_{m_p}^{(k_p)} = \sum_{i=1}^{N} a_i \boldsymbol{\phi}_i \tag{2.70}$$

where $\sum_i a_i^2 = 1$. We then have

$$e^{-2\Delta_p} = \left| \begin{bmatrix} 1 & \cdots & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & 1 & 0 \\ 0 & \cdots & \cdots & \frac{\gamma_N}{1+\gamma_N} \end{bmatrix} + \begin{bmatrix} \frac{a_1}{\sqrt{\gamma_1}} \\ \frac{a_2}{\sqrt{\gamma_2}} \\ \vdots \\ \frac{a_{N-1}}{\sqrt{\gamma_{N-1}}} \\ \frac{a_N}{\sqrt{1+\gamma_N}} \end{bmatrix} \begin{bmatrix} \frac{a_1}{\sqrt{\gamma_1}} & \frac{a_2}{\sqrt{\gamma_2}} & \cdots & \frac{a_{N-1}}{\sqrt{\gamma_{N-1}}} & \frac{a_N}{\sqrt{1+\gamma_N}} \end{bmatrix} \right| \tag{2.71}$$

Now, we note that there are $N-2$ vectors of the form

$$\boldsymbol{\psi}_i = \begin{bmatrix} \psi_{i1} \\ \psi_{i2} \\ \vdots \\ \psi_{i(N-1)} \\ 0 \end{bmatrix} \tag{2.72}$$

$i = 1, 2, \ldots, N-2$ for which

$$\boldsymbol{\psi}_i^\top \begin{bmatrix} \frac{a_1}{\sqrt{\gamma_1}} \\ \frac{a_2}{\sqrt{\gamma_2}} \\ \vdots \\ \frac{a_{N-1}}{\sqrt{\gamma_{N-1}}} \\ \frac{a_N}{\sqrt{1+\gamma_N}} \end{bmatrix} = 0 \tag{2.73}$$

We then note that if we define

$$\mathbf{Z} = \begin{bmatrix} 1 & \cdots & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & 1 & 0 \\ 0 & \cdots & \cdots & \frac{\gamma_N}{1+\gamma_N} \end{bmatrix} + \begin{bmatrix} \frac{a_1}{\sqrt{\gamma_1}} \\ \frac{a_2}{\sqrt{\gamma_2}} \\ \vdots \\ \frac{a_{N-1}}{\sqrt{\gamma_{N-1}}} \\ \frac{a_N}{\sqrt{1+\gamma_N}} \end{bmatrix} \begin{bmatrix} \frac{a_1}{\sqrt{\gamma_1}} & \frac{a_2}{\sqrt{\gamma_2}} & \cdots & \frac{a_{N-1}}{\sqrt{\gamma_{N-1}}} & \frac{a_N}{\sqrt{1+\gamma_N}} \end{bmatrix} \tag{2.74}$$

then

$$\mathbf{Z}\boldsymbol{\psi}_i = \boldsymbol{\psi}_i \tag{2.75}$$

$i = 1, 2, \ldots, N-2$ so the $\boldsymbol{\psi}_i$ are eigenvectors of $\mathbf{Z}$ and the remaining eigenvectors reside in a space spanned by the orthogonal vectors

$$\mathbf{x}_1 = \begin{bmatrix} \frac{a_1}{\sqrt{\gamma_1}} \\ \frac{a_2}{\sqrt{\gamma_2}} \\ \vdots \\ \frac{a_{N-1}}{\sqrt{\gamma_{N-1}}} \\ 0 \end{bmatrix} \tag{2.76}$$

and

$$\mathbf{x}_2 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{2.77}$$

This allows us to define a unitary matrix

$$\mathbf{U} = \begin{bmatrix} \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \cdots & \boldsymbol{\psi}_{N-2} & \mathbf{x}_1/\|\mathbf{x}_1\| & \mathbf{x}_2 \end{bmatrix} \tag{2.78}$$

such that

$$|\mathbf{U}^\top \mathbf{Z} \mathbf{U}| = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \cdots & \cdots & \vdots \\ \vdots & \vdots & 1 & \cdots & 0 \\ \vdots & \vdots & 0 & \|\mathbf{x}_1\|^2 + 1 & \|\mathbf{x}_1\| \frac{a_N}{\sqrt{1+\gamma_N}} \\ 0 & \cdots & \cdots & \|\mathbf{x}_1\| \frac{a_N}{\sqrt{1+\gamma_N}} & \frac{\gamma_N + a_N^2}{1+\gamma_N} \end{bmatrix} \tag{2.79}$$

Since $|\mathbf{Z}| = |\mathbf{U}^\top \mathbf{Z} \mathbf{U}|$ we have

$$|\mathbf{Z}| = \frac{1}{1+\gamma_N}\left(\gamma_N + \sum_{i=1}^{N} a_i^2 \frac{\gamma_N}{\gamma_i}\right) \tag{2.80}$$

We now consider the constraints on the a_i when Δ_p is small. We have

$$e^{-2\Delta_p} = 1 - \epsilon = \frac{1}{1+\gamma_N}\left(\sum_{i=1}^{N} a_i^2 \frac{\gamma_N}{\gamma_i} + \gamma_N\right) \tag{2.81}$$

where $\epsilon > 0$. We can rewrite this as

$$(1-\epsilon)(1+\gamma_N) = 1 - \epsilon' = \sum_{i=1}^{N} a_i^2 \frac{\gamma_N}{\gamma_i} \tag{2.82}$$

which we can further rewrite as

$$\epsilon' = \sum_{i=1}^{N} a_i^2 (1 - \frac{\gamma_N}{\gamma_i}) \tag{2.83}$$

since $\sum_{i=1}^{N} a_i^2 = 1$. We then note that the coefficients $\gamma_N/\gamma_i < 1$ unless $\gamma_i = \gamma_N$ since we assumed eigenvalues ordered from largest to smallest so that all terms in the sum are non-negative. Therefore,

$$\epsilon' \geq a_i^2 (1 - \frac{\gamma_N}{\gamma_i}) \tag{2.84}$$

$i = 1, 2, \ldots, N$. So as $\epsilon' \to 0$ we must have each term in the sum approach zero as well. Thus, for large enough p, we cannot simultaneously have large a_i and γ_N/γ_i differing greatly from 1 – which implies that in the limit of $p \to \infty$ ($\epsilon' \to 0$), the codeword to be replaced, $\mathbf{s}_{m_p}^{(k_p)}$ approaches a minimum eigenvector of $\mathbf{R}_{m_p}^{(k_p)}$. Finally, since it was assumed that $\mathbf{s}_{m_p}^{(k_p)}$ was chosen to maximally increase the sum capacity, then for any other codeword $\mathbf{s}_m^{(k)}$ the corresponding change in sum capacity could only be smaller were that codeword replaced. Therefore, if at algorithm step p the a_i associated with $\mathbf{s}_{m_p}^{(k_p)}$ are tightly bound by equation (2.84), then the corresponding coefficients for every other codeword in the system are similarly bound at step p as well. Thus, all the codewords in the ensemble simultaneously approach minimum eigenvalue eigenvectors of their corresponding $\mathbf{R}_m^{(k)}$. And since $\mathbf{R}^{(k)} = \mathbf{R}_m^{(k)} + \mathbf{s}_m^k(\mathbf{s}_m^k)^\top$, the maximum capacity increase algorithm produces codeword ensembles in the limit for which all codewords $\mathbf{s}_m^{(k)}$ are eigenvectors of $\mathbf{R}^{(k)}$ – which proves the theorem. ■

Theorem 2.8 : *The maximum capacity increase algorithm for interference avoidance produces codeword matrices* $\mathbf{S}_m$ *which water fill their associated inverted channel problem defined by equation (2.34).*

Proof: There may exist fixed points where all codewords are eigenvectors of their associated inverted channel covariance matrices, but which do not correspond to water filling solutions [63]. However, by [62], these suboptimal fixed points can be escaped as part of the algorithm and sum capacity further increased. Since the maximum sum capacity increase algorithm for interference avoidance does not stop unless each user codeword ensemble is a water filling solution to that user's inverted channel problem, its application must result in water filling codeword ensembles $\mathbf{S}_m$ for all users. ■

So in summary, there are at least two interference avoidance algorithms for use with dispersive channels which guarantee convergence to maximum sum capacity ensembles. The multiuser eigen-algorithm is similar to iterative water filling [94] while the maximum capacity increase algorithm is not, though the end result of sum capacity maximization is the same.

2.7 Additional Properties and Generalization

In this section, properties of optimal codeword ensembles that maximize sum capacity are derived using properties of interference avoidance algorithms under a loose set of constraints on the channel gain matrices $\{\mathbf{\Lambda}_i\}$. We note that these properties have been mentioned informally for multiuser DMT systems [17] and can also be found in recent work [44].

We start with the following theorem:

Theorem 2.9 : *If the codewords of two given users each span the whole signal space* $\mathbb{R}^N$ *then these users must have identical channels.*

Proof: At the fixed point of the eigen-algorithm we can write for users i and j

$$\begin{aligned} \mathbf{\Lambda}_i^{-1/2}\mathbf{R}\mathbf{\Lambda}_i^{-1/2}\mathbf{S}_i\mathbf{S}_i^\top &= \mu_i\mathbf{S}_i\mathbf{S}_i^\top \\ \mathbf{\Lambda}_j^{-1/2}\mathbf{R}\mathbf{\Lambda}_j^{-1/2}\mathbf{S}_j\mathbf{S}_j^\top &= \mu_j\mathbf{S}_j\mathbf{S}_j^\top \end{aligned} \tag{2.85}$$

Since both users span the signal space, matrices $\mathbf{S}_i\mathbf{S}_i^\top$ and $\mathbf{S}_j\mathbf{S}_j^\top$ are invertible, and by post-multiplying by their inverses followed by appropriate multiplication by the channel eigenvalue matrix we get

$$\mathbf{R} = \mu_i \mathbf{\Lambda}_i = \mu_j \mathbf{\Lambda}_j \tag{2.86}$$

which implies that the channel eigenvalue matrix of user i is a scaled version of the channel eigenvalue matrix of user j

$$\mathbf{\Lambda}_i = \frac{\mu_j}{\mu_i} \mathbf{\Lambda}_j \tag{2.87}$$

Therefore, both users i and j must see the same channel if each user's codeword ensemble spans N dimensions. ■

Note that Theorem 2.9 is a first indication that the signal space is frequency partitioned at a fixed point. That is, codeword matrices of users with different channel eigenvalue matrices cannot contain all frequency components.

Next, we make the following assumption:

Assumption I: *The ratio of channel eigenvalues for any pair of users $i \neq j$ is different for different dimensions $r \neq s$ corresponding to sinusoids with different frequencies $f_r \neq f_s$.*

$$\frac{\lambda_r^{(i)}}{\lambda_r^{(j)}} \neq \frac{\lambda_s^{(i)}}{\lambda_s^{(j)}} \quad \forall i \neq j \in \{1, \ldots, L\},\ r \neq s,\ f_r \neq f_s \tag{2.88}$$

This is a reasonable assumption for some level of precision ε in the representation of channel eigenvalue matrices since a small perturbation $O(\varepsilon)$ will spoil any potential equality. We note however, that for two dimensions which are sinusoids of the same frequency (sine and cosine), the ratio of channel eigenvalues must be *identical* since we presume real-valued signals.

Using equation (2.52) the eigenvalue $\sigma_r^{(i)}$ for dimension r in the inverted channel signal space of user i can be related to the "water level" μ_j corresponding to dimension r in the inverted channel signal space of user j (see Figure 2.4) by

$$\sigma_r^{(i)} = \mu_j \frac{\lambda_r^{(j)}}{\lambda_r^{(i)}} \tag{2.89}$$

Theorem 2.10 : *No two users can reside in subspaces that overlap in more than one frequency.*

Proof: For any dimension r in which users i and j overlap equation (2.52) implies

$$\mu_i \lambda_r^{(i)} = \mu_j \lambda_r^{(j)} \tag{2.90}$$

which can also be rewritten as

$$\frac{\lambda_r^{(i)}}{\lambda_r^{(j)}} = \frac{\mu_j}{\mu_i} \tag{2.91}$$

Under Assumption I (equation (2.88)) that the ratio of channel eigenvalues for users i and j is different for different dimensions corresponding to different frequencies, then equation (2.91) is true for one and only one frequency f_r. Hence, any pair of users with codeword matrices $\mathbf{S}_i \neq \mathbf{S}_j$ can overlap at most in one frequency at equilibrium.

Note that the theorem can also be proven by using the inverted channel signal space water filling equations for users i and j. That is, assuming both users put power in the same two frequencies results in a relation which contradicts Assumption I. ■

Use of the eigen-algorithm for interference avoidance implies the following property for the fixed point of the interference avoidance algorithm for multiaccess dispersive channels.

Theorem 2.11 : *If two users i and j reside in overlapping subspaces, then the overlap occurs in a minimum gain ratio dimension.*

Proof: Under the assumption of Theorem 2.4 that each user takes full advantage of the signal space dimensionality, i.e. the number of symbols for each user M_k is at least N, it follows from [62] that after convergence of interference avoidance, for any user i the eigenvalue μ_i is the smallest of all the eigenvalues of $\mathbf{R}^{(i)}$. Therefore

$$\mu_i \leq \sigma_r^{(i)} \quad \forall\, r = 1, \ldots, N \tag{2.92}$$

Hence, for any pair of users i and j we can write that

$$\mu_j \leq \frac{\lambda_s^{(i)}}{\lambda_s^{(j)}} \mu_i \quad \forall\, s = 1, \ldots, N \tag{2.93}$$

If users i and j overlap in dimension r then

$$\mu_j = \frac{\lambda_r^{(i)}}{\lambda_r^{(j)}} \mu_i \leq \frac{\lambda_s^{(i)}}{\lambda_s^{(j)}} \mu_i \tag{2.94}$$

Let us denote by

$$g_s(i,j) = \frac{\lambda_s^{(i)}}{\lambda_s^{(j)}} \tag{2.95}$$

the *gain ratio* of user i over user j in dimension s. Then equation (2.94) indicates that overlap occurs in a dimension r which corresponds to a *minimum* gain ratio for user i over user j. Since equation (2.94) can be rewritten as

$$\mu_i = \frac{\lambda_r^{(j)}}{\lambda_r^{(i)}}\mu_j \leq \frac{\lambda_s^{(j)}}{\lambda_s^{(i)}}\mu_j \tag{2.96}$$

the same is true for user j, i.e. dimension r corresponds to *minimum* gain ratio for user j over user i as well. Note that the minimum gain ratio for a given user is taken only over those dimensions spanned by that user. ■

Some remarks in connection with Theorem 2.10 can be made at this point. The fact that in the optimal codeword ensemble two users cannot overlap in more than one frequency generalizes (for a fixed number of users) as the number of signal space dimensions $N \to \infty$ to distinct frequency bands for different users.[6] Such Frequency Division Multiple Access (FDMA) is well-known to maximize the total capacity of multiple access channels with ISI [11].

2.8 Chapter Summary

Interference avoidance methods have been applied to codeword optimization in the uplink of a CDMA system. For a single user, by using a sufficiently long symbol interval so that intersymbol interference is essentially eliminated, along with a sinusoidal basis for the signal space, the problem is cast as a multiuser detection problem for which application of interference avoidance methods to codeword optimization becomes straightforward. We note that the real-valued signal space representation using separate dimensions for sine and cosine of the same frequency introduced in section 2.3 is the natural framework for interference avoidance methods, and that complex-valued representations fail to give meaningful results (see Appendix 2.C).

The use of uniform energy codewords which maximize sum capacity allows matched filters to be used as the optimal linear receivers [84, 85]. In our context this implies a simple receiver structure consisting of a matched filter bank for each user ℓ and identical independent modulation of the M_ℓ symbol streams associated with user ℓ. Such receivers composed of many identical structures might

[6] As the number of dimensions, (implicitly frequencies in our case) that span the signal space increases to infinity, overlap on a set of dimensions of zero measure implies that different users do not overlap at all.

be good candidates for integration. We also note that the outputs of the matched filters might be further processed using various multiuser detection techniques [83]).

For multiple users we show, using the same signal space approach, that interference avoidance also yields optimal ensemble of codewords which maximize sum capacity. Although the results were presented in terms of sequential application for each set of user signatures, or in terms of a global selection of the codeword which will improve sum capacity most at each step, we have found empirically that optimal ensembles are achieved under a variety of codeword update selection methods, including random update. Though this is not *too* surprising given Theorem 2.3, it does bear mentioning since it suggests that interference avoidance in dispersive systems is robust. Also worthy of mention is the observation that empirically, the codewords also seem to converge even though the interference avoidance algorithms presented only guarantee *convergence in class* – where codeword ensemble properties converge, but not necessarily the individual codewords within the ensemble [62]. Next we note that various properties of optimal codeword ensembles are implied from fixed-point properties of interference avoidance algorithms and are derived in the paper[7]. Of particular note, we find that under reasonably loose assumptions on the channel gain matrices, two given users can overlap in at most one frequency. And when taken together with the optimality of matched filter receivers, single frequency overlap can drastically reduce the receiver complexity, an issue which will be further explored in chapter 7.

Finally, results in this chapter suggest that interference avoidance might also be used in related problems where multiple users are constrained to different portions of the signal space, with possible overlap between spaces. Of particular interest are MIMO channels which result when multiple inputs and outputs are used for communications. Example of such channels are multiple antenna systems in which transmitter *and* receiver diversity are used to combat multipath [59] and fading effects [18, 19, 39]. Application of interference avoidance to such channels is presented in detail in Chapter 5.

[7] Some of these properties have also been independently derived by other researchers. The fact that users cannot span the whole signal space and they overlap in at most one frequency was mentioned in [17]. Recent work on multicarrier systems [44] mentions also the property of carrier sharing implied by the relative channel gain ratios.

2.A Proof of Lemma 2.1

Let us denote by $\boldsymbol{\mu}$ the vector of eigenvalues of $\mathbf{V}$ before the replacement, indexed in decreasing order $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_N$, and by $\boldsymbol{\mu}'$ the eigenvalues after replacement, also in decreasing order. Let us also denote by $\boldsymbol{\nu}$ the eigenvalues of $\mathbf{Q}$, indexed in decreasing order as well.

Note that by the spectral factorization theorem [72, p. 296] we have

$$\mathbf{Q} = \sum_{n=1}^{N} \nu_n \mathbf{v}_n \mathbf{v}_n^\top \tag{2.97}$$

where the $\mathbf{v}_n,\ n = 1, \ldots, N$, are the eigenvectors of $\mathbf{Q}$.

After application of the eigen-algorithm for interference avoidance and replacement of $\mathbf{x}$ by $\mathbf{v}_N$, the minimum eigenvector of $\mathbf{Q}$, the resulting matrix can be expressed as

$$\mathbf{V}' = \mathbf{Q} + \mathbf{v}_N \mathbf{v}_N^\top = \sum_{n=1}^{N-1} \nu_n \mathbf{v}_n \mathbf{v}_n^\top + (\nu_N + 1)\mathbf{v}_N \mathbf{v}_N^\top \tag{2.98}$$

Therefore, the eigenvalues of $\mathbf{V}'$ after replacement $\boldsymbol{\mu}'$ will be $\nu_1, \ldots, \nu_{N-1}, \nu_N + 1$. However, they are no longer in decreasing order, since $\nu_N + 1$ may no longer correspond to the smallest eigenvalue, but can appear in the sequence at some index k^*. Hence, the partial sums of eigenvalues of $\mathbf{V}'$ can be written as follows

$$\begin{aligned}
\mu'_1 &= \max(\nu_1, \nu_N + 1) \\
\mu'_1 + \mu'_2 &= \max(\nu_1 + \nu_2, \nu_1 + \nu_N + 1) \\
&\ \vdots \\
\mu'_1 + \mu'_2 + \ldots + \mu'_n &= \max(\nu_1 + \nu_2 + \ldots + \nu_{n-1} + \nu_n, \nu_1 + \nu_2 + \ldots + \nu_{n-1} + \nu_N + 1) \\
&\ \vdots
\end{aligned} \tag{2.99}$$

By using the Rayleigh quotient [72] we can bound partial sums of eigenvalues of matrix $\mathbf{V}$ before replacement as follows. The largest eigenvalue

$$\mu_1 = \max_{\|\mathbf{y}\|=1} \mathbf{y}^\top (\mathbf{Q} + \mathbf{x}\mathbf{x}^\top)\mathbf{y} \tag{2.100}$$

can be lower bounded by

$$\mu_1 \geq \max_{\|\mathbf{y}\|=1} (\mathbf{y}^\top \mathbf{Q} \mathbf{y}) + \min_{\|\mathbf{y}\|=1} (\mathbf{y}^\top \mathbf{x}\mathbf{x}^\top \mathbf{y}) = \nu_1 \tag{2.101}$$

The first term in equation (2.101) is ν_1 (the maximum eigenvalue of $\mathbf{Q}$) while the second[8] is 0. On the other hand, we can also lower bound equation (2.100) by

$$\mu_1 \geq \min_{\|\mathbf{y}\|=1}(\mathbf{y}^\top\mathbf{Q}\mathbf{y}) + \max_{\|\mathbf{y}\|=1}(\mathbf{y}^\top\mathbf{x}\mathbf{x}^\top\mathbf{y}) = \nu_N + 1 \tag{2.102}$$

with the first term corresponding to ν_N (the minimum eigenvalue of $\mathbf{Q}$) and the second term being equal to 1. By combining equations (2.101) and (2.102) we get

$$\mu_1 \geq \max(\nu_1, \nu_N + 1) \tag{2.103}$$

which implies that

$$\mu_1 \geq \mu_1' \tag{2.104}$$

Following [72] we can write the sum of the two largest eigenvalues of $\mathbf{Q}$ before replacement as

$$\mu_1 + \mu_2 = \max_{\|\mathbf{y}_1\|=1, \|\mathbf{y}_2\|=1, \mathbf{y}_2 \perp \mathbf{y}_1} \mathbf{y}_1^\top(\mathbf{Q} + \mathbf{x}\mathbf{x}^\top)\mathbf{y}_1 + \mathbf{y}_2^\top(\mathbf{Q} + \mathbf{x}\mathbf{x}^\top)\mathbf{y}_2 \tag{2.105}$$

Reasoning as before, we can lower bound equation (2.105) by

$$\begin{aligned} \mu_1 + \mu_2 \quad \geq \quad & \max_{\|\mathbf{y}_1\|=1, \|\mathbf{y}_2\|=1, \mathbf{y}_2 \perp \mathbf{y}_1} (\mathbf{y}_1^\top\mathbf{Q}\mathbf{y}_1 + \mathbf{y}_2^\top\mathbf{Q}\mathbf{y}_2) \\ & + \min_{\|\mathbf{y}_1\|=1, \|\mathbf{y}_2\|=1, \mathbf{y}_2 \perp \mathbf{y}_1} (\mathbf{y}_1^\top\mathbf{x}\mathbf{x}^\top\mathbf{y}_1 + \mathbf{y}_2^\top\mathbf{x}\mathbf{x}^\top\mathbf{y}_2) = \nu_1 + \nu_2 \end{aligned} \tag{2.106}$$

with the first term representing the sum of the two largest eigenvalues of $\mathbf{Q}$ and the last term being zero as the sum of two smallest eigenvalues of rank 1 matrix $\mathbf{x}\mathbf{x}^\top$. Also

$$\begin{aligned} \mu_1 + \mu_2 \quad \geq \quad & \max_{\|\mathbf{y}_1\|=1}(\mathbf{y}_1^\top\mathbf{Q}\mathbf{y}_1) + \min_{|\mathbf{y}_2\|=1, \mathbf{y}_2 \perp \mathbf{y}_1}(\mathbf{y}_2^\top\mathbf{Q}\mathbf{y}_2) \\ & + \max_{\|\mathbf{y}_1\|=1, \|\mathbf{y}_2\|=1, \mathbf{y}_2 \perp \mathbf{y}_1} (\mathbf{y}_1^\top\mathbf{x}\mathbf{x}^\top\mathbf{y}_1 + \mathbf{y}_2^\top\mathbf{x}\mathbf{x}^\top\mathbf{y}_2) = \nu_1 + \nu_N + 1 \end{aligned} \tag{2.107}$$

since the first term represents the largest eigenvalue of $\mathbf{Q}$, the second represents the smallest eigenvalue of $\mathbf{Q}$, and the last term is one, as the sum of first two largest eigenvalues of rank 1 matrix $\mathbf{x}\mathbf{x}^\top$. By combining equations (2.106) and (2.107) we get

$$\mu_1 + \mu_2 \geq \max(\nu_1 + \nu_2, \nu_1 + \nu_N + 1) \tag{2.108}$$

[8] $\mathbf{x}\mathbf{x}^\top$ is a matrix of rank 1 with one eigenvalue equal to one and the rest equal to zero.

which implies in a similar way that

$$\mu_1 + \mu_2 \geq \mu_1' + \mu_2' \tag{2.109}$$

Equations (2.104) and (2.109) can be extended to

$$\sum_{i=1}^{n} \mu_i \geq \sum_{i=1}^{n} \mu_i', \quad n = 1, \ldots, N-1 \tag{2.110}$$

For the case $n = N$ we have that

$$\sum_{i=1}^{N} \mu_i = \text{Trace}\left[\mathbf{Q} + \mathbf{x}\mathbf{x}^\top\right] = \text{Trace}\left[\mathbf{Q}\right] + 1 \tag{2.111}$$

and

$$\sum_{i=1}^{N} \mu_i' = \text{Trace}\left[\mathbf{Q} + \mathbf{v}\mathbf{v}^\top\right] = \text{Trace}\left[\mathbf{Q}\right] + 1 \tag{2.112}$$

which implies that

$$\sum_{i=1}^{N} \mu_i = \sum_{i=1}^{N} \mu_i' \tag{2.113}$$

Equations (2.110) and (2.113) imply the desired result

$$\boldsymbol{\mu} \succ \boldsymbol{\mu}' \tag{2.114}$$

i.e. the eigenvalues of $\mathbf{V}$ after replacement $\mu_1', \ldots, \mu_N'$ are majorized [38] by the eigenvalues $\mathbf{V}$ before the replacement $\mu_1, \ldots, \mu_N$. ■

2.B Proof of Theorem 2.6

In order to solve the constrained optimization problem defined by equations (2.58)-(2.60) we use the Lagrange multipliers method and construct

$$J = \frac{1}{2} \sum_{n=1}^{N} \log\left(1 + \frac{1}{\sigma_n} \sum_{\ell=1}^{L} \lambda_n^{(\ell)} \varrho_n^{(\ell)}\right) + \xi_1 \left(\sum_{n=1}^{N} \varrho_n^{(1)} - M_1\right) + \ldots + \xi_L \left(\sum_{n=1}^{N} \varrho_n^{(L)} - M_L\right) \tag{2.115}$$

and then compute the derivative for user i, dimension n, i.e.

$$\frac{\partial J}{\partial \varrho_n^{(i)}} = \frac{1}{2} \cdot \frac{1}{1 + \frac{1}{\sigma_n} \sum_{\ell=1}^{L} \lambda_n^{(\ell)} \varrho_n^{(\ell)}} \cdot \frac{\lambda_n^{(i)}}{\sigma_n} + \xi_i \tag{2.116}$$

According to the Kuhn-Tucker theorem [7, p. 429] the necessary and sufficient conditions for maximizing J are

$$\frac{\partial J}{\partial \varrho_n^{(i)}} \geq 0 \tag{2.117}$$

with equality if $\varrho_n^{(i)} \neq 0$. For those dimensions n for which $\varrho_n^{(i)} \neq 0$ equation (2.116) can be rewritten as

$$\frac{\lambda_n^{(i)}}{\sigma_n + \sum_{\ell=1}^{L} \lambda_n^{(\ell)} \varrho_n^{(\ell)}} + 2\xi_i = 0 \tag{2.118}$$

By taking all the derivatives for $n = 1, \ldots, N$ for user i we get the equalities

$$\frac{\lambda_{n_1}^{(i)}}{\sigma_{n_1} + \sum_{\ell=1}^{L} \lambda_{n_1}^{(\ell)} \varrho_{n_1}^{(\ell)}} = \ldots = \frac{\lambda_{N_i}^{(i)}}{\sigma_{N_i} + \sum_{\ell=1}^{L} \lambda_{N_i}^{(\ell)} \varrho_{N_i}^{(\ell)}} \tag{2.119}$$

for all those dimensions $n = n_1, \ldots, N_i$ corresponding to non-zero $\varrho_n^{(i)}$. This can also be rewritten as

$$\frac{1}{\varrho_{n_1}^{(i)} + \frac{\sigma_{n_1}}{\lambda_{n_1}^{(i)}} + \frac{1}{\lambda_{n_1}^{(i)}} \sum_{\ell=1,\ell\neq i}^{L} \lambda_{n_1}^{(\ell)} \varrho_{n_1}^{(\ell)}} = \ldots = \frac{1}{\varrho_{N_i}^{(i)} + \frac{\sigma_{N_i}}{\lambda_{N_i}^{(i)}} + \frac{1}{\lambda_{N_i}^{(i)}} \sum_{\ell=1,\ell\neq i}^{L} \lambda_{N_i}^{(\ell)} \varrho_{N_i}^{(\ell)}} \tag{2.120}$$

The equalities in equation (2.120) define water filling under channel inversion for user i and they yield the distribution of power along signal space dimensions for user i. Note that equation (2.120) is exactly the classical water filling equation (see for example [15]) for the case when user i has a flat channel, and both noise and interference coming from other users in each signal space dimension is scaled by the corresponding channel eigenvalue of user i. ■

2.C Complex Channel Models and Interference Avoidance

The real channel model introduced in section 2.3 can be easily extended to accommodate complex-valued channels. However, there are serious pitfalls to naive application of the usual complex vector formulation seen typically in the OFDM literature. Furthermore, even more sophisticated application results in difficulties with signal realization.

To proceed, we first note that any signal at the channel input

$$x(t) = \sum_{n=1}^{N} x_{2n-1} \cos(2\pi f_n t) + x_{2n} \sin(2\pi f_n t) \tag{2.121}$$

can be rewritten as

$$x(t) = \mathfrak{Re}\left\{\sum_{n=1}^{N}(x_{2n-1} + jx_{2n})e^{-j2\pi f_n t}\right\} \tag{2.122}$$

where by $\mathfrak{Re}\{\cdot\}$ we have denoted the real part of the complex argument. Using the following vector notation for the real, respectively complex, bases of the signal space

$$\mathbf{\Psi}(t) = \begin{bmatrix} \cos(2\pi f_1 t) \\ \sin(2\pi f_1 t) \\ \vdots \\ \cos(2\pi f_n t) \\ \sin(2\pi f_n t) \\ \vdots \\ \cos(2\pi f_N t) \\ \sin(2\pi f_N t) \end{bmatrix} \in \mathbb{R}^{2N} \qquad \tilde{\mathbf{\Psi}}(t) = \begin{bmatrix} e^{-j2\pi f_1 t} \\ \vdots \\ e^{-j2\pi f_n t} \\ \vdots \\ e^{-j2\pi f_N t} \end{bmatrix} \in \mathbb{C}^N \tag{2.123}$$

and

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{2n-1} \\ x_{2n} \\ \vdots \\ x_{2N-1} \\ x_{2N} \end{bmatrix} \in \mathbb{R}^{2N} \qquad \tilde{\mathbf{x}} = \begin{bmatrix} x_1 + jx_2 \\ \vdots \\ x_{2n-1} + jx_{2n} \\ \vdots \\ x_{2N-1} + jx_{2N} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_n \\ \vdots \\ \tilde{x}_N \end{bmatrix} \in \mathbb{C}^N \tag{2.124}$$

we can rewrite equations (2.121) and (2.122) in vector form as

$$x(t) = \mathbf{\Psi}(t)^\top \mathbf{x} = \mathfrak{Re}\{\tilde{\mathbf{\Psi}}(t)^\top \tilde{\mathbf{x}}\} \tag{2.125}$$

Following the usual representation of signals [27, p. 725] one can regard the N-dimensional complex signal vector $\tilde{\mathbf{x}}$ as the complex equivalent of the $2N$-dimensional real signal vector $\mathbf{x}$ with x_{2n-1} and x_{2n} being the n-th in-phase and quadrature components.

A linear and time invariant dispersive channel characterized by impulse response $h(t)$ or equivalently by transfer function $H(f) = \mathfrak{F}\{h(t)\}$ will introduce amplitude distortion $\lambda_n = |H(f_n)|$ and phase shift $\varphi_n = \arg\{H(f_n)\}$ for all frequencies $n = 1, \ldots, N$. Thus, the output $y(t)$ corresponding to input $x(t)$ is written as

$$\begin{aligned} y(t) &= \sum_{n=1}^{N} \lambda_n x_{2n-1} \cos(2\pi f_n t + \varphi_n) + \lambda_n x_{2n} \sin(2\pi f_n t + \varphi_n) \\ &= \mathfrak{Re}\left\{ \sum_{n=1}^{N} \lambda_n (x_{2n-1} + j x_{2n}) e^{-j2\pi f_n t} e^{j\varphi_n} \right\} \end{aligned} \tag{2.126}$$

By defining the complex channel gains $\tilde{\lambda}_n = \lambda_n e^{j\varphi_n} = H(f_n),\ n = 1, \ldots, N$ and the complex channel gain matrix $\tilde{\mathbf{\Lambda}} = \text{diag}\{\tilde{\lambda}_1, \ldots, \tilde{\lambda}_N\}$ the channel output can be rewritten in complex vector form as

$$\tilde{\mathbf{y}} = \tilde{\mathbf{\Lambda}} \tilde{\mathbf{x}} \tag{2.127}$$

At the receiver, the output signal $y(t)$ is corrupted also by additive Gaussian noise $n(t)$ so that the complex received vector is

$$\tilde{\mathbf{r}} = \tilde{\mathbf{\Lambda}} \tilde{\mathbf{x}} + \tilde{\mathbf{n}} \tag{2.128}$$

with $\tilde{\mathbf{n}}$ containing the "in-phase" and "quadrature" noise components corresponding to $n(t)$.

Now, assume that the input signal $x(t)$ is generated by sending information using the multicode CDMA approach

$$x(t) = \sum_{k=1}^{M} b_k s_k(t) = \sum_{k=1}^{M} b_k \left[\sum_{n=1}^{N} s_{k,2n-1} \cos(2\pi f_n t) + s_{k,2n} \sin(2\pi f_n t) \right] \tag{2.129}$$

In vector form the transmitted signal is written as

$$x(t) = \mathbf{\Psi}(t)^{\top} \mathbf{S} \mathbf{b} \tag{2.130}$$

with $2N \times M$ real codeword matrix $\mathbf{S}$ having as columns the $2N$-dimensional real signal vectors corresponding to the signature waveforms used for transmission. Using the outlined approach the real codeword matrix $\mathbf{S}$ has the complex equivalent $\tilde{\mathbf{S}}$ which is an $N \times M$ complex matrix having as columns the N-dimensional complex signal vectors corresponding to the signature waveforms used for transmission. Thus, the input signal can be rewritten as

$$x(t) = \mathfrak{Re}\{\tilde{\mathbf{\Psi}}(t)^{\top} \tilde{\mathbf{S}} \mathbf{b}\} \tag{2.131}$$

Accordingly, the received signal is equivalent to the complex signal vector given by the matrix equation

$$\tilde{\mathbf{r}} = \tilde{\mathbf{\Lambda}}\tilde{\mathbf{S}}\mathbf{b} + \tilde{\mathbf{n}} \tag{2.132}$$

However, we note that even though equation (2.132) is identical in form to the single user dispersive channel equation (2.18) *the eigen-algorithm cannot be applied.* In illustration, consider the simplest case, $N = 1$. In the real channel model of section 2.3 this corresponds to a 2-dimensional signal space spanned by the sine and cosine functions and for $M = 2$ symbols one can always find two orthogonal signal vectors. However, the complex channel model has only one dimension and it is impossible to find two orthogonal signal vectors so that $M = 2$ symbols will not interfere with each other during the transmission. Fixing this problem would require awkward redefinition of inner products and orthogonality which seems not worth pursuing.

Alternatively, one can represent equation (2.121) using the complex Fourier series approach as

$$x(t) = \sum_{n=1}^{N} \left[\frac{1}{2}(x_{2n-1} + jx_{2n})e^{-j2\pi f_n t} + \frac{1}{2}(x_{2n-1} - jx_{2n})e^{j2\pi f_n t} \right] \tag{2.133}$$

which results in a $2N$-dimensional complex signal space representation spanned by $\tilde{\mathbf{\Psi}}(t)$ with an equivalent $2N$-dimensional complex codeword vector $\tilde{\mathbf{x}}$

$$\tilde{\mathbf{\Psi}}(t) = \begin{bmatrix} e^{-j2\pi f_1 t} \\ e^{j2\pi f_1 t} \\ \vdots \\ e^{-j2\pi f_n t} \\ e^{j2\pi f_n t} \\ \vdots \\ e^{-j2\pi f_N t} \\ e^{j2\pi f_N t} \end{bmatrix} \in \mathbb{C}^{2N} \qquad \tilde{\mathbf{x}} = \begin{bmatrix} \frac{1}{2}(x_1 + jx_2) \\ \frac{1}{2}(x_1 - jx_2) \\ \vdots \\ \frac{1}{2}(x_{2n-1} + jx_{2n}) \\ \frac{1}{2}(x_{2n-1} - jx_{2n}) \\ \vdots \\ \frac{1}{2}(x_{2N-1} + jx_{2N}) \\ \frac{1}{2}(x_{2N-1} - jx_{2N}) \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_1^* \\ \vdots \\ \tilde{x}_n \\ \tilde{x}_n^* \\ \vdots \\ \tilde{x}_N \\ \tilde{x}_N^* \end{bmatrix} \in \mathbb{C}^{2N} \tag{2.134}$$

where z^* denotes the complex conjugate of z. We note the complex conjugate symmetry in complex signal vector $\tilde{\mathbf{x}}$ which is implied by the real signal $x(t)$.

In this approach, the response of a linear and time invariant channel characterized by the same transfer function $H(f)$ due to input signal $x(t)$ is

$$
\begin{aligned}
y(t) &= \sum_{n=1}^{N} \lambda_n x_{2n-1} \cos(2\pi f_n t + \varphi_n) + \lambda_n x_{2n} \sin(2\pi f_n t + \varphi_n) \\
&= \sum_{n=1}^{N} \left[\frac{1}{2}(x_{2n-1} + jx_{2n})\lambda_n e^{-j2\pi f_n t} e^{j\varphi_n} + \frac{1}{2}(x_{2n-1} - jx_{2n})\lambda_n e^{j2\pi f_n t} e^{j\varphi_n} \right]
\end{aligned}
\tag{2.135}
$$

With the same complex channel gain definition $\tilde{\lambda}_n = \lambda_n e^{j\varphi_n} = H(f_n)$ the channel output can be written in the vector notation

$$\tilde{\mathbf{y}} = \tilde{\mathbf{\Lambda}} \tilde{\mathbf{x}} \tag{2.136}$$

with complex gains in conjugate pairs

$$\tilde{\mathbf{\Lambda}} = \mathrm{diag}\{\tilde{\lambda}_1, \tilde{\lambda}_1^*, \ldots, \tilde{\lambda}_N, \tilde{\lambda}_N^*\}$$

With the input signal generated according to the multicode CDMA equation (2.129) the complex received signal vector will be given a matrix equation identical in form to equation (2.132) but in which the complex codeword matrix $\tilde{\mathbf{S}}$ is of dimension $2N \times M$, with each column having the complex conjugate symmetry property implied by real signature waveforms. We note that even though application of interference avoidance is possible in this case and would result in optimal codeword ensembles, the codewords can correspond to non-realizable signature waveforms since there is no guarantee in general that minimum eigenvectors of Hermitian matrices display the complex conjugate symmetry necessary for real signals.

Thus, the sine/cosine formulation used in this chapter is the natural one for application of interference avoidance to dispersive systems.

Chapter 3

Interference Avoidance and Fading Channels

Application of interference avoidance to dispersive channels has been presented in detail in Chapter 2 where it has been shown that interference avoidance algorithms yield codeword ensembles which maximize sum capacity. However, the time-varying nature of fading and multipath environments characteristic of wireless channels introduces additional challenges. As time variations appear to be unpredictable, wireless channels have randomly time variant impulse responses and are characterized statistically – which further implies that the sum capacity as defined in Chapter 2 will also be a random variable.

In a survey paper on fading channels [5] it was emphasized that capacity is the most important information-theoretic measure for characterization of communication over fading channels, although other information-theoretic measures like error exponents and cutoff rates are also used in the literature. However, as pointed out in [19], determining capacity for fading channels is still an open research topic. Specifically, the uncertainty of the channel realization introduces another unknown so that relatively recent results (e.g., [21]) only provide an upper bound for capacity since complete channel information is assumed. Furthermore, the extent to which such information is accurate can be a critical consideration.

In [19] capacity was considered as a random variable and Complementary Cumulative Distribution Functions (CCDFs) were found. Such CCDFs allow outage probability evaluation for given signaling schemes and fading channel models. In [41] a different approach was taken by connecting time variation of the channel with an error in channel measurement. This connection is reasonable since a time-invariant channel can be measured with arbitrary precision without rate penalty, while stochastic time variation implies that extra information (the channel state) must also be conveyed – resulting in a rate penalty. In this framework, the effect of imperfect knowledge of the channel on mutual information is analyzed, and lower and upper bounds on channel capacity are established.

It is also shown that as the measurement error vanishes these bounds become tight.

In this chapter we analyze sum capacity of multiaccess fading channels in the context of interference avoidance algorithms. In the case of quasistatic fading channels with large coherence times, channel parameters can be estimated and are assumed to be known for the duration of the transmission. Then, interference avoidance algorithms can be applied and optimal codeword ensembles that maximize sum capacity can be computed and used for transmitting information. From time to time, codewords are eventually updated according to variations of the channel.

When the coherence time of the channel is small, implying a dynamic channel, it may not be possible to estimate channel characteristics and perform interference avoidance for each and every channel realization. Specifically, by the time the channel has been estimated sufficiently well and new optimal codewords calculated, the channel may have changed to a new realization. A more realistic approach in this case is to use the average characteristics of the channel and compute codeword ensembles that are optimal for the average channel. These codeword ensembles would be used for the duration of the transmission, regardless of the intervening channel realizations.

To evaluate the efficacy of choosing codewords for the average channel as compared to choosing random codewords or optimal codewords for each channel instance, we treat sum capacity as a random variable and find CCDFs via Monte Carlo simulation similar to those in [19]. Based on the sum capacity CCDFs, the performance of random codeword ensembles, codeword ensembles optimal for the average channel, and codeword ensembles optimal for each channel realization are compared. This enables us to determine the performance improvement to be had over random codewords if we choose codewords for the average channel or optimally for each channel instance.

3.1 Fading Channel Models and Interference Avoidance

In this section we show how application of interference avoidance can be extended from dispersive channels, which are deterministic and time-invariant, to fading channels, which are time-varying with random realizations. We concentrate our attention on indoor wireless channels for which extensive studies have been performed (see [26] and references therein). An analysis of a multicarrier CDMA scheme, similar to the one used in Chapter 2, for indoor wireless channels can also be found in [92][1].

Similar to [92] we assume a frequency selective fading channel model with flat fading of the carriers. In a Rayleigh fading environment the amplitude scaling $\kappa_n^{(\ell)}$ of the n-th carrier due to the channel between user ℓ and basestation is a Rayleigh random variable with the probability density function

$$f_{\kappa_n^{(\ell)}}(\kappa_n^{(\ell)}) = \frac{\kappa_n^{(\ell)}}{\sigma_n^{(\ell)^2}} e^{-\frac{\kappa_n^{(\ell)^2}}{2\sigma_n^{(\ell)^2}}} \tag{3.1}$$

where the parameter $\sigma_n^{(\ell)^2}$ is related to the second moment of the Rayleigh random variable $E[\kappa_n^{(\ell)^2}] = 2\sigma_n^{(\ell)^2}$. The second moment of this random variable characterizes the average channel and represents the eigenvalue corresponding to carrier n of user ℓ average channel, i.e. $E[\kappa_n^{(\ell)^2}] = 2\sigma_n^{(\ell)^2} = \lambda_n^{(\ell)}$.

3.1.1 Slowly Fading Channels

A slowly fading channel has a large coherence time T_c, or equivalently a small Doppler spread [55]. This is a measure of the channel's temporal variation and for residential buildings or office environments with reduced mobility one can assume that a channel is stable for a few seconds [26]. For high data rates this implies that the channel is stable for a large number of symbols thus allowing identification of channel parameters. In our case these are the amplitude scalings of different carriers (or channel eigenvalues) and can be determined by probing the channel with appropriate pilot symbols [43].

To apply interference avoidance algorithms, an estimate of the received signal autocorrelation matrix must be obtained in addition to channel identification. By subtracting the contribution of a given symbol/user the autocorrelation matrix of the corresponding interference-plus-noise process needed by the minimum eigenvector algorithm is then computed. This problem has been studied extensively in [73] where it has been shown that the minimum number of symbol intervals needed for accurate estimation of the autocorrelation matrix of the received signal depends on dimensionality (number of carriers in our case) and on the number of codewords/users in the system. The use of

[1] As previously mentioned, though applied via a multicarrier modulation scheme, interference avoidance methods are general. An example of applying interference avoidance to discrete time fading channel models is provided in Appendix 3.A.

training sequences to speed up estimation is also presented in [73].

In order for interference avoidance to be efficient, the overhead associated with identification, estimation, and application of the interference avoidance algorithm, should be small compared to the actual number of information symbols sent. A symbol duration $\mathcal{T} = 2\mu$s (which corresponds to 0.5 Msymbols/s and also satisfies the condition imposed by the multicarrier modulation scheme[2]) implies that approximately 1,000,000 symbols can be transmitted in 2 s, which is of the order of the channel coherence time. With a 10% overhead factor this implies that up to 100,000 symbols may be used for estimation, which seems to cover a lot of potential combinations of dimensions/users/codewords [73].

In conclusion, for slowly fading indoor channels, it may be possible to apply interference avoidance in a straightforward way. After channel eigenvalues are determined, interference avoidance algorithms are applied to compute optimal codeword ensembles that maximize sum capacity. These codeword ensembles are then used for transmission, with periodic updates coupled to channel variation.

3.1.2 Interference Avoidance for the Average Channel

Now suppose the channel is dynamic and cannot be estimated rapidly enough to satisfy the quasistatic conditions necessary to apply interference avoidance for each channel instance. That is, suppose that by the time the channel is estimated and the codewords calculated, the channel has already changed to a new realization.

In this case only average characteristics of the channel can be measured and we seek to determine the performance of applying interference avoidance techniques to the average channel. For the multicarrier modulation scheme proposed, with a frequency selective fading channel model, we use the average values of the gains in equation (3.1) to perform interference avoidance and determine optimal codeword ensembles for the average channel. Let us denote the ensemble of user codeword matrices by $\mathcal{S} = \{\mathbf{S}_1, \ldots, \mathbf{S}_L\}$ and the ensemble of channel eigenvalue matrices by $\mathbf{\Lambda} = \{\mathbf{\Lambda}_1, \ldots, \mathbf{\Lambda}_L\}$. Then, for a given ensemble of codeword and channel eigenvalue matrices the

[2] $\mathcal{T}$ must be larger than the duration of all channel impulse responses. In the case of multipath fading channels this is equivalent to the duration of the multipath intensity profile which for indoor environments (medium size office or residential buildings) is of the order of 400 ns [26].

sum capacity is obtained from equation (2.39) as

$$C_{\text{sum}}(\mathbf{\Lambda}, \mathcal{S}) = \frac{1}{2}\log\left[\det\left(\sum_{\ell=1}^{L}\mathbf{\Lambda}_{\ell}^{1/2}\mathbf{S}_{\ell}\mathbf{S}_{\ell}^{\top}\mathbf{\Lambda}_{\ell}^{1/2} + \mathbf{W}\right)\right] - \frac{1}{2}\log\left(\det\mathbf{W}\right) \tag{3.2}$$

For a fixed ensemble of codeword matrices $\mathcal{S}$, due to the randomness of channel realizations $\mathbf{\Lambda}$, the resulting sum capacity is a random variable. Using Jensen's inequality [15, p. 25] and the fact that the function $\log(\det\mathbf{A})$ is concave for positive semidefinite matrices $\mathbf{A}$ [31, p. 466] we can write that

$$E_{\mathbf{\Lambda}}[C_{\text{sum}}(\mathbf{\Lambda}, \mathcal{S})] \leq C_{\text{sum}}(\bar{\mathbf{\Lambda}}, \mathcal{S}) \tag{3.3}$$

where $\bar{\mathbf{\Lambda}}$ denotes the average value of the channel eigenvalue matrices ensemble. Moreover, for any ensemble of user codeword matrices $\mathcal{S}$ we can also write that

$$C_{\text{sum}}(\bar{\mathbf{\Lambda}}, \mathcal{S}) \leq C_{\text{sum}}(\bar{\mathbf{\Lambda}}, \mathcal{S}^*) \tag{3.4}$$

where

$$\mathcal{S}^* = \arg\max_{\mathcal{S}} C_{\text{sum}}(\bar{\mathbf{\Lambda}}, \mathcal{S}) \tag{3.5}$$

is the ensemble of codeword matrices that maximizes the sum capacity corresponding to the ensemble of average channel eigenvalue matrices $\bar{\mathbf{\Lambda}}$, and is obtained by applying interference avoidance algorithms with average values of channel parameters. ensemble of codewords.

We note that the codeword ensemble which is optimal for the average channel is

$$\mathcal{S}^\dagger = \arg\max_{\mathcal{S}} E_{\mathbf{\Lambda}}[C_{\text{sum}}(\mathbf{\Lambda}, \mathcal{S})] \tag{3.6}$$

and that, although $\mathcal{S}^*$ is not actually the codeword ensemble optimal for the average channel, empirically we have seen improvements when using $\mathcal{S}^*$. The improvements can be observed by looking at sum capacity CCDFs, which show what capacity can be achieved with a given probability for a given ensemble of codewords. An outage occurs whenever capacity is below a given value, and the probability of outage P_{out} can be identified from the corresponding CCDF.

The sum capacity CCDFs presented in figures 3.1 and 3.2, which are typical for the experiments we have performed, correspond to a system using $N = 10$ carriers, with $L = 2$ users, and white noise at the basestation receiver with a signal-to-noise ratio of 10 dB. A set of 1,000 randomly chosen codeword ensembles have been generated and 10,000 realizations of Rayleigh fading channels

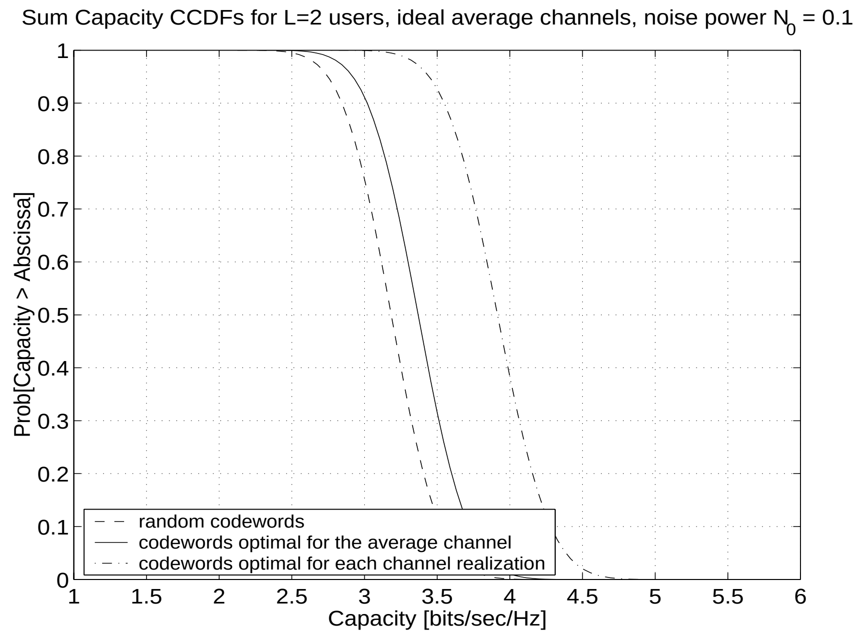

Figure 3.1: Sum Capacity CCDFs for multiaccess fading channels comparing random codeword ensembles with codeword ensembles optimal for the average channel and codeword ensembles optimal for each channel realization. Average channels are assumed to be ideal.

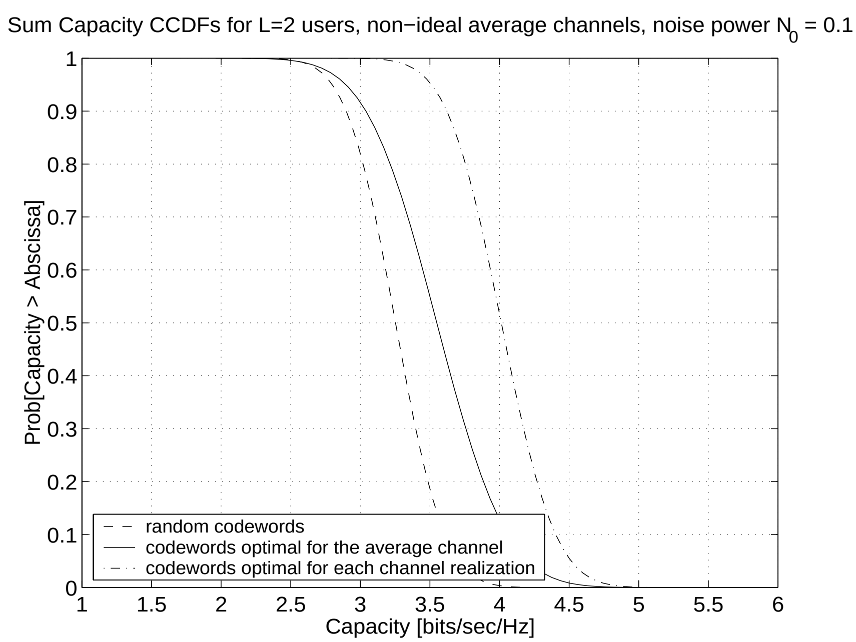

Figure 3.2: Sum Capacity CCDFs for multiaccess fading channels comparing random codeword ensembles with codeword ensembles optimal for the average channel and codeword ensembles optimal for each channel realization. Average channels are non-ideal.

corresponding to the two users have been considered. Codeword ensembles optimal for the average channel and codeword ensembles optimal for each channel realization were also computed. In figure 3.1 the average channel was considered ideal, with all channel eigenvalues equal to 1, which implies that second moments of Rayleigh random variables $\kappa_n^{(\ell)^2}$ in equation (3.1) are also equal to 1. In the case of a non-ideal average channel, channel eigenvalues can be obtained from the frequency spectrum corresponding to the average channel. For the CCDFs presented in figure 3.2 a very general case of non-ideal average channel was considered, with channels eigenvalues uniformly distributed between 0.5 and 1.5 corresponding to an average channel that attenuates some frequencies and boosts others.

From both figures (3.1 and 3.2) we note that for the same probability of outage the sum capacity when using codeword ensembles yielded by application of interference avoidance for the average channel is always larger than the corresponding value for random codeword ensembles. Of course, the best performance is obtained with codeword ensembles optimal for each channel realization, but under rapid channel variation this may not always be possible. Thus, our simulation results confirm that using codeword ensembles tuned to the average channel is better than using random codewords but worse than using per-channel codewords. However, we also note that the capacity gain obtained from optimal codeword ensembles for each channel realization as compared to that obtained using average channel optimal codewords might not justify the computational burden of doing interference avoidance for each channel realization. Thus, selecting codeword sets optimal for the average channel may be a reasonable compromise.

3.2 Chapter Summary

Application of interference avoidance in the context of fading channels was analyzed in this chapter. A frequency selective channel model was assumed which is suited for the multicarrier modulation scheme proposed. In the case of slowly fading the channels which are assumed to be known and stable for the whole duration of the transmission, application of interference avoidance is straightforward. Otherwise, if channels variation is so rapid that optimal codeword ensembles cannot be computed before channels change, then interference avoidance can be used to compute codeword ensembles with average channel paramenters and still realize a performance improvement. Examples of sum capacity CCDFs obtained via Monte Carlo simulation are also presented.

3.A Discrete-Time Fading Channel Models

We have previously used a multicarrier formulation for interference avoidance. To show generality, here we provide an explicit formulation for interference avoidance using discrete time channel models.

Let us consider the discrete-time multiple access channel model characterizing the uplink of a system with L users communicating with a common receiver (basestation)

$$r(n) = \sum_{\ell=1}^{L} \sqrt{h_\ell(n)}x_\ell(n) + w(n) \tag{3.7}$$

where $x_\ell(n)$ and $h_\ell(n)$ are the transmitted signal and the fading process of the ℓ^{th} user respectively, and $w(n)$ is additive Gaussian noise that corrupts the received signal. This is a basic frequency non-selective multi-access fading channel model also used in [19, 74]. By stacking together values of the received signal $r(n)$ in equation (3.7) for $n = 0, \dots, N-1$ corresponding to an observation interval, we obtain the vector-matrix equation

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell^{1/2}\mathbf{x}_\ell + \mathbf{w} \tag{3.8}$$

with $\mathbf{r} = [r(0) \dots r(N-1)]^\top$, $\mathbf{H} = \text{diag}\{h_\ell(0), \dots, h_\ell(N-1)\}$, $\mathbf{x}_\ell = [x_\ell(0) \dots x_\ell(N-1)]^\top$, and $\mathbf{w} = [w(0) \dots w(N-1)]^\top$. Equation (3.8) can be further written as

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell^{1/2}\mathbf{S}_\ell\mathbf{b}_\ell + \mathbf{w} \tag{3.9}$$

by assuming that transmitted waveforms are obtained from

$$\mathbf{x}_\ell = \mathbf{S}_\ell\mathbf{b}_\ell \qquad \ell = 1, \dots, L \tag{3.10}$$

where $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^\top$ is the symbol sequence to be transmitted by user ℓ through a codeword matrix $\mathbf{S}_\ell$. Note that equation (3.10) can be regarded as a spreading scheme in which each symbol in $\mathbf{b}_\ell$ is "spread" over N "time chips".

Equation (3.9) is identical in form with the multiaccess dispersive channels equation (2.31). There are however two main differences:

- The spreading scheme in equation (3.10) implies a time representation of waveforms as superposition of "time chips" specified by the corresponding codewords as opposed to the dispersive channels case where waveforms were represented in frequency as a superposition of sinusoids.

- Channels are described by random matrices containing values of the fading processes (channel states) as opposed to the dispersive channels case where they were described by channel eigenvalues.

Nevertheless, by working with average values for $\{\mathbf{H}_\ell\}$ matrices one can directly apply the eigen-algorithm for interference avoidance and derive an ensemble of optimal codeword matrices $\{\mathbf{S}_\ell\}$ for the average channel.

It is also worth noting here that the signal space partitioning induced by the optimal codeword ensemble presented in chapter 2 leads to a time division multiple access (TDMA) scheme, in which users transmit information in distinct time slots with possible overlaps in at most one slot, using time slots with best channel states. Similar conclusions with regard to codeword ensembles apply for this case as well.

Chapter 4

Interference Avoidance and Multiaccess Vector Channels

Vector channels arise in communication systems as a consequence of representing a continuous time (waveform) channel in a finite dimensional signal space [20, Ch. 8]. The study of vector channels is important since it provides a theoretical framework for the analysis of a wide variety of communication channels such as multiple access channels, channels with memory, or channels with multiple antennas in the transmitter and/or receiver, and has received increasing attention from the research community lately.

Results on capacity of multiaccess vector channels have been derived in [87], and an asymptotically optimal water filling algorithm for multiaccess vector channels can be found in [86]. A characterization of the capacity region for multiaccess vector channels and an iterative water filling algorithm to evaluate optimal transmit spectra that maximize the sum capacity of the channel can be found in [94].

CDMA schemes have a natural vector channel representation implied by the signature sequences (or codewords) corresponding to distinct users in the system. In this context, optimal signature sequences (or codeword ensembles) that maximize sum capacity of a multiaccess channel are analyzed in [84], and an algorithm for obtaining such optimal sequences is also presented. We have seen in the previous chapters that such optimal codeword ensembles which maximize sum capacity can also be obtained through application of interference avoidance methods.

In this chapter we discuss application of interference avoidance to multiaccess vector channels. Using a general multiaccess vector channel model along with a multicode CDMA scheme for transmission of information, we generalize application of interference avoidance methods. We first show that application of greedy interference avoidance for any codeword/user is guaranteed not to decrease sum capacity, and then generalize the eigen-algorithm showing that it is equivalent to iterative water filling [94]. Thus, application of the eigen-algorithm will always yield an optimal ensemble of codewords that maximizes the sum capacity of the multiaccess vector channel. This result clears the way for application of interference avoidance methods to other communication problems in which the underlying model is a multiaccess vector channel.

4.1 The Vector Multiple Access Channel

In a very broad sense, a single user vector channel is defined by [94]

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{4.1}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the input and output vectors of dimension N_x, respectively N_y, $\mathbf{n}$ is the additive noise vector, and $\mathbf{H}$ is the $N_y \times N_x$ channel matrix. The vector channel defined in equation (4.1) represents a linear transformation from an input signal space of dimension N_x to an output signal space of dimension N_y and the channel matrix $\mathbf{H}$ can be viewed as relating the bases of the input and output signal spaces [72, p. 116]. Note that the channel is considered memoryless, but a similar model applies to channels with memory[1].

Extending the definition in equation (4.1) to multiple users, a multiaccess vector channel is obtained in which a set of L users communicate with a common receiver (basestation). The multiaccess vector channel is defined by the equation

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} \tag{4.2}$$

where $\mathbf{x}_\ell$ of dimension N_ℓ is the input vector corresponding to user ℓ, $\ell = 1, \ldots, L$, $\mathbf{r}$ of dimension N is the received vector at the common receiver corrupted by additive noise vector $\mathbf{n}$ of the same dimension, and $\mathbf{H}_\ell$ is the $N \times N_\ell$ channel matrix corresponding to user ℓ. It is assumed that $N \geq N_\ell, \forall \ell = 1, \ldots, L$. This is a general approach to a multiuser communication system in which different users reside in different signal spaces, with different dimensions and potential overlap between them, and all being subspaces of the receiver signal space. We note that each user's signal space as well as the receiver signal space are of finite dimension and are implied by a finite time interval $\mathcal{T}$ and finite bandwidths W_ℓ for each user ℓ, respectively and W (which includes all W_ℓ's corresponding to all users) for the receiver [34]. As has already been mentioned, for memoryless

[1] We have already seen an example in Chapter 2. Additional examples can be found in section 4.3 and [94].

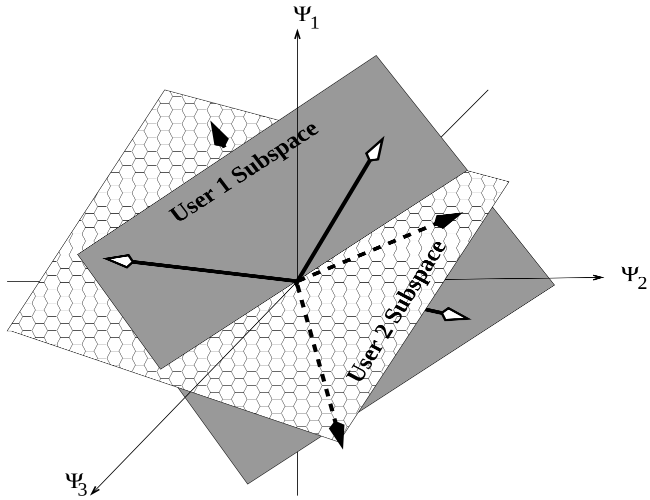

Figure 4.1: 3-dimensional receiver signal space with 2 users residing in 2-dimensional subspaces. Vectors represent particular signals in user 1 (continuous line), respectively user 2 (dashed line) signal spaces.

channels, the channel matrix $\mathbf{H}_\ell$ merely relates the bases of user ℓ's signal space and receiver signal space, but a similar model applies to channels with memory in which case the channel matrix $\mathbf{H}_\ell$ also incorporates channel attenuation and multipath. Figure 4.1 provides a graphical illustration of such a signal space configuration for 2 users residing in 2-dimensional subspaces with a 3-dimensional receiver signal space.

In this signal space setting we assume that in the finite time interval of duration $\mathcal{T}$ each user ℓ sends a "frame" of data using a multicode CDMA approach similar to the one used in Chapter 2 (see Figure 2.1). Each symbol is transmitted using a distinct signature waveform which spans the frame that is, the sequence $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^\top$ is transmitted as a linear superposition of distinct, unit-energy waveforms $s_m^{(\ell)}(t)$ as in

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t) \tag{4.3}$$

In the N_ℓ-dimensional signal space corresponding to user ℓ, each waveform can be represented as an N_ℓ-dimensional vector, thus the input vector $\mathbf{x}_\ell$ corresponding to user ℓ is equivalent to a linear superposition of unit norm codeword column vectors $\mathbf{s}_m^{(\ell)}$ scaled by the corresponding $b_m^{(\ell)}$. That is, each user uses an $N_\ell \times M_\ell$ codeword matrix $\mathbf{S}_\ell$

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \cdots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \tag{4.4}$$

so that

$$\mathbf{x}_\ell = \mathbf{S}_\ell \mathbf{b}_\ell \tag{4.5}$$

Therefore, the received signal at the common receiver can be rewritten as

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{4.6}$$

Note that under the assumption that $M_\ell \geq N_\ell$ the $N_\ell \times N_\ell$ covariance matrix of $\mathbf{x}_\ell$ defined as $\mathbf{X}_\ell = E[\mathbf{x}_\ell \mathbf{x}_\ell^\top] = \mathbf{S}_\ell \mathbf{S}_\ell^\top$ has full rank and spans user ℓ signal space.

It is well known that when the noise signal is Gaussian, the mutual information is maximized when the information signals $x_\ell(t)$ are jointly Gaussian [15, p. 405]. All that remains is to determine the exact spectral composition of each $x_\ell(t)$. The capacity region for the multiaccess vector channel defined in equation (4.2) has been established in [94]. In the same paper [94], maximization of sum capacity for the multiaccess vector channel (4.2)

$$C_s = \frac{1}{2} \log \left[\det \left(\sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{X}_\ell \mathbf{H}_\ell^\top + \mathbf{W} \right) \right] - \frac{1}{2} \log(\det \mathbf{W}) \tag{4.7}$$

is formulated as a convex optimization problem

$$\max_{\mathbf{X}_\ell} C_s \quad \text{subject to} \quad \text{Trace}\,[\mathbf{X}_\ell] = P_\ell, \quad \mathbf{X}_\ell \geq 0, \quad \ell = 1, \ldots, L \tag{4.8}$$

and it is shown that optimal transmit covariance matrices $\mathbf{X}_\ell$, $\ell = 1, \ldots, L$ satisfy a simultaneous water filling condition and can be found through an iterative water filling procedure.

4.2 Sum Capacity Maximization and Interference Avoidance

Since in our approach transmit covariance matrices are expressed in terms of user codeword matrices as $\mathbf{X}_\ell = \mathbf{S}_\ell \mathbf{S}_\ell^\top$, $\ell = 1, \ldots, L$ sum capacity can be written as

$$C_s = \frac{1}{2} \log \left[\det \left(\sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \right) \right] - \frac{1}{2} \log(\det \mathbf{W}) \tag{4.9}$$

Furthermore, in the context of interference avoidance we are interested in sum capacity maximization through codeword adaptation. Therefore, in our case the problem of maximizing sum capacity has a slighlty different formulation than that in [94] which is mentioned in equation (4.8). More precisely, in our case we are interested in finding the codeword ensemble which maximizes sum capacity, that is

$$\max_{\mathbf{S}_\ell} C_s \quad \text{subject to} \quad \text{Trace}\left[\mathbf{S}_\ell \mathbf{S}_\ell^\top\right] = M_\ell, \quad \ell = 1, \ldots, L \tag{4.10}$$

An additional constraint in our case is given by the fact that all matrices $\mathbf{S}_\ell$ have unit norm columns.

We note that, the original framework for interference avoidance presented in Chapter 1 and introduced in [63], assumes an arbitrary N-dimensional signal space for the receiver *and* all users in the multiuser system, as opposed to the multiuser system in equation (4.2) where different users are allowed to reside in different signal spaces. With a CDMA access scheme each user ℓ is assigned a finite duration unit-energy signature waveform $S_\ell(t)$, or equivalently a unit norm N-dimensional codeword $\mathbf{s}_\ell$, to convey a symbol, b_ℓ. In this setting, the received signal vector at the common receiver is described by equations (1.2) or (1.4). As it has already been noted in Chapter 1, in a colored noise background the eigen-algorithm[2] for interference avoidance converges to an optimal fixed point where the resulting codeword ensemble water fills the signal space and maximizes sum capacity defined in equation (1.23). In this case, by observing that the input covariance matrix of user ℓ is a rank one matrix [93] $\mathbf{X}_\ell = \mathbf{s}_\ell \mathbf{s}_\ell^\top$, the problem of finding the optimal codeword ensemble that maximizes sum capacity in equation (1.23) can also be formulated as a spectral optimization problem. More precisely, the $\mathbf{X}_\ell$ are the solution of the following optimization problem:

$$\text{maximize} \quad \frac{1}{2}\log\left[\det\left(\sum_{\ell=1}^{L}\mathbf{X}_\ell + \mathbf{W}\right)\right] - \frac{1}{2}\log(\det \mathbf{W}) \tag{4.11}$$

$$\text{subject to} \quad \begin{cases} \text{Trace}\left[\mathbf{X}_\ell\right] = 1 & \\ \mathbf{X}_\ell \geq 0 & \ell = 1, \ldots, L \\ \text{rank}(\mathbf{X}_\ell) = 1 & \end{cases} \tag{4.12}$$

However, because the rank constraint is not a convex constraint, the optimization problem defined in equation (4.11) subject to the constraints in equation (4.12) does not enjoy the usual global

[2]Augumented eventually with the "class warfare" procedure [62]

convergence properties of convex optimization problems. Thus, suboptimal fixed points of the eigen-algorithm for interference avoidance are theoretically possible, even though they have never been observed in practice when starting with a randomly chosen codeword ensemble [63]. Suboptimal fixed points of the eigen-algorithm for interference avoidance have been analyzed in detail in [62] where it has been shown that the eigen-algorithm augmented with a so-called “class warfare” procedure escapes suboptimal points and always reaches the optimal fixed point for which sum capacity is maximized.

Returning to our general sum capacity maximization problem in equation (4.10) we note that this is a convex optimization problem only when each user has at least as many codewords M_ℓ as signal space dimensions N_ℓ, so that its corresponding transmit covariance matrix $\mathbf{X}_\ell$ is allowed to span the whole signal space. We also note that sum capacity maximization subject to additional constraints on the rank of $\mathbf{X}_\ell$ matrices is still an open research problem.

The fact that algorithms based on greedy interference avoidance have been shown to yield codeword ensembles that maximize sum capacity for dispersive channels – a particular case of the general multiaccess vector channel in equation (4.6) – suggests that greedy interference avoidance may also be used in the general context of multiaccess vector channels to obtain such optimal codeword ensembles. Since the second term in the sum capacity expression in equation (4.9) is fixed one needs only to maximize the determinant of the received signal covariance matrix

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \tag{4.13}$$

In order to establish application of the eigen-algorithm for interference avoidance let us rewrite the received signal in equation (4.6) from the perspective of user k

$$\mathbf{r} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \sum_{\ell=1, \ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \mathbf{z}_k \tag{4.14}$$

where $\mathbf{z}_k$ represents the interference-plus-noise seen by user k

$$\mathbf{z}_k = \sum_{\ell=1, \ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{4.15}$$

with covariance matrix

$$\mathbf{Z}_k = E[\mathbf{z}_k \mathbf{z}_k^\top] = \sum_{\ell=1, \ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \tag{4.16}$$

Since $\mathbf{Z}_k$ is symmetric it can be diagonalized

$$\mathbf{Z}_k = \mathbf{E}_k \mathbf{\Delta}_k \mathbf{E}_k^\top \tag{4.17}$$

Furthermore, because $\mathbf{Z}_k$ is a positive definite covariance matrix we can define the whitening transformation

$$\mathbf{T}_k = \mathbf{\Delta}_k^{-1/2} \mathbf{E}_k^\top \tag{4.18}$$

such that, in transformed coordinates equation (4.14) is equivalent to

$$\tilde{\mathbf{r}} = \mathbf{T}_k \mathbf{r} = \mathbf{T}_k \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \mathbf{T}_k \mathbf{z}_k = \tilde{\mathbf{H}}_k \mathbf{S}_k \mathbf{b}_k + \mathbf{w}_k \tag{4.19}$$

where $\tilde{\mathbf{H}}_k = \mathbf{T}_k \mathbf{H}_k$ is the channel matrix seen by user k in the new coordinates and $\mathbf{w}_k = \mathbf{T}_k \mathbf{z}_k$ is the equivalent "white noise" with covariance matrix $E[\mathbf{w}_k \mathbf{w}_k^\top] = \mathbf{T}_k \mathbf{Z}_k \mathbf{T}_k^\top = \mathbf{I}$ equal to the identity matrix. We note that the received signal covariance matrix in the transformed coordinates is related to the original signal covariance matrix by the equation

$$\tilde{\mathbf{R}} = E[\tilde{\mathbf{r}} \tilde{\mathbf{r}}^\top] = \mathbf{T}_k \mathbf{R} \mathbf{T}_k^\top \tag{4.20}$$

and any procedure that attempts to maximize $\det \tilde{\mathbf{R}}$ through adaptation of user k codeword matrix $\mathbf{S}_k$ will also maximize $\det \mathbf{R}$ since they are related by

$$\det \tilde{\mathbf{R}} = \det(\mathbf{T}_k \mathbf{R} \mathbf{T}_k^\top) = \det \mathbf{R} (\det \mathbf{T}_k)^2 \tag{4.21}$$

We now apply the singular value decomposition (SVD) [72, p. 442] to the transformed channel matrix corresponding to user k

$$\tilde{\mathbf{H}}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^\top \tag{4.22}$$

where matrix $\mathbf{U}_k$ of dimension $N \times N$ has as columns the eigenvectors of $\tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^\top$, matrix $\mathbf{V}_k$ of dimension $N_k \times N_k$ has as columns the eigenvectors of $\tilde{\mathbf{H}}_k^\top \tilde{\mathbf{H}}_k$, and matrix $\mathbf{D}_k$ of dimension $N \times N_k$ contains the singular values of $\tilde{\mathbf{H}}_k$ on the main diagonal and zero elsewhere. We note that because $\mathbf{T}_k$ is invertible the rank of $\tilde{\mathbf{H}}_k$ will be equal to that of $\mathbf{H}_k$. Without loss of generality we assume that $\mathbf{H}_k$ has full rank[3] N_k. Thus, the singular value matrix $\mathbf{D}_k$ can be partitioned as

$$\mathbf{D}_k = \begin{bmatrix} \tilde{\mathbf{D}}_k \\ \mathbf{0} \end{bmatrix} \tag{4.23}$$

[3]This is not a restriction since if $\mathbf{H}_k$ is not full rank then some dimensions the user k signal space will have zero projection on the output space. Therefore we can redefine a reduced codeword matrix $\mathbf{S}_k$ which uses only dimensions with nonzero projections on the output space.

with $\tilde{\mathbf{D}}_k$ an $N_k \times N_k$ diagonal matrix containing the non-zero singular values along the diagonal and zeros in rest. The left inverse of $\mathbf{D}_k$ is defined as

$$\mathbf{D}_k^\dagger = \begin{bmatrix} \tilde{\mathbf{D}}_k^{-1} & \mathbf{0} \end{bmatrix} \tag{4.24}$$

and it is obvious that

$$\mathbf{D}_k^\dagger \mathbf{D}_k = \mathbf{I}_{N_k} \tag{4.25}$$

Returning to equation (4.19) in which the SVD for transformed channel matrix $\tilde{\mathbf{H}}_k$ has been applied we have

$$\tilde{\mathbf{r}} = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^\top \mathbf{S}_k \mathbf{b}_k + \mathbf{w}_k \tag{4.26}$$

We pre-multiply by $\mathbf{U}_k^\top$

$$\mathbf{r}_k = \mathbf{U}_k^\top \mathbf{r} = \mathbf{D}_k \mathbf{V}_k^\top \mathbf{S}_k \mathbf{b}_k + \mathbf{U}_k^\top \mathbf{w}_k \tag{4.27}$$

and define $\tilde{\mathbf{S}}_k = \mathbf{V}_k^\top \mathbf{S}_k$ and $\tilde{\mathbf{w}}_k = \mathbf{U}_k^\top \mathbf{w}_k$. Note that because both $\mathbf{U}_k$ and $\mathbf{V}_k$ are orthogonal matrices they preserve norms of vectors. Thus, columns of $\tilde{\mathbf{S}}_k$ are also unit norm as were the columns of $\mathbf{S}_k$. Also, because the equivalent noise term $\mathbf{w}_k$ is white, then $\tilde{\mathbf{w}}_k$ will remain white with the same covariance matrix equal to the identity matrix.

$$\mathbf{r}_k = \mathbf{D}_k \tilde{\mathbf{S}}_k \mathbf{b}_k + \tilde{\mathbf{w}}_k \tag{4.28}$$

with covariance matrix given by

$$\mathbf{R}^{(k)} = E[\mathbf{r}_k \mathbf{r}_k^\top] = \mathbf{U}_k^\top \tilde{\mathbf{R}} \mathbf{U}_k \tag{4.29}$$

for which $\det \mathbf{R}^{(k)} = \det \tilde{\mathbf{R}}$ since $\mathbf{U}_k$ is an orthogonal transformation. Following a similar line of reasoning as above we note that any procedure which will attempt to maximize $\det \mathbf{R}^{(k)}$ through adaptation of $\tilde{\mathbf{S}}_k$ will also maximize $\det \mathbf{R}$. We also note that the partitioning of the singular value matrix in equation (4.23) implies the following partition on $\mathbf{R}^{(k)}$

$$\mathbf{R}^{(k)} = \begin{bmatrix} \tilde{\mathbf{D}}_k \tilde{\mathbf{S}}_k \tilde{\mathbf{S}}_k^\top \tilde{\mathbf{D}}_k + \mathbf{I}_{N_k} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N-N_k} \end{bmatrix} \tag{4.30}$$

in which $\mathbf{I}_\rho$ denotes the identity matrix of order ρ and $\mathbf{0}$ denotes a matrix with all elements equal to zero.

At this point we define an equivalent problem for user k by pre-multiplying with the left inverse of $\mathbf{D}_k$ and obtain

$$\tilde{\mathbf{r}}_k = \mathbf{D}_k^{\dagger}\mathbf{r}_k = \tilde{\mathbf{S}}_k\mathbf{b}_k + \tilde{\mathbf{z}}_k \tag{4.31}$$

which is identical in form with equation (1.4) and allows straightforward application of interference avoidance methods to optimizing user k codeword matrix $\tilde{\mathbf{S}}_k$ as. The "noise" term $\tilde{\mathbf{z}}_k$ in equation (4.31) represents the interference-plus-noise from the rest of the system that is present in user k signal space and has covariance matrix

$$\tilde{\mathbf{Z}}_k = E[\tilde{\mathbf{z}}_k\tilde{\mathbf{z}}_k^{\top}] = \mathbf{D}_k^{\dagger}E[\tilde{\mathbf{w}}_k\tilde{\mathbf{w}}_k^{\top}]\mathbf{D}_k^{\dagger\top} = \tilde{\mathbf{D}}_k^{-2} \tag{4.32}$$

The transformed codeword matrix $\tilde{\mathbf{S}}_k$ in equation (4.31) is completely equivalent with the original codeword matrix $\mathbf{S}_k$ since they are related through an orthogonal transformation $\mathbf{V}_k^{\top}$. The covariance matrix of $\tilde{\mathbf{r}}_k$ is given by

$$\tilde{\mathbf{R}}^{(k)} = E[\tilde{\mathbf{r}}_k\tilde{\mathbf{r}}_k^{\top}] = \tilde{\mathbf{S}}_k\tilde{\mathbf{S}}_k^{\top} + \tilde{\mathbf{D}}_k^{-2} \tag{4.33}$$

and using the partition in equation (4.30) we have

$$\det \mathbf{R}^{(k)} = \det(\tilde{\mathbf{D}}_k\tilde{\mathbf{S}}_k\tilde{\mathbf{S}}_k^{\top}\tilde{\mathbf{D}}_k + \mathbf{I}_{N_k}) = \det \mathbf{D}_k^{-2} \det \tilde{\mathbf{R}}^{(k)} \tag{4.34}$$

which implies that maximizing $\det \tilde{\mathbf{R}}^{(k)}$ will also maximize $\det \mathbf{R}^{(k)}$ which in turn will imply maximization of $\det \mathbf{R}$ the determinant of the original received signal covariance matrix.

4.2.1 Greedy Interference Avoidance

We recall that greedy interference avoidance consists of replacing codeword corresponding to symbol m of user k by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process in the inverted channel space, that is

$$\mathbf{R}_m^{(k)} = \tilde{\mathbf{S}}_k\tilde{\mathbf{S}}_k^{\top} - \tilde{\mathbf{s}}_m^{(k)}\tilde{\mathbf{s}}_m^{(k)\top} + \tilde{\mathbf{D}}_k^{-2} \tag{4.35}$$

Furthermore, since $\mathbf{R}^{(k)} = \mathbf{R}_m^{(k)} + \tilde{\mathbf{s}}_m^{(k)}\tilde{\mathbf{s}}_m^{(k)\top}$, using Lemma 2.1 we note that the eigenvalues of $\mathbf{R}^{(k)}$ before application of the greedy interference avoidance procedure are majorized by those after application. Thus, $\det \mathbf{R}^{(k)}$ and implicitly sum capacity cannot be decreased by application of greedy interference avoidance.

We also note that, empirically we have observed that repeated application of greedy interference avoidance with various codeword replacement procedures with respect to codewords/users usually reaches a fixed point. Although we have not been able to prove this result in general, simulations have shown that, when users have at least as many codewords as signal space dimensions, this fixed point corresponds to a simultaneous water filling solution for all users in their respective signal spaces and is identical to that in [94], thus corresponding to maximum sum capacity.

4.2.2 The Eigen-Algorithm and Iterative Water Filling

Repeated sequential application of the greedy interference avoidance procedure for all codewords of a given user k defines the eigen-algorithm, whose application for user k equivalent problem in equation (4.31) will water fill its signal space regarding all other users as noise, and maximize $\det \mathbf{R}^{(k)}$ and implicitly $\det \mathbf{R}$. Thus, iterative application of the eigen-algorithm by all users in the system is an instance of an iterative water filling procedure [94] in which each user adapts its corresponding codeword matrix, regarding all other users' signals as noise while maximizing $\det \mathbf{R}$. Such a procedure is guaranteed to converge to an optimal fixed point where sum capacity is maximized.

We can now formally state the eigen-algorithm for interference avoidance for vector multiaccess channels:

The Generalized Eigen-Algorithm for Multiaccess Vector Channels

1. Start with a randomly chosen codeword ensemble specified by the codeword matrices $\{\mathbf{S}_k\}_{k=1}^{L}$

2. For each user $k = 1 \dots L$

   (a) Using equations (4.17) and (4.18) compute the transformation matrix $\mathbf{T}_k$ that whitens the interference-plus-noise seen by user k

   (b) Change coordinates and compute transformed user k channel matrix $\tilde{\mathbf{H}}_k = \mathbf{T}_k \mathbf{H}_k$

   (c) Apply SVD for $\tilde{\mathbf{H}}_k$ and project the problem onto user k signal space to obtain $\tilde{\mathbf{r}}_k$ in equation (4.31)

(d) adjust user k codewords sequentially: the codeword corresponding to symbol m of user k is replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process $\mathbf{R}_m^{(k)}$

(e) Iterate previous step until convergence (making use of escape methods [62] if the procedure stops in suboptimal points)

(f) After all N_k codewords have been changed the new codeword matrix corresponding to user k is $\mathbf{S}_k = \mathbf{V}_k \tilde{\mathbf{S}}_k$

3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

We note that steps $2(d) - (e)$ of the algorithm correspond to water filling of user k's signal space considering all other users' signals as noise. The water filling process is done for all users sequentially and repeated (step 3) until convergence to maximum sum capacity value implied by the iterative water filling procedure [94]. We also note that convergence of the algorithm is in terms of sum capacity and not in terms of individual codeword matrices, and conclude that the eigen-algorithm will always yield a codeword ensemble that maximizes sum capacity.

4.3 Application to Dispersive Channels

In Chapter 2 application of interference avoidance to dispersive channels was presented in a multicarrier modulation framework in which all users reside in the same signal space and signature waveforms corresponding to distinct bits/users are represented as superposition of sinusoids specified by the corresponding codewords. The result derived in the previous section, namely that interference avoidance can be used to derive codeword ensembles that maximize sum capacity for any type of multiple access vector channels, suggests that interference avoidance methods can be applied to other models of dispersive communication channels. The idea is simple and is based on deriving a mathematical representation of the communication system in terms of a vector multiple access channel. This will characterize the system through an equation identical in form with equation (4.6) for which application of interference avoidance is straightforward. The difference between (4.6) and the resulting equation will be given by the fact that channel matrices will incorporate also channel memory in addition to relating transmitter and receiver signal spaces. A general way of deriving such a representation is presented in this section, along with an example that shows application for multipath channels for direct sequence CDMA (DS-CDMA) systems.

Let us consider a multiuser system with L users communicating with a common receiver over distinct, dispersive channels, characterized by impulse responses $h_\ell(t),\ \ell = 1, \ldots, L$. We assume that each user's signal space of dimension N_ℓ is spanned by the vector of functions $\mathbf{\Psi}^{(\ell)}(t) = [\Psi_1^{(\ell)}(t) \ldots \Psi_{N_\ell}^{(\ell)}(t)]^\top$ and that the receiver signal space of dimension N is spanned by the vector of functions $\mathbf{\Psi} = [\Psi_1(t) \ldots \Psi_N(t)]^\top$. Let also $\mathbf{H}_\ell$ be the $N \times N_\ell$ matrix with orthonormal columns relating the bases of the two signal spaces, that is

$$\mathbf{\Psi}^{(\ell)}(t) = \mathbf{H}_\ell^\top \mathbf{\Psi}(t) \tag{4.36}$$

Each user ℓ transmits the signal $x_\ell(t)$ of duration $\mathcal{T}$ corresponding to a frame containing M_ℓ symbols, and is composed of a superposition of waveforms scaled by the symbols

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} \mathbf{s}_m^{(\ell)}(t) \tag{4.37}$$

or in terms of the basis functions

$$x_\ell(t) = \mathbf{\Psi}^{(\ell)}(t)^\top \mathbf{x}_\ell = \mathbf{\Psi}^{(\ell)}(t)^\top \mathbf{S}_\ell \mathbf{b}_\ell \tag{4.38}$$

At the receiver, the signal sent by user ℓ corresponds to $y_\ell(t)$, the convolution of $x_\ell(t)$ with the corresponding channel ℓ impulse response $h_\ell(t)$

$$y_\ell(t) = \int_0^{\mathcal{T}} h_\ell(\tau) x_\ell(t-\tau) d\tau \tag{4.39}$$

Projected onto the basis functions of the receiver signal space the vector $\mathbf{y}_\ell$ is obtained such that component n of $\mathbf{y}_\ell$ is

$$\begin{aligned} y_n^{(\ell)} &= \int_0^{\mathcal{T}} y_\ell(t) \Psi_n(t) dt \\ &= \int_0^{\mathcal{T}} \int_0^{\mathcal{T}} h_\ell(\tau) x_\ell(t-\tau) \Psi_n(t) d\tau dt \qquad (4.40) \\ &= \int_0^{\mathcal{T}} \int_0^{\mathcal{T}} h_\ell(\tau) \mathbf{\Psi}^{(\ell)}(t-\tau)^\top \mathbf{x}_\ell \Psi_n(t) d\tau dt \end{aligned}$$

Thus, vector $\mathbf{y}_\ell$ can be written as

$$\mathbf{y}_\ell = \mathbf{F}_\ell \mathbf{H}_\ell \mathbf{x}_\ell \tag{4.41}$$

where the $N \times N$ matrix $\mathbf{F}_\ell$ is obtained from the following equation

$$\mathbf{F}_\ell = \int_0^{\mathcal{T}} \int_0^{\mathcal{T}} h_\ell(\tau) \boldsymbol{\Psi}(t) \boldsymbol{\Psi}(t-\tau)^\top dt d\tau \tag{4.42}$$

In order to obtain the expression for $\mathbf{F}_\ell$ equation (4.36) was used to express the basis of user ℓ signal space in terms of the basis of the receiver signal space. Note that all the calculations were done under the implicit assumption that the frame duration $\mathcal{T}$ is longer than the duration of all $h_\ell(t),\ \ell = 1, \ldots, L$, so that settling of all channel responses is allowed and intersymbol interference (ISI) between succesive frames can be safely ignored. Equation (4.41) can also be rewritten as

$$\mathbf{y}_\ell = \boldsymbol{\mathcal{H}}_\ell \mathbf{S}_\ell \mathbf{b}_\ell \tag{4.43}$$

where $\boldsymbol{\mathcal{H}}_\ell = \mathbf{F}_\ell \mathbf{H}_\ell$ is the $N \times N_\ell$ channel matrix corresponding to user ℓ and incorporates, as it has already been mentioned, both the transformation between user ℓ's signal space and receiver signal space (matrix $\mathbf{H}_\ell$), and the channel $h_\ell(t)$ memory (matrix $\mathbf{F}_l$).

Therefore, the equation of the received signal at the common receiver, consisting of the sum of all received signals from all users plus noise, is written as

$$\mathbf{r} = \sum_{\ell=1}^{L} \boldsymbol{\mathcal{H}}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{4.44}$$

and is identical in form to equation (4.6). The eigen-algorithm for interference avoidance can be applied now as in the previous section to derive optimal codeword ensembles that maximize the sum capacity of the multiaccess channel.

In order to illustrate the above procedure on a specific example we consider a DS-CDMA system with "time chips" spanning the signal space and processing gain N. In this case all users and the receiver reside in the same signal space of dimension N spanned by the functions[4]

$$\Psi_n(t) = \begin{cases} \dfrac{N}{\mathcal{T}} & \text{for} \quad \dfrac{n-1}{N}\mathcal{T} \le t \le \dfrac{n}{N}\mathcal{T} \\ 0 & \text{elsewhere} \end{cases} \qquad n = 1, \ldots, N \tag{4.45}$$

[4]Normalization of "time chips" is done only to be consistent with the definition of basis functions assumed to have unit energy.

As a consequence, all matrices $\mathbf{H}_\ell = \mathbf{I}_N$ and we only have to worry about computing the matrices $\mathbf{F}_\ell$ in order to derive channel matrices.

We assume a multipath channel, with paths spaced at the chip duration and impulse response given by

$$h_\ell(t) = \sum_{i=0}^{p_\ell - 1} h_i^{(\ell)} \delta\left(t - i\frac{\mathcal{T}}{N}\right) \qquad \ell = 1, \dots, L \tag{4.46}$$

with p_ℓ being the number of paths corresponding to user ℓ and $h_i^{(\ell)}$ being the path gains. A similar channel model has been considered in [57,58] in a discrete-time setting, and as we will see the same channel matrices will be obtained through our approach.

The channel matrix corresponding to user ℓ is obtained from equation (4.42) as

$$\begin{aligned}
\mathbf{F}_\ell &= \int_0^{\mathcal{T}} \mathbf{\Psi}(t) \int_0^{\mathcal{T}} \sum_{i=0}^{p_\ell - 1} h_i^{(\ell)} \delta\left(t - i\frac{\mathcal{T}}{N}\right) \mathbf{\Psi}(t-\tau)^\top d\tau dt \\
&= \int_0^{\mathcal{T}} \mathbf{\Psi}(t) \sum_{i=0}^{p_\ell - 1} h_i^{(\ell)} \int_0^{\mathcal{T}} \delta\left(t - i\frac{\mathcal{T}}{N}\right) \mathbf{\Psi}(t-\tau)^\top d\tau dt \\
&= \sum_{i=0}^{p_\ell - 1} h_i^{(\ell)} \int_0^{\mathcal{T}} \mathbf{\Psi}(t) \mathbf{\Psi}\left(t - i\frac{\mathcal{T}}{N}\right)^\top dt = \sum_{i=0}^{p_\ell - 1} h_i^{(\ell)} \mathbf{F}_i^{(\ell)}
\end{aligned} \tag{4.47}$$

with $N \times N$ matrices $\mathbf{F}_i^{(\ell)}$ given by

$$\mathbf{F}_i^{(\ell)} = \int_0^{\mathcal{T}} \mathbf{\Psi}(t) \mathbf{\Psi}\left(t - i\frac{\mathcal{T}}{N}\right)^\top dt \tag{4.48}$$

Due to the particular form of basis functions $\mathbf{\Psi}(t)$ (non-overlapping rectangular pulses) matrices $\mathbf{F}_i^{(\ell)}$ will be sparse, with ones only in certain positions and zeros in rest. Starting with the first term, this is equal to the identity matrix

$$\mathbf{F}_0^{(\ell)} = \int_0^{\mathcal{T}} \mathbf{\Psi}(t) \mathbf{\Psi}(t)^\top dt = \mathbf{I}_N \tag{4.49}$$

since time chips do not overlap and only the product of a given function with itself is not equal to zero. As we delay the basis functions the ones along the main diagonal will be shifted downwards: a delay of a chip duration $\mathcal{T}/N$ is equivalent with a downshift of the 1 along the corresponding column. This is due to the fact that, by delaying time chips, we will have less and less overlap between them. Thus, the second term is

$$\mathbf{F}_1^{(\ell)} = \int_0^{\mathcal{T}} \mathbf{\Psi}(t)\mathbf{\Psi}\left(t - \frac{\mathcal{T}}{N}\right)^\top dt = \begin{bmatrix} 0 & & & & \\ 1 & 0 & & & \\ 0 & 1 & \ddots & & \\ \vdots & \vdots & \ddots & 0 & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \tag{4.50}$$

and so on, until the last term in which 1's will be shifted down p_ℓ positions. Hence, the channel matrix for user ℓ has the form

$$\mathbf{F}_\ell = \begin{bmatrix} h_0^{(\ell)} & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ h_1^{(\ell)} & h_0^{(\ell)} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ h_2^{(\ell)} & h_1^{(\ell)} & h_0^{(\ell)} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{p_\ell-1}^{(\ell)} & h_{p_\ell-2}^{(\ell)} & \cdots & \ddots & h_0^{(\ell)} & 0 & \cdots & 0 & 0 \\ 0 & h_{p_\ell-1}^{(\ell)} & h_{p_\ell-2}^{(\ell)} & \cdots & h_1^{(\ell)} & h_0^{(\ell)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \cdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \ddots & h_0^{(\ell)} & 0 \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & h_1^{(\ell)} & h_0^{(\ell)} \end{bmatrix} \tag{4.51}$$

and is identical with with the channel matrix used in [57, 58].

4.4 Chapter Summary

Application of interference avoidance to multiaccess vector channels was presented. The vector channel is regarded as a linear transformation between the input signal space corresponding to the transmitter and the output signal space corresponding to the receiver and is characterized by a channel gain matrix. The approach is general and allows different, possibly overlapping, signal spaces for distinct users in a multiuser system, all being subspaces of the signal space at the common receiver(s).

Information is transmitted using a generalized form of CDMA where symbols for a given user is conveyed by a distinct signature waveform in some arbitrary signal space. The set of waveforms corresponding to a given user is equivalent to a codeword matrix in the signal space, and optimal codeword matrices that maximize the sum capacity of the multiaccess channel can be obtained by application of interference avoidance methods.

By whitening the interference-plus-noise and using the SVD greedy interference avoidance is applied in a given user's signal space. We show that application of greedy interference avoidance for any codeword/user does not decrease sum capacity. Furthermore, we show that *sequential application of the eigen-algorithm for interference avoidance by all users in a multiuser system is equivalent to iterative water filling* in which each user water fills its corresponding signal space considering other user's signals as noise.

The result is powerful and clears the way for application of interference avoidance methods to a variety of problems including arbitrary dispersive channel models, multiuser multiple antenna systems, and asynchronous multiuser systems.

Chapter 5

Application to Multiple Antenna Systems

Multiple antenna systems, used in wireless communications to provide spatial diversity, have been shown to improve system performance by mitigating the effects of multipath fading. Traditionally, spatial diversity was implemented only at one side of the communication system (mainly at the receiver). However, recent research indicates that performance can be significantly improved by using spatial diversity both at the transmitter and at the receiver.

Performance of multiple antenna systems in fading environments has been analyzed in several papers which have shown a potentially large increase in capacity. Since standard approaches are not close in performance to the theoretical limits [19, 39], new modulation schemes have been proposed and analyzed for multiple antenna systems [18,28]. It has also been shown that presence of multipath can improve performance with an appropriate multiple antenna structure [59]. Recently, an optimal power control algorithm for multiple antenna systems [95] has been proposed based on an iterative water filling scheme [94].

In this chapter we present application of interference avoidance methods to multiuser systems with multiple inputs and multiple outputs (MIMO) such as those associated with the uplink of a wireless system in which users and the basestation are equipped with multiple antennas. Our approach is based on application of interference avoidance to general multiaccess vector channels presented in Chapter 4 for the particular multiaccess vector channel corresponding to a multiuser MIMO system. We note that a vector channel characterized by a channel matrix between a given transmitter and the receiver is a natural representation in the case of MIMO systems and several models can be found in the literature [18, 19, 39, 59]. We also note that the approach is completely general and applicable to any MIMO system models regardless of the choice of signal space basis functions.

Information is transmitted over the MIMO channel using the multicode CDMA approach described in Chapter 4 in which the sequence of information symbols that make up the frame to be sent by a given user over the MIMO channel is "spread" over the available dimensions using a precoding matrix. More precisely, the columns of the precoding matrix are used as codewords, or spreading sequences, for the information symbols in the frame. Formulation of the MIMO channel problem in this CDMA context allows direct application of interference avoidance techniques to determine optimal precoding matrices in both the single user and multiple user cases. These are optimal in the sense that the common signal-to-interference plus noise-ratio (SINR) for all symbols is maximized, as well as in the information theoretic sense of maximizing sum capacity [63, 85].

The chapter is organized as follows. We introduce the multiuser multiple antenna system with the equivalent multiuser vector channel model and the multicode CDMA approach for transmission of information in section 5.1. In section 5.2 we show how interference avoidance can be applied to optimize precoder matrices in both single and multiple user cases. Numerical simulations are presented in section 5.3 and performance is analyzed in terms of signal-to-noise ratios (SNRs) at the receiver and sum capacity. For a single-user case we note that with random precoding matrices the SNR at receiver antennas has a bell-shaped distribution. However, after interference avoidance is performed optimal precoding matrices imply that the SNR is approximately the same for all receive antennas even though no prior assumption about equal SNRs at each antenna has been made as in [19]. We also note that for the same average noise power at the receiver, doubling the number of antennas (both in the transmitter and in the receiver) translates to an approximately 3 dB increase in the SNR.

Next we look at sum capacity in the context of fading channels. Precoding matrices optimal for the average channel are obtained for all users in the system using interference avoidance algorithms and these are used to compute sum capacity for actual realizations of the fading channel models. Sum capacity is treated as a random variable in this case and complementary cumulative distribution functions (CCDFs) similar to those in [19] are plotted. The plots show what capacity can be achieved with a given probability of outage when precoders optimal for the average channel are used, and are consistent with those in [19], supporting the generally accepted idea that the use of multiple antennas in both the transmitter and the receiver is beneficial for system performance.

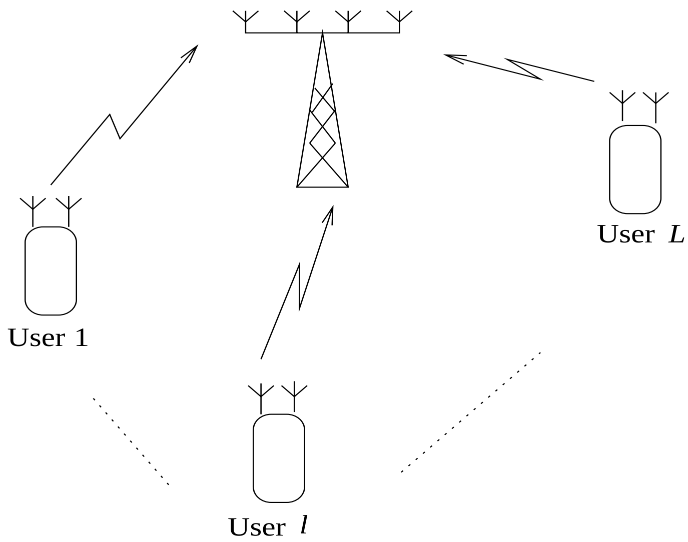

Figure 5.1: Multiuser MIMO system in which all users and the basestation are equipped with antenna arrays for transmission/reception.

5.1 The MIMO System Model

We consider a multiuser system with L users communicating with a common receiver (basestation) in which all users and the basestation are equipped with antenna arrays for transmission/reception. We denote by T_ℓ the number of transmit antennas corresponding to user ℓ, $\ell = 1, \ldots, L$ and by R the number of receive antennas at the basestation. The system is described schematically in Figure 5.1.

Each user ℓ transmits information in frames of duration $\mathcal{T}$. The channel between user ℓ's transmit antenna i and receive antenna j at the basestation is characterized by the causal impulse response $h_{ij}^{(\ell)}(t)$ of duration $T_{ij}^{(\ell)}$ assumed stable (time-invariant) over the duration $\mathcal{T}$ of the frame. The transmitted waveform $x_i^{(\ell)}(t)$ at transmit antenna i of user ℓ convolved with this impulse response yields the corresponding waveform $y_{ij}^{(\ell)}(t)$ due to user ℓ transmit antenna i at receive antenna j

$$y_{ij}^{(\ell)}(t) = x_i^{(\ell)}(t) * h_{ij}^{(\ell)}(t) \tag{5.1}$$

The received waveform at receive antenna j is then a superposition of all such received waveforms from all transmit antennas of all users plus additive Gaussian noise

$$r_j(t) = \sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} y_{ij}^{(\ell)}(t) + n_j(t) = \sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} x_i^{(\ell)}(t) * h_{ij}^{(\ell)}(t) + n_j(t) \tag{5.2}$$

If we approached the problem using the waveform representation above, the mathematics involved would become very complex and would artificially increase its difficulty. The use of a signal space approach overcomes the difficulties associated with the waveform representation above by working with equivalent signal vectors and vector channels, and making use of much simpler linear algebra methods. We note that different MIMO channel models may be obtained by using different basis functions for the signal space. For example, by using sinc functions one ends up with models similar to those in [18, 59] obtained by Nyquist sampling. Our simulation results in section 5.3 are obtained for a MIMO channel model based on a multicarrier modulation scheme similar to that used in Chapter 2. Nevertheless, once the MIMO channel matrix $\mathbf{H}_\ell$ is obtained, application of interference avoidance methods remains unchanged.

In the equivalent signal space representation, the N-dimensional received signal vector at the basestation is given by [18, 19, 59]

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} \tag{5.3}$$

with $\mathbf{x}_\ell$ being the N_ℓ-dimensional signal vector transmitted by user ℓ, $\mathbf{H}_\ell$ the $N \times N_\ell$ MIMO channel matrix corresponding to user ℓ, and $\mathbf{n}$ the noise vector at the receiver. We note that equation (5.3) is identical in form with the general multiaccess vector channel equation (4.2), and that the dimensions of the transmitter and receiver signal spaces will depend on the number of transmit and receive antennas employed[1]. Similar to Chapter 4 users transmit sequences of symbols consisting of zero-mean unit variance Gaussian random variables as frames, and for a given user ℓ the N_ℓ-dimensional transmitted vector $\mathbf{x}_\ell$ is obtained from the sequence of symbols to be sent $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^\top$ through a spreading operation specified by the $N_\ell \times M_\ell$ precoding matrix

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \cdots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \tag{5.4}$$

[1] For the MIMO channel models in [18, 19, 59] these are actually equal to the number of antennas employed.

whose columns have unit norm and determine the “spreading” of corresponding symbols in the frame over the N_ℓ available dimensions. Therefore, the transmitted vector sent by user ℓ is written as

$$\mathbf{x}_\ell = \mathbf{S}_\ell \mathbf{b}_\ell \tag{5.5}$$

which implies that the received signal vector at the basestation is given by an equation identical in form with equation (4.6), that is

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{5.6}$$

Our problem is now to optimize the precoder matrices such that the sum capacity of the multiple access vector channel defined in equation (5.6) corresponding to the multiuser MIMO system is maximized.

5.2 Precoder Optimization Through Interference Avoidance

The fact that equation (5.6) is identical to equation (4.6) of the received signal corresponding to the multiaccess vector channel in Chapter 4 suggests that the generalized eigen-algorithm for vector multiple access channels can be directly applied to precoder optimization in the case of multiuser MIMO systems. However we note that in Chapter 4 it is assumed that all channel matrices have full rank which may not always be the case for the MIMO channel models presented in section 5.1. Therefore, caution should be exercised when applying the generalized eigen-algorithm for precoder optimization.

Similar to Chapter 4 we start by rewriting the received signal in equation (5.6) from the perspective of user k

$$\mathbf{r} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{5.7}$$

and denoting by

$$\mathbf{z}_k = \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{5.8}$$

the interference-plus-noise seen by user k with the corresponding covariance matrix

$$\mathbf{Z}_k = E[\mathbf{z}_k \mathbf{z}_k^\top] = \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \tag{5.9}$$

Note that if user k channel matrix is not full-rank, some of the signal space dimensions will need to be discarded as they do not carry useful information for user k. However, this can only be done if the interference-plus-noise present in these dimensions is statistically independent from that in the remaining dimensions. This is accomplished by whitening the interference-plus-noise seen by user k through the transformation

$$\mathbf{T}_k = \mathbf{\Delta}_k^{-1/2}\mathbf{E}_k^\top \tag{5.10}$$

in which matrices $\mathbf{\Delta}_k$ and $\mathbf{E}_k$ are obtained according to the generalized eigen-algorithm from the eigenvalue decomposition of matrix $\mathbf{Z}_k = \mathbf{E}_k\mathbf{\Delta}_k\mathbf{E}_k^\top$.

In the transformed coordinates, equation (5.7) is equivalent to

$$\tilde{\mathbf{r}} = \mathbf{T}_k\mathbf{r} = \mathbf{T}_k\mathbf{H}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{T}_k\mathbf{z}_k = \tilde{\mathbf{H}}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{w}_k \tag{5.11}$$

where $\tilde{\mathbf{H}}_k = \mathbf{T}_k\mathbf{H}_k$ is the MIMO channel matrix seen by user k in the new coordinates and $\mathbf{w}_k = \mathbf{T}_k\mathbf{z}_k$ is the equivalent "white noise" term with covariance matrix $E[\mathbf{w}_k\mathbf{w}_k^\top] = \mathbf{T}_k\mathbf{Z}_k\mathbf{T}_k^\top = \mathbf{I}$ equal to the identity matrix. Following the generalized eigen-algorithm, we apply the SVD and obtain

$$\tilde{\mathbf{H}}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\top \tag{5.12}$$

where matrix $\mathbf{U}_k$ of dimension $N \times N$ has as columns the eigenvectors of $\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^\top$, matrix $\mathbf{V}_k$ of dimension $N_k \times N_k$ has as columns the eigenvectors of $\tilde{\mathbf{H}}_k^\top\tilde{\mathbf{H}}_k$, and matrix $\mathbf{D}_k$ of dimension $N \times N_k$ contains the singular values of $\tilde{\mathbf{H}}_k$ on the main diagonal and zeros elsewhere. Any vector in the N_k-dimensional input space of user k can then be represented in terms of the orthonormal set of vectors $\{\mathbf{v}_i^{(k)}\}$ representing the columns of $\mathbf{V}_k$. Similarly, any vector in the N-dimensional receiver space is representable in terms of the orthonormal set of vectors $\{\mathbf{u}_i^{(k)}\}$ representing the columns of $\mathbf{U}_k$. Furthermore, because these sets of vectors come from the SVD decomposition (5.12) we have

$$\mathbf{v}_i^{(k)\top}\mathbf{v}_j^{(k)} = \delta_{ij} \;\Rightarrow \mathbf{v}_i^{(k)\top}\tilde{\mathbf{H}}_k^\top\tilde{\mathbf{H}}_k\mathbf{v}_j^{(k)} = d_i^{(k)\,2}\delta_{ij} \tag{5.13}$$

Therefore, user k should only put energy into those vectors $\mathbf{v}_i^{(k)}$ that correspond to non-zero singular values $d_i^{(k)} \neq 0$.

Also, let us denote by ρ_k the rank of user k's transformed MIMO channel matrix $\tilde{\mathbf{H}}_k$, equal to the number of non-zero singular values. It is obvious that

$$\rho_k = \text{rank}(\tilde{\mathbf{H}}_k) \leq \min(N, N_k) \tag{5.14}$$

Then, the dimension of the column space of matrix $\tilde{\mathbf{H}}_k$ will be equal to ρ_k. Also, the dimension of the null space of $\mathbf{H}_k$ is $N_k - \rho_k$ and the dimension of the left null space is $N - \rho_k$. Because there are only ρ_k non-zero singular values and we are interested only in their corresponding eigenvectors, we can partition matrix $\mathbf{D}_k$ containing the singular values as

$$\mathbf{D}_k = \begin{bmatrix} \bar{\mathbf{D}}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{5.15}$$

with a $\rho_k \times \rho_k$ diagonal matrix $\bar{\mathbf{D}}_k$ which contains the nonzero singular values and zero matrices of appropriate dimensions.

Returning to equation (5.11) in which we apply the SVD for matrix $\tilde{\mathbf{H}}_k$ we obtain

$$\tilde{\mathbf{r}} = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^\top \mathbf{S}_k \mathbf{b}_k + \mathbf{w}_k \tag{5.16}$$

We can premultiply by $\mathbf{U}_k^\top$

$$\tilde{\mathbf{r}}_k = \mathbf{U}_k^\top \mathbf{r}_k = \mathbf{D}_k \mathbf{V}_k^\top \mathbf{S}_k \mathbf{b}_k + \mathbf{U}_k^\top \mathbf{w}_k \tag{5.17}$$

By defining $\tilde{\mathbf{S}}_k = \mathbf{V}_k^\top \mathbf{S}_k$ and $\tilde{\mathbf{w}}_k = \mathbf{U}_k^\top \mathbf{w}_k$ we have

$$\tilde{\mathbf{r}}_k = \mathbf{D}_k \tilde{\mathbf{S}}_k \mathbf{b}_k + \tilde{\mathbf{w}}_k \tag{5.18}$$

The partition in equation (5.15) on the singular value matrix $\mathbf{D}_k$ induces the following partition of transformed precoder matrix $\tilde{\mathbf{S}}_k$

$$\tilde{\mathbf{S}}_k = \begin{bmatrix} \tilde{\mathbf{S}}_{k1} \\ \tilde{\mathbf{S}}_{k2} \end{bmatrix} \tag{5.19}$$

with $\tilde{\mathbf{S}}_{k1}$ of dimension $\rho_k \times M_k$ and $\tilde{\mathbf{S}}_{k2}$ of dimension $(N_k - \rho_k) \times M_k$.

Note that because both $\mathbf{U}_k$ and $\mathbf{V}_k$ are orthogonal matrices they preserve norms of vectors. Thus, columns of $\tilde{\mathbf{S}}_k$ are also unit norm as were the columns of $\mathbf{S}_k$. Also, because the equivalent noise term $\mathbf{w}_k$ is white, then $\tilde{\mathbf{w}}_k$ will also be white.

In light of the partitions in equations (5.15) and (5.19) we can safely ignore the last $N - \rho_k$ dimensions of the received vector $\tilde{r}_k$ in equation (5.18) and reduce dimensionality of the problem to the rank of the channel matrix ρ_k. This can be done because, on one hand, no transmitted signal due to user k will be observed on these dimensions, and on the other hand noise components on these dimensions are statistically independent of the remaining noise components. Thus, the reduced dimension problem in which only the first ρ_k components of the received vector appear can be written as

$$\bar{\mathbf{r}}_k = [\mathbf{I}_{\rho_k} \ \ \mathbf{0}]\tilde{\mathbf{r}}_k = \bar{\mathbf{D}}_k\bar{\mathbf{S}}_k\mathbf{b}_k + \bar{\mathbf{w}}_k \tag{5.20}$$

with $\bar{\mathbf{S}}_k = \tilde{\mathbf{S}}_{k1}$ and $\bar{\mathbf{w}} = [\mathbf{I}_{\rho_k} \ \ \mathbf{0}]\tilde{\mathbf{w}}_k$. The covariance matrix of the "new" noise vector is also an identity matrix of dimension ρ_k.

Equation (5.20) is identical to the single user dispersive channel equation (2.18) and the eigen-algorithm can now be directly applied to optimization of matrix $\bar{\mathbf{S}}_k$. With the matrix $\bar{\mathbf{S}}_k$ yielded by the eigen-algorithm for interference avoidance one can obtain the full dimension precoder matrix

$$\mathbf{S}_k = \mathbf{V}_k \begin{bmatrix} \bar{\mathbf{S}}_k \\ \mathbf{0} \end{bmatrix} \tag{5.21}$$

so that each input codeword vector is a linear combination of only those $\mathbf{v}_i^{(k)}$ which actually appear at the channel output.

The precoder optimization algorithm through interference avoidance for multiuser MIMO systems is formally stated below:

The Eigen-Algorithm for Multiuser MIMO Systems

1. Start with a randomly chosen set of precoder matrices $\{\mathbf{S}_\ell\}_{\ell=1}^{L}$
2. For each user $k = 1 \ldots L$
   (a) Compute the transformation matrix $\mathbf{T}_k$ in equation (5.10) that whitens the interference-plus-noise seen by user k
   (b) Change coordinates and compute transformed user k's MIMO channel matrix $\tilde{\mathbf{H}}_k = \mathbf{T}_k\mathbf{H}_k$
   (c) Apply SVD for $\tilde{\mathbf{H}}_k$ and project the problem onto user k's signal space to obtain $\tilde{\mathbf{r}}_k$ in equation (5.18)
   (d) Reduce dimensionality to ρ_k the rank of the MIMO channel matrix and obtain the reduced dimension problem in equation (5.20)

(e) Define the equivalent inverted channel problem similar to equation (2.19) for user k by premultiplying with $\bar{\mathbf{D}}_k^{-1}$

$$\bar{\mathbf{r}}_{k,inv} = \bar{\mathbf{D}}_k^{-1}\bar{\mathbf{r}}_k = \bar{\mathbf{S}}_k\mathbf{b}_k + \bar{\mathbf{D}}_k^{-1}\bar{\mathbf{w}}_k \tag{5.22}$$

(f) Adjust user k's transformed precoder matrix by replacing its columns sequentially: column m of $\bar{\mathbf{S}}_k$ ($\bar{\mathbf{s}}_m^{(k)}$) is replaced by the minimum eigenvector of corresponding interference-plus-noise covariance matrix

$$\mathbf{R}_m^{(k)} = \bar{\mathbf{S}}_k\bar{\mathbf{S}}_k^\top - \bar{\mathbf{s}}_m^{(k)}\bar{\mathbf{s}}_m^{(k)\top} + \bar{\mathbf{D}}_k^{-2} \tag{5.23}$$

(g) Iterate previous step until convergence (making use of escape methods [62] if the procedure stops in suboptimal points)

3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

We note that steps 2(f)-(g) represent application of the basic eigen-algorithm and "water fill" user k's signal space while regarding the remaining users in the system as noise. Therefore, applied iteratively by each user the procedure is an instance of *iterative water filling* and is guaranteed to converge to a fixed point [94] where sum capacity of the multiple access vector channel in equation (5.6) is maximized. We again note that empirically, the algorithm seemed robust with respect to the order of codeword replacement.

We also note, that from a practical point of view the dimensionality of the problem can be further reduced in step 2(d) of the algorithm by taking advantage of the water filling result implied by the eigen-algorithm. More precisely, the noise levels over which water filling occurs in steps 2(f)-(g) of the algorithm are given by the inverse of the non-zero singular values of the MIMO channel matrix $\tilde{\mathbf{H}}_k$, that is

$$\bar{\mathbf{D}}_k^{-2} = \begin{bmatrix} d_1^{(k)^{-2}} & & & & \\ & \ddots & & & \\ & & d_i^{(k)^{-2}} & & \\ & & & \ddots & \\ & & & & d_{\rho_k}^{(k)^{-2}} \end{bmatrix} \tag{5.24}$$

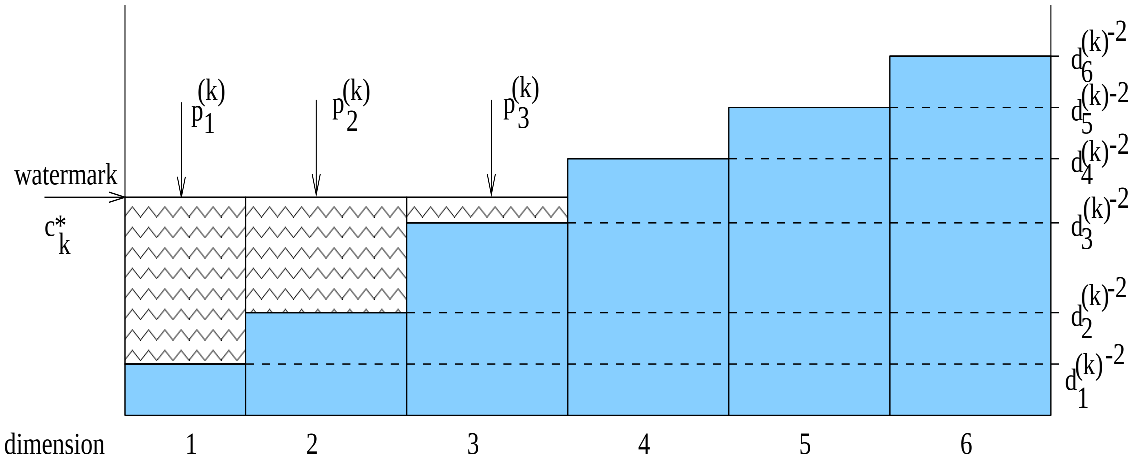

Figure 5.2: An example of water filling diagram in a signal space with 6 dimensions for which the total power M_k of user k is split only on the first 3 dimensions with minimum "noise" energy. The identities implied by water filling $c_k^* = p_1^{(k)} + d_1^{(k)^{-2}} = p_2^{(k)} + d_2^{(k)^{-2}} = p_3^{(k)} + d_3^{(k)^{-2}}$ determine $c_k^* = [M_k + (d_1^{(k)^{-2}} + d_2^{(k)^{-2}} + d_3^{(k)^{-2}})]/3 \leq d_4^{(k)^{-2}}$.

in decreasing order of their magnitudes as yielded by the SVD

$$d_1^{(k)} \geq \ldots \geq d_i^{(k)} \geq \ldots \geq d_{\rho_k}^{(k)} \tag{5.25}$$

This implies that their inverses will be in increasing order of their magnitudes, i.e.

$$d_1^{(k)^{-2}} \leq \ldots \leq d_i^{(k)^{-2}} \leq \ldots \leq d_{\rho_k}^{(k)^{-2}} \tag{5.26}$$

Because for each user k we are limited to a fixed amount of transmitted power equal to Trace $\left[\mathbf{S}_k \mathbf{S}_k^\top\right] = M_k$ (coming from the fact that each user k sends M_k symbols each with unit energy) we can determine how many of the ρ_k dimensions will actually carry information by looking at the "watermark" in the corresponding water filling diagram (see Figure 5.2). If we denote by n the number of dimensions water-filled by the transmitted power then the "watermark" is defined as

$$c_k^* = \frac{M_k + \sum_{i=1}^{n} d_i^{(k)^{-2}}}{n} \tag{5.27}$$

and can be found algorithmically by checking the following inequalities

$$\left.\begin{array}{r}c_m^{(k)^*} = \dfrac{1}{m}\left(M_k + \displaystyle\sum_{i=1}^{m} d_i^{(k)^{-2}}\right) > d_{m+1}^{(k)^{-2}} \\ \\ c_{m+1}^{(k)^*} = \dfrac{1}{m+1}\left(M_k + \displaystyle\sum_{i=1}^{m+1} d_i^{(k)^{-2}}\right) \leq d_{m+2}^{(k)^{-2}}\end{array}\right\} \implies n = m+1 \tag{5.28}$$

The first inequality in (5.28) tells us that if we were to use only the first m dimensions to do water filling, the resulting watermark $c_m^{(k)^*}$ will be larger than the $(m+1)^{\text{st}}$ "noise level". However, the next inequality assures that when the $(m+1)^{\text{st}}$ dimension is used then the resulting watermark $c_{m+1}^{(k)^*}$ will be less than or equal to the corresponding "noise level" and therefore no additional dimensions will be water-filled.

Reducing the number of dimensions to only those which are actually water-filled by user k is advantageous from a numerical point of view since the eigen-algorithm will be performed on a lower dimensional problem. We note however, that sum capacity of the MIMO channel does not change since it is still obtained by water filling over the maximum number of dimensions with smallest "background" noise energy in user k's inverted channel signal space.

5.3 Simulation Results

In this section we perform simulations and look at performance in terms of signal-to-noise ratios (SNRs) at the receiver and channel capacity. Capacity is analyzed in the context of fading channels which are characteristic of wireless communications. Due to the randomness of channel realizations, the resulting capacity is a random variable and we plot Complementary Cumulative Distribution Functions (CCDFs) similar to those in [19] from which one can see what capacity can be achieved with a given probability. More precisely, we say that an outage occurs whenever capacity is below a given value, and identify the probability of outage P_{out} from the corresponding CCDF.

The MIMO channel model used for simulations is derived by using the same set of basis functions for the signal space as in Chapter 2 consisting of real sinusoids (sine and cosine functions). Similar to Chapter 2 we assume that the frame duration $\mathcal{T} \gg T_{ij}^{(\ell)}$, $\forall \ell,\ i,\ j$. Consequently, sinusoids are eigenfunctions for all the channels in the multiple antenna link, and let us denote by N_c the number of frequencies used. The number of transmit antennas for user ℓ is T_ℓ and the number of receiver antennas is R.

Decomposition of each channel into orthogonal sub-channels implied by the sinusoidal basis functions leads to multicarrier modulation for transmission of information on each pair of transmit/receive antennas. In this context the equivalent vector channel representation for equation (5.1) is

$$\mathbf{y}_{ij}^{(\ell)} = \mathbf{\Lambda}_{ij}^{(\ell)1/2}\mathbf{x}_i^{(\ell)} \tag{5.29}$$

where $\mathbf{x}_i^{(\ell)}$ and $\mathbf{y}_{ij}^{(\ell)}$ are the $2N_c$-dimensional input and output vectors, and $\mathbf{\Lambda}_{ij}^{(\ell)}$ the $2N_c \times 2N_c$ matrix containing the eigenvalues corresponding to channel having impulse response $h_{ij}^{(\ell)}(t)$. We note that in the context of multicarrier modulation when real sinusoids are approximately channel eigenfunctions, channel eigenvalues correspond to the real channel gains for the frequencies that span the signal space. The received vector at receive antenna j corresponding to the received waveform in equation (5.2) is then given by

$$\mathbf{r}_j = \sum_{\ell=1}^{L}\sum_{i=1}^{T_\ell}\mathbf{y}_{ij}^{(\ell)} + \mathbf{n}_j = \sum_{\ell=1}^{L}\sum_{i=1}^{T_\ell}\mathbf{\Lambda}_{ij}^{(\ell)1/2}\mathbf{x}_i^{(\ell)} + \mathbf{n}_j \tag{5.30}$$

where $\mathbf{n}_j$ is the additive noise vector at receive antenna j with covariance matrix $E[\mathbf{n}_j\mathbf{n}_j^\top] = \mathbf{W}_j$.

By stacking together all received signal vectors from all receive antennas we can write

$$\begin{bmatrix} \mathbf{r}_1 \\ \vdots \\ \mathbf{r}_j \\ \vdots \\ \mathbf{r}_R \end{bmatrix} = \sum_{\ell=1}^{L} \begin{bmatrix} \mathbf{\Lambda}_{11}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{i1}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{T_\ell 1}^{(\ell)1/2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{\Lambda}_{1j}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{ij}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{T_\ell j}^{(\ell)1/2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{\Lambda}_{1R}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{iR}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{T_\ell R}^{(\ell)1/2} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1^{(\ell)} \\ \vdots \\ \mathbf{x}_i^{(\ell)} \\ \vdots \\ \mathbf{x}_{T_\ell}^{(\ell)} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_j \\ \vdots \\ \mathbf{n}_R \end{bmatrix} \tag{5.31}$$

or simply

$$\mathbf{r} = \sum_{\ell=1}^{L}\mathbf{H}_\ell\mathbf{x}_\ell + \mathbf{n} \tag{5.32}$$

where we have denoted by $\mathbf{H}_\ell$ the $2N_cT_\ell \times 2N_cR$ matrix containing channel eigenvalue matrices of all channels in the multiple antenna link between user ℓ and the basestation, $\mathbf{x}_\ell$ the $2N_cN_\ell$-dimensional transmitted vector of user ℓ, and $\mathbf{n}$ the $2N_cR$-dimensional noise vector at the basestation. Under the assumption that noise vectors at different antennas are independent then the noise covariance matrix will be block diagonal, each block containing the covariance of the noise that corrupts the received signal at the corresponding receive antenna

$$E[\mathbf{n}\mathbf{n}^{\top}] = \mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & & \\ & \ddots & \\ & & \mathbf{W}_{N_r} \end{bmatrix}$$

with $\mathbf{W}_j = E[\mathbf{n}\mathbf{n}^{\top}]$.

The above MIMO channel model has been derived under the implicit assumption that the N_c periods of spanning sinusoids are large compared to the propagation delays between antenna elements so that the sine and cosine components are still approximately synchronized at the receiver even in the presence of multiple transmit and receive antennas. In addition, for simplicity, carrier synchronization for all users has also been assumed.

We start with the single user multiple antenna case which was the case considered in previous approaches. For this case we analyze SNR distribution when random precoding matrices are used as well as when the optimal precoding matrices yielded by the eigen-algorithm are used, and we also look at improvement in SNR generated by an increase in number of antennas. CCDFs for sum capacity for single user as well as multiple user cases are also plotted.

5.3.1 Receiver SNR Distribution

In order to compute the SNRs at the receiver antennas we return to equation (5.30) which for only one user becomes

$$\mathbf{r}_j = \sum_{i=1}^{T} \mathbf{y}_{ij} + \mathbf{n}_j = \sum_{i=1}^{T} \mathbf{\Lambda}_{ij}^{1/2} \mathbf{x}_i + \mathbf{n}_j \tag{5.33}$$

Note that we have dropped the user index ℓ and replaced the number of transmit antennas by T since there is only one user in the system. The average energy of the signal at receive antenna j

$$\begin{aligned} E[\mathbf{r}_j^{\top}\mathbf{r}_j] &= \text{Trace}\left[E[\mathbf{r}_j\mathbf{r}_j^{\top}]\right] \\ &= \text{Trace}\left[\sum_{p=1}^{T}\sum_{r=1}^{T} \mathbf{\Lambda}_{pj}^{1/2} E[\mathbf{x}_p\mathbf{x}_r^{\top}]\mathbf{\Lambda}_{rj}^{1/2} + \mathbf{W}_j\right] \\ &= \text{Trace}\left[\sum_{p=1}^{T}\sum_{r=1}^{T} \mathbf{\Lambda}_{pj}^{1/2} E[\mathbf{x}_p\mathbf{x}_r^{\top}]\mathbf{\Lambda}_{rj}^{1/2}\right] + \text{Trace}\left[\mathbf{W}_j\right] \end{aligned} \tag{5.34}$$

where $\mathbf{x}_p,\ \mathbf{x}_r$ represent signals coming from transmit antennas p and r respectively, which can be obtained explicitly by partitioning the $2N_cT \times M$ precoding matrix $\mathbf{S}$ in T blocks of dimensions $2N_c \times M$ stacked together

$$\mathbf{x} = \mathbf{S}\mathbf{b} = \begin{bmatrix} \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_i \\ \vdots \\ \mathbf{S}_T \end{bmatrix} \mathbf{b} = \begin{bmatrix} \mathbf{S}_1\mathbf{b} \\ \vdots \\ \mathbf{S}_i\mathbf{b} \\ \vdots \\ \mathbf{S}_T\mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_i \\ \vdots \\ \mathbf{x}_T \end{bmatrix} \tag{5.35}$$

With these partitions we get $E[\mathbf{x}_p\mathbf{x}_r^\top] = \mathbf{S}_p\mathbf{S}_r^\top$ and we compute the SNR at receive antenna j as the ratio of the first term in the right hand side of equation (5.34) to the second term

$$\mathrm{SNR}_j = \frac{\mathrm{Trace}\left[\sum_{p=1}^{T}\sum_{r=1}^{T} \mathbf{\Lambda}_{pj}^{1/2}\mathbf{S}_p\mathbf{S}_r^\top \Lambda_{rj}^{1/2}\right]}{\mathrm{Trace}\left[\mathbf{W}_j\right]} \tag{5.36}$$

Note that knowledge of channels composing the multiple antenna link implies different SNRs at different receive antennas even in the context of white noise with the same average power. This is different from [19] where the same average SNR at all receive antennas has been assumed. However, we have a similar constraint on the total transmitted power which is constant regardless of the number of transmit antennas used.

With random precoding matrices and for a particular set of channels comprising the multiple antenna link, the SNRs at different receive antennas have the bell-shaped distribution shown in 5.3 (upper plots). The SNR at all receiver antennas has been recorded for a number of $N_c = 5$ spannig frequencies (corresponding to 10 real sinusoids), with equal number of antennas at transmitter and receiver $T = R = 1,\ 2,\ 4,\ 8$, and average power of the white noise $N_0 = 0.5$ at each receive antenna for the same set of channels but for different (random) precoding matrices. However, application of interference avoidance water fills the channels appropriately and results in a fixed set of SNRs at each receive antenna for given instances of the channel(s). As it can be seen from Figure 5.3 doubling the number of antenna elements in both transmitter and receiver results in about a 3 dB improvement in the SNR. Also note that after interference avoidance the SNR is approximately the same for all receive antennas, even though no a priori assumption about equal SNRs at each antenna [19] has been made.

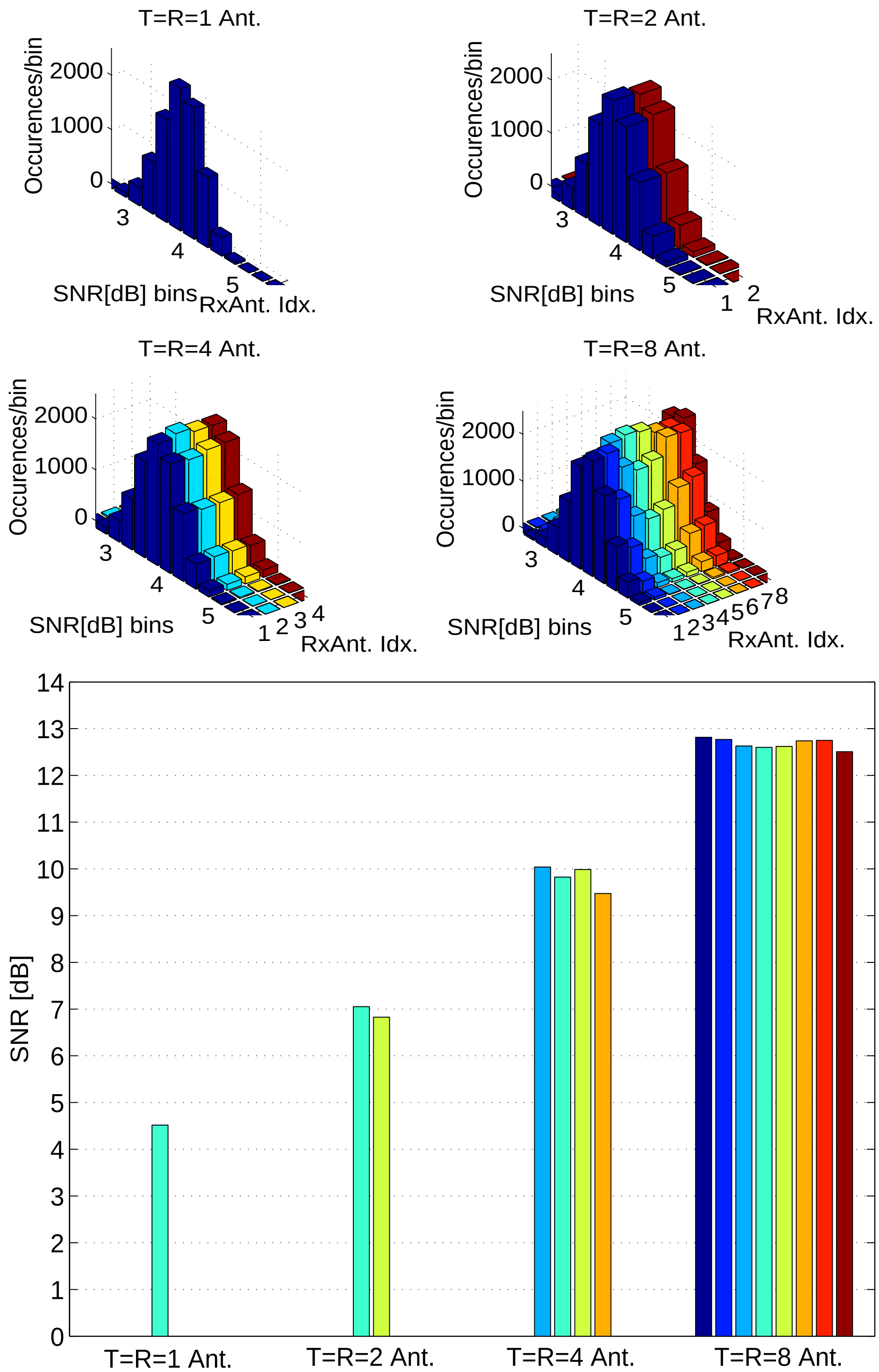

Figure 5.3: SNR distributions for a single user multiple antenna system with random precoding matrices (upper plot). SNRs with optimal precoding matrices yielded by the interference avoidance algorithm for a single user multiple antenna system (lower plot). Signal space dimension is $N = 10$ and average power of white noise is $N_0 = 0.5$ at each receive antenna.

5.3.2 Fading Channels and Outage Capacity

In the case of fading environments, which is characteristic of wireless communications, the impulse responses of channels in the multiple antenna link change over time and it becomes difficult, if not impossible, to apply interference avoidance to determine optimal precoding matrices corresponding to all channel realizations that occur during the duration of the transmission. However, in such cases interference avoidance is applied using average characteristics of the channels as described in Chapter 3 to determine precoder matrices which are optimal for the average channel.

We analyze the effect of fading on the sum capacity of our MIMO channel model by using precoding matrices which are optimal for the average channel (obtained by application of the eigen-algorithm) and computing the corresponding sum capacity for various realizations of the fading channel models assumed. The results are used to plot the corresponding CCDFs.

We assume the same frequency-selective fading channel model as in Chapter 3 which is suited to the multicarrier modulation scheme used for transmission [92].

For a single user system, we perform a set of experiments in which we first determine a precoding matrix which is optimal for the average channel defined in terms of a set of average values of the Rayleigh random variables, and then compute capacity values for distinct realizations of these Rayleigh random variables using equation

$$C = \frac{1}{2}\log[\det(\mathbf{H}\mathbf{S}\mathbf{S}^\top\mathbf{H}^\top + \mathbf{W})] - \frac{1}{2}\log(\det\mathbf{W}) \tag{5.37}$$

where $\mathbf{H}$ is the MIMO channel matrix of the considered user. Again there is no need for user index ℓ as there is only one user in the system. The resulting set of capacity values are used to derive the CCDFs presented in Figure 5.4. In these plots we compare the case of only one transmit and one receive antenna with the case of two transmit and two receive antennas, and four transmit and four receive antennas respectively. Simulations are done with a set of $N = 5$ carrier frequencies for different average white noise power at the receiver $N_0 = 1,\ 0.5,\ 0.25,\ 0.1$. These values correspond to a 3 dB increase in the SNR at receiver antennas and for the average channel imply SNRs of approximately[2] 1dB, 4dB, 7dB, and 10 dB for the $T = R = 1$ antenna case, 4 dB, 7dB, 10dB and

[2] As it has been seen, the SNRs at receive antennas are not necessarily the same. However, when optimal codewords are used they are approximately equal.

13dB for the $T = R = 2$ antenna case, and 7 dB, 10 dB, 13 dB, and 16 dB for the $T = R = 4$ antenna case.

From the CCDFs in Figure 5.4 see that for the analyzed scenario, the use of multiple antennas at both transmitter and receiver improves outage capacity. For example, for an outage probability $P_{out} = 1\%$, capacity is increased for low SNR from less than 0.5 bits/sec/Hz to approximately 1.1 bits/sec/Hz for two transmit and receive antennas, and almost 2.7 bits/sec/Hz for four transmit and receive antennas. For high SNR, the rates increase from from about 1.5 bits/sec/Hz to about 3.6 bits/sec/Hz for two transmit and receive antennas and 5.7 bits/sec/Hz for four transmit and receive antennas.

Next we consider a multiuser MIMO system with $L = 2$ users and perform similar simulations as in the single user case with $N = 5$ carrier frequencies for different average white noise power at the receiver $N_0 = 1,\ 0.5,\ 0.25,\ 0.1$. Optimal precoding matrices for the average channel for two users are determined, and then sum capacity values for distinct realizations of these Rayleigh random variables is computed using equation

$$C = \frac{1}{2}\log\left[\det\left(\sum_{\ell=1}^{L}\mathbf{H}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{H}^\top + \mathbf{W}\right)\right] - \frac{1}{2}\log(\det\mathbf{W}) \tag{5.38}$$

and the resulting set of capacity values are used to derive the CCDFs presented in Figure 5.5. In these plots we compare sum capacity for the two user system in the case of only one transmit antenna per user and one receive antenna with the case of two transmit antennas per user and two receive antennas, and four transmit antennas per user and four receive antennas respectively. From the CCDFs in Figure 5.5 we see similar improvements in sum capacity for the analyzed scenario and conclude that the use of multiple antennas in the transmitters as well as in the receiver improves outage capacity.

5.4 Chapter Summary

Application of interference avoidance methods to multiuser MIMO systems systems has been presented and analyzed in this chapter. Such systems are associated with the uplink of a wireless system in which users and the basestation have multiple antennas.

Our approach is based on application of interference avoidance to general multiaccess vector

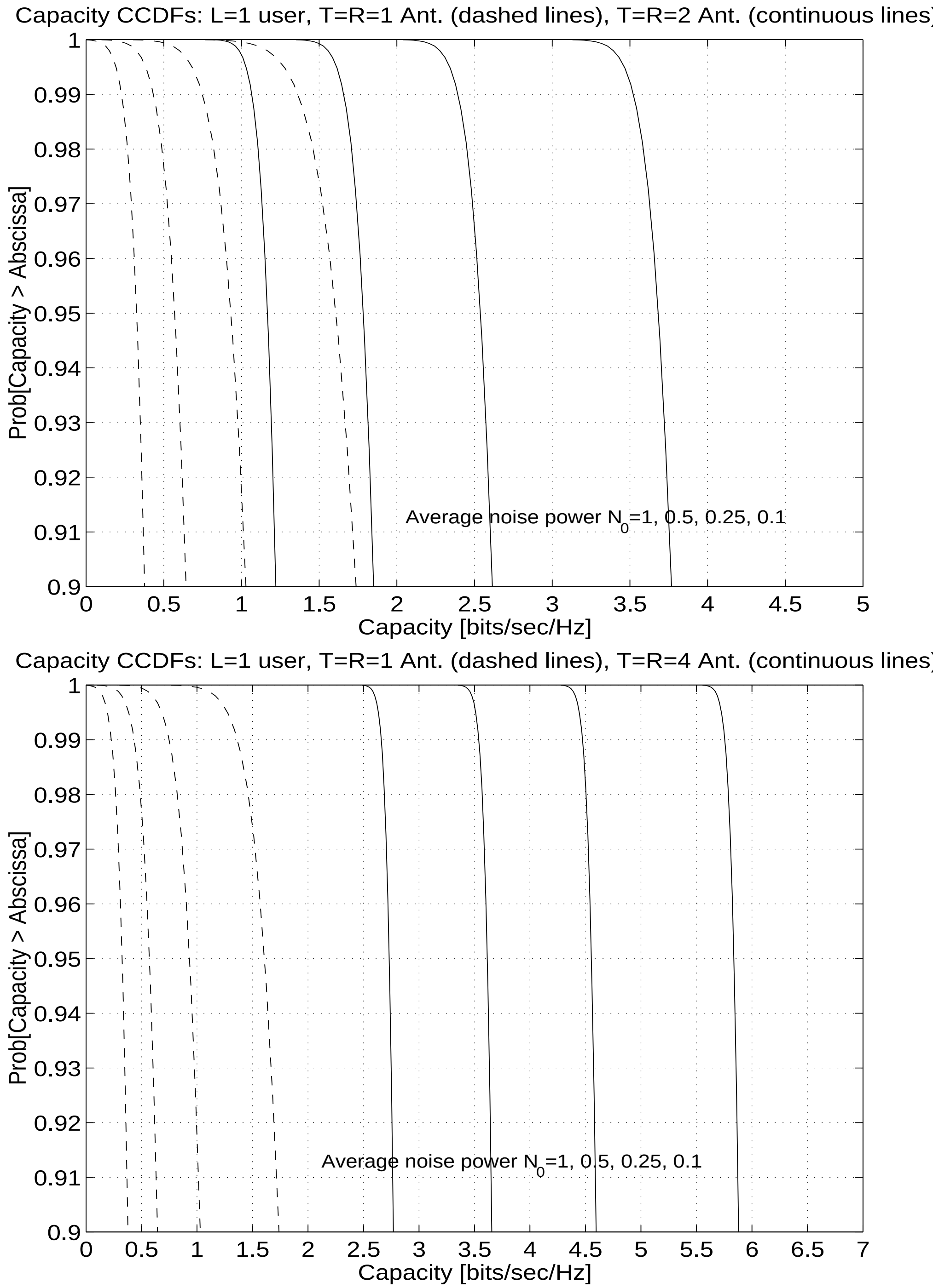

Figure 5.4: Capacity CCDFs for single user MIMO system. The $T = R = 1$ antenna case is compared with the $T = R = 2$ antenna case (upper plot) and with the $T = R = 4$ antenna case (lower plot).

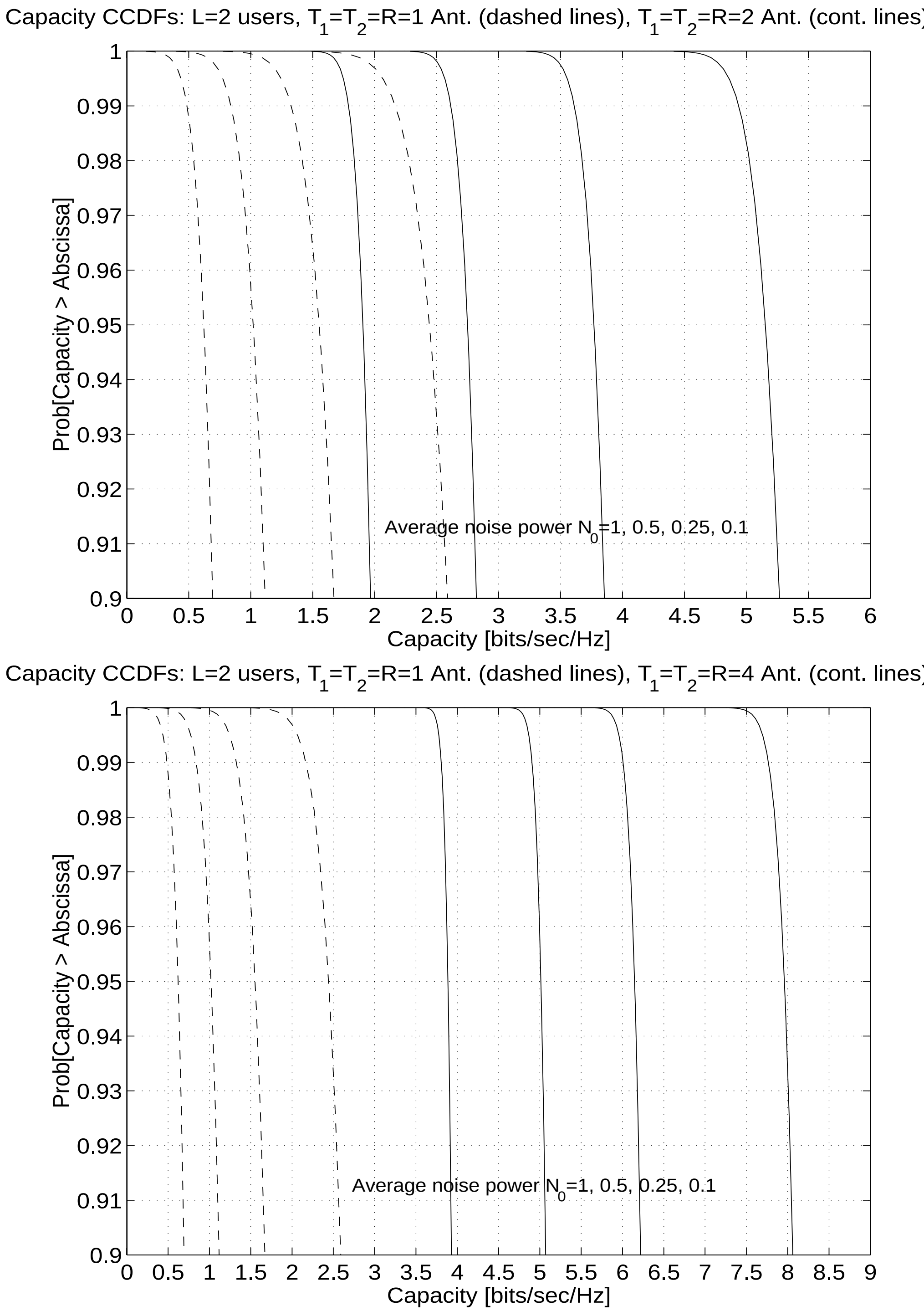

Figure 5.5: Sum capacity CCDFs for a two-user MIMO system. The $T_1 = T_2 = R = 1$ antenna case is compared with the $T_1 = T_2 = R = 2$ antenna case (upper plot) and with the $T_1 = T_2 = R = 4$ antenna case (lower plot).

channels presented in Chapter 4 for the particular multiaccess vector channel corresponding to the multiuser MIMO system. We note that the approach is completely general and applicable to any MIMO system models regardless of the choice of signal space basis functions. Information is sent in frames using a multicode CDMA approach with spreading over the available dimensions implied by a precoding matrix. Optimal precoding matrices for which sum capacity is maximized are obtained for all users in the system through sequential application of the eigen-algorithm for interference avoidance. The procedure, which is an instance of iterative water filling [94], works in the single user case as well as in the multiuser case.

Numerical results based on simulations were also presented in the paper. We note that these results are consistent with the well known results in the multiple antenna literature, namely that the use of multiple antennas in both the transmitter and the receiver is beneficial for system performance.

For a single user system, the SNR distribution at receiver antennas was computed with both random precoding matrices as well as with optimal precoding matrices yielded by the eigen-algorithm, and it has been noted that with the latter the SNR is approximately the same for all receive antennas in the absence of any a priori assumptions (like those made for example in [19]). It has also been noted that doubling the number of antennas (both in the transmitter and in the receiver) translates to a gain of about 3 dB in the SNR when optimal precoding matrices are used.

Sum capacity of the multiuser MIMO channel was also investigated in the context of fading environments. A frequency-selective fading channel model was assumed and precoding matrices optimal for the average channel were used and sum capacity was treated as a random variable in this case for which CCDF curves were plotted. We note that single user CCDFs are comparable to those in [19] showing similar capacity improvements when multiple antennas are used in both the transmitter and the receiver. The CCDFs for multiple users show also improvement in sum capacity when multiple antennas are used in conjunction with designing precoding matrices that are optimal for the average channel for all users in the system.

Chapter 6

Interference Avoidance for Asynchronous Systems

An asynchronous multiuser system consists of a number users received at a common receiver with non-coincident symbol intervals. This can be a consequence of lack of synchronization for systems with users having the same data rate, or can be generated by differences in data rates for systems where distinct users are allowed to transmit at different data rates. The latter case is especially interesting for future wireless systems which may have to support users with different data rates. Relaxing synchronization constraints simplifies the system design by relaxing timing control.

The capacity region of symbol-asynchronous Gaussian multiple access channels has been derived in [82] using an equivalent multiple access channel model with memory and frame synchronism [81]. Recently, for chip-based DS-CDMA systems user capacity of the asynchronous system has been analyzed and compared to that of the synchronous system [33] and a class of optimum signature sequences has been identified [77] for which there is no loss in user capacity due to asynchrony. However, the analysis is restricted to symbol-asynchronous but chip-synchronous DS-CDMA systems.

In this chapter we present application of interference avoidance methods to codeword optimization in an asynchronous CDMA system. Similar to [82] we consider users in the system frame synchronous and use a multicode CDMA transmission scheme. Symbols of a frame are sent in parallel using distinct signature waveforms of extended duration. This approach relaxes the synchronization requirements for the multiuser system since the frame duration will be larger than the duration of individual symbol intervals would have been if sequential transmission of symbols in a frame had been used.

An equivalent discrete time vector channel model is obtained for which application of interference avoidance is straightforward and results in codeword (or equivalently waveform) ensembles that maximize sum capacity.

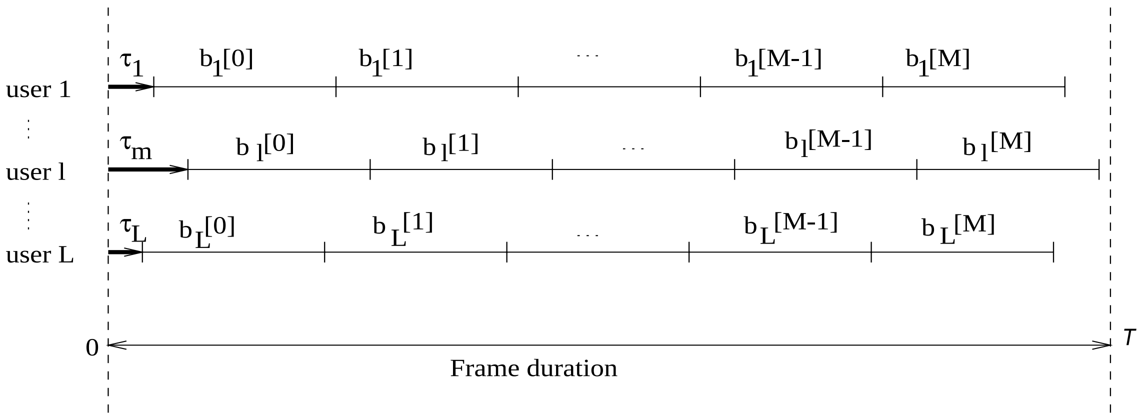

Figure 6.1: Symbol-asynchronous users with the same data rate $1/T$ modeled as frame synchronous. Within the frame duration each user sends M symbols.

6.1 Problem Statement

We start by considering an asynchronous CDMA system with users transmitting at the same data rate $1/T$ for which symbol intervals corresponding to different users are not synchronized at the common receiver. Following the methodology described in [83, p. 21] one needs to introduce offsets that model the lack of alignment of symbol intervals at the receiver. As opposed to the synchronous model for which the received signal can be written by taking only one-shot of the model over the symbol interval $[0, T]$ as in equation (1.2) for the asynchronous case one needs to include the offsets $\tau_\ell \in [0, T)$, $\ell = 1, \ldots, L$, and consider the fact that users send frames with many symbols $\mathbf{b}_\ell = [b_1^{(\ell)}, \ldots, b_M^{(\ell)}]$ of duration $\mathcal{T}$ as shown in Figure 6.1. This implies that frame-synchronism rather than symbol-synchronism is assumed, and users start and finish their transmissions within T units of each other. The received signal is then written as

$$R(t) = \sum_{\ell=1}^{L} \sum_{m=1}^{M} b_m^{(\ell)} S_\ell(t - mT - \tau_\ell) + n(t) \tag{6.1}$$

Similar to [82] we assume that the offsets τ_ℓ, $\ell = 1, \ldots, L$, are known to the receiver, but unknown to the transmitters so that they cannot advance or retard their transmissions. As noted in [82] the assumption of frame synchronism can be made at any rate with some sort of channel feedback which is an underlying assumption for systems using interference avoidance methods [63].

Since each symbol transmitted by a given user overlaps also with past/future symbols transmitted by the other users the symbol asynchronous multiple access channel has memory and can be modeled through an equivalent multiple access channel with memory [81, 83]. Thus, one can think of approaching the asynchronous system similar to a system where symbols are subject to ISI (dispersive channels): the use of a multicode CDMA scheme for transmission of symbols using signature waveforms of extended duration to convey each symbol in the frame. While in the case of dispersive channels this approach makes ISI inconsequential, for asynchronous systems this relaxes the synchronization requirement, making it easier to be obtained in practice. Furthermore, this approach can be easily generalized to users with different data rates by considering a different number of symbols M_ℓ in a frame for distict user ℓ. The transmitted signal corresponding to user ℓ is written as

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t) \tag{6.2}$$

with $s_m^{(\ell)}(t)$ being the signature waveform corresponding to symbol m of user ℓ of duration $\mathcal{T}$. We note here that a similar approach has been used in [83, Ch. 4] in the analysis of multiuser detectors for asynchronous CDMA systems. The received signal at the basestation is the sum of signals transmitted by all users plus additive Gaussian noise and is written as

$$R(t) = \sum_{\ell=1}^{L} x_\ell(t-\tau_\ell) + n(t) = \sum_{\ell=1}^{L}\sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t-\tau_\ell) + n(t) \tag{6.3}$$

Our goal is then to find an optimal ensemble of waveforms $s_m^{(\ell)}(t)$, $\ell = 1, \ldots, L$, $m = 1, \ldots, M_\ell$, for which the sum capacity of the multiple access channel is maximized.

6.2 The Equivalent Vector Multiple Access Channel

The multicode CDMA scheme in the previous section implies a vector multiple access channel representation for which application of interference avoidance as presented in Chapter 4 is straightforward. In order to derive the equivalent vector channel model, we assume that each user ℓ resides in a signal space of finite dimension N_ℓ implied by the frame duration $\mathcal{T}$ and finite bandwidth W_ℓ, and spanned by the vector of functions $\mathbf{\Psi}^{(\ell)}(t) = [\Psi_1^{(\ell)}(t) \ldots \Psi_{N_\ell}^{(\ell)}(t)]^\top$. Furthermore, we assume that the receiver signal space of dimension N is spanned by $\mathbf{\Phi}(t) = [\Phi_1(t) \ldots \Phi_N(t)]^\top$ and is implied by bandwidth W that includes all W_ℓ's corresponding to all users and observation interval

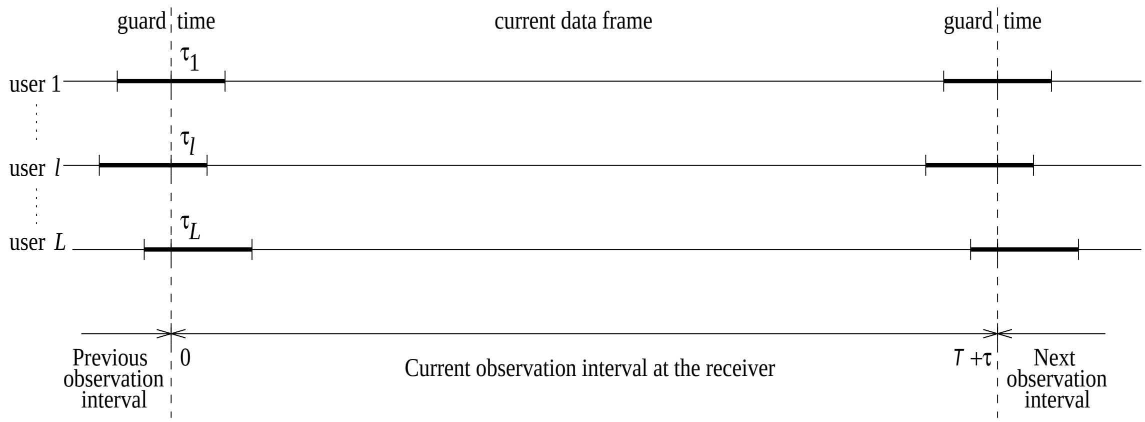

Figure 6.2: Users sending data frames of duration $\mathcal{T}$ with guard intervals.

$\mathcal{T} + \tau$ with $\tau = \max_\ell\{\tau_\ell\}$ being the largest user delay measured with respect to the begining of the observation interval at the receiver (see Figure 6.2).

We note that, by adding the maximum delay τ to the frame duration we ensure that all waveforms corresponding to the current frame are completely observed during the observation interval at the receiver. We also note that, in order to avoid overlap between succesive frames at the receiver, time guard intervals are inserted at the begining and end of each transmitted frames as it can be seen in Figure 6.2.

Each user ℓ transmits the signal $x_\ell(t)$ given by equation (6.2) which is written in terms of user ℓ basis functions as

$$x_\ell(t) = \mathbf{\Psi}^{(\ell)}(t)^\top \mathbf{x}_\ell = \mathbf{\Psi}^{(\ell)}(t)^\top \mathbf{S}_\ell \mathbf{b}_\ell \tag{6.4}$$

with

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \dots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \qquad \ell = 1, \dots, L$$

the codeword matrix of user ℓ. The received signal at the common receiver contains signals of all users with corresponding delays plus additive noise

$$r(t) = \sum_{\ell=1}^{L} x_\ell(t - \tau_\ell) + n(t) \tag{6.5}$$

and is observed over the interval $[0, \mathcal{T} + \tau]$. By projecting the received signal onto the basis functions of the receiver signal space an equivalent vector channel model is obtained for which the received signal vector is

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{6.6}$$

with the $N \times N_\ell$ matrix $\mathbf{H}_\ell$ relating the user ℓ signal space and receiver signal space defined by

$$\mathbf{H}_\ell = \int_0^{\mathcal{T}+\tau} \mathbf{\Phi}(t) \mathbf{\Psi}^{(\ell)}(t - \tau_\ell)^\top dt \tag{6.7}$$

Note that equation (6.6) is identical in form to equation (4.6), thus allowing straightforward application of the generalized eigen-algorithm to determine optimal codeword ensembles that maximize sum capacity as described in Chapter 4. For completness we formally state the generalized eigen-algorithm as it applies to codeword optimization for asynchronous systems:

The Generalized Eigen-Algorithm for Asynchronous CDMA Systems

1. Start with a randomly chosen codeword ensemble specified by the codeword matrices $\{\mathbf{S}_k\}_{k=1}^{L}$ and a specified set of user offsets $\{\tau_k\}_{k=1}^{L}$
2. Determine the set of channel matrices $\{\mathbf{H}_k\}_{k=1}^{L}$ using equation (6.7)
3. For each user $k = 1 \ldots L$
   (a) compute the transformation matrix $\mathbf{T}_k$ that whitens the interference-plus-noise seen by user k
   (b) Change coordinates and compute user k's transformed channel matrix $\tilde{\mathbf{H}}_k = \mathbf{T}_k \mathbf{H}_k$
   (c) Apply SVD for $\tilde{\mathbf{H}}_k$ and project the problem onto user k's signal space to obtain $\tilde{\mathbf{r}}_k$ in equation (4.31)
   (d) adjust user k codewords sequentially: the codeword corresponding to symbol m of user k is replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process in the inverted channel space
   (e) Iterate step (d) until convergence (making use of escape methods [62] if the procedure stops in suboptimal points)
4. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

So, in summary, by using parallel transmission of symbols in a frame with signature waveforms of extended duration the asynchronous multiuser system can be modeled as a multiaccess vector channel for which application of the generalized eigen-algorithm of is straightforward. We note that, using the resulting optimal codeword ensembles implies a very simple structure at the receiver with matched filters as optimal linear receivers [84, 85].

6.3 Chapter Summary

Application of interference avoidance for asynchronous CDMA systems was presented in this chapter. The asynchronous system is modeled as frame-synchronous and a multicode CDMA approach is used for transmission of symbols in a frame through signature waveforms of extended duration. A vector multiple access channel model is derived to which interference avoidance applies in a straightforward way and yields optimal codeword ensembles that maximize sum capacity.

Chapter 7

Empirical Studies

In this chapter we present some empirical studies on various issues of the eigen-algorithm for interference avoidance. Since power is a main factor in determining interference, in a practical implementation one could also think of incorporating some power control mechanism along with interference avoidance in order to provide better performance and possibly minimize transmitted power. In section 7.1 we present some theoretical considerations leading to an algorithm that combines interference avoidance with a power control mechanism in which each user adjusts both codeword and power so as to achieve a target SINR with minimum power.

In section 7.2 the SINR variation during the transient phase of the greedy algorithm for interference avoidance is analyzed, and empirical evidence showing that at any step of the algorithm the SINR cannot go below the minimum starting value is provided.

Quantization issues for interference avoidance are explored in section 7.3. Codewords are calculated based on information at the receiver and must be fed back to the transmitter either directly or indirectly (by feeding back covariance information for example). In either case, the accuracy of the feedback affects performance. We therefore examine the level of quantization necessary to achieve almost optimal performance, noting that high accuracy entails high demands on the feedback channel.

In section 7.4 we look at complexity issues. We evaluate operational complexity of the eigen-algorithm and explore the reduction in receiver complexity implied by the channelization produced by application of interference avoidance for dispersive channels.

7.1 Interference Avoidance and Power Control

We consider a synchronous multiuser communication system with M users, similar to the one considered in Chapter 1 but in which users are received with different powers at the basestation. The received signal at the basestation is described by a relation similar to equation (1.2) modified to reflect the unequal user powers

$$\mathbf{r} = \sum_{i=1}^{M} b_i \sqrt{p_i}\mathbf{s}_i + \mathbf{n} \tag{7.1}$$

All terms have the same meaning as in equation (1.2), and p_i is the received power of user i. We define the $M \times M$ diagonal matrix $\mathbf{P}$ containing received powers of all users

$$\mathbf{P} = \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_M \end{bmatrix} \tag{7.2}$$

and rewrite the received signal similar to equation (1.4)

$$\mathbf{r} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n} \tag{7.3}$$

Following the same line of reasoning as in Chapter 1 we assume simple matched filters at the receiver for all users and compute the SINR for user k

$$\begin{aligned} \gamma_k &= \frac{\text{power of user } k}{\text{power of interference-plus-noise seen by user } k} = \frac{p_k}{i_k} \\ &= \frac{(\sqrt{p_k}\mathbf{s}_k^\top \mathbf{s}_k)^2}{\sum_{j=1, j\neq k}^{M} (\mathbf{s}_k^\top \mathbf{s}_j \sqrt{p_j})^2 + E[(\mathbf{s}_k^\top \mathbf{n})^2]} = \frac{p_k}{\mathbf{s}_k^\top \left(\sum_{j=1, j\neq k}^{M} p_j \mathbf{s}_j \mathbf{s}_j^\top + \mathbf{W} \right) \mathbf{s}_k} \end{aligned} \tag{7.4}$$

We define the correlation matrix of the interference-plus-noise seen by user k in this case

$$\mathbf{R}_k = \sum_{j=1, j\neq k}^{M} p_j \mathbf{s}_j \mathbf{s}_j^\top + \mathbf{W} = \mathbf{S}\mathbf{P}\mathbf{S}^\top - p_k \mathbf{s}_k \mathbf{s}_k^\top + \mathbf{W} \tag{7.5}$$

which implies that the SINR for user k becomes

$$\gamma_k = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} \tag{7.6}$$

When the powers of users are fixed but unequal and only the codewords are modified, greedy interference avoidance algorithms have also been introduced and analyzed in [63]. We note that using the SINR as metric maximizing it through adaptation of user codewords implies decreasing the term $\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k$ in equation (7.6). Therefore, similar to the equal power case presented in Chapter 1, replacement of user k's codeword $\mathbf{s}_k$ by a new codeword $\mathbf{x}_k$ such that equation (1.15) is satisfied would define a valid interference avoidance algorithm. We again note, that the quantity $\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k$ which is the Rayleigh quotient of matrix $\mathbf{R}_k$, is absolutely minimized by choosing $\mathbf{x}_k$ to be the minimum eigenvector of $\mathbf{R}_k$. Replacement of user k's codeword $\mathbf{s}_k$ by minimum eigenvector $\mathbf{x}_k$ of the interference-plus-noise correlation matrix $\mathbf{R}_k$ defines greedy interference avoidance for unequal powers, and its properties and convergence analysis can also be found in [62, 63].

For a combined interference avoidance and power control algorithm, we first note that when the goal is to achieve a specified target SINR for each user, it may not necessarily imply maximizing or even increasing the SINR. For example a user might have a better SINR than required and might like to adjust both codeword and power so that the target SINR value is met.

We also note that if user power were the only adjustable parameter, then standard power control algorithms [91] could be employed to meet the target SINR for each user in the system. However, interference avoidance methods provide additional degrees of freedom, by allowing users to change their codewords in addition to their power. Therefore, our goal in combining interference avoidance with power control is *to achieve a target SINR with minimum user power by decreasing effective interference through codeword adaptation.*

Let us denote by γ_k^* the target SINR for user k which is achieved for a given power p_k^* and codeword $\mathbf{x}_k$. According to equation (7.6) this can be written as

$$\gamma_k^* = \frac{p_k^*}{\mathbf{x}_k^\top \mathbf{R}_k \mathbf{x}_k} \tag{7.7}$$

It is obvious that when both the power and the codeword are adjustable, the target SINR for user k can be met by a multitude of combinations $\{p_k^*,\ \mathbf{x}_k\}$. However, since power is a precious resource in a wireless system, we are interested in that combination for which the target SINR is achieved with minimum user power. This is possible when effective interference seen by user k is minimum as well and this can be obtained by replacing user k codeword with the minimum eigenvector of $\mathbf{R}_k$. The resulting SINR after codeword replacement will be proportional to the inverse minimum eigenvalue λ_k^* of $\mathbf{R}_k$, and the target γ_k^* for user k will be obtained by adjusting power to

$$p_k^* = \gamma_k^* \lambda_k^* \tag{7.8}$$

This leads to the following algorithm of combined interference avoidance and power control:

The Combined Interference Avoidance and Power Control Algorithm

1. For a given background noise with covariance matrix $\mathbf{W}$ start with a random set of user codewords and powers specified by matrices $\mathbf{S}$ and $\mathbf{P}$ respectively.

2. Specify the desired target SINRs $\gamma_1^*, \ldots, \gamma_M^*$.

3. For each user $k = 1, \ldots, M$ do

   (a) Compute covariance matrix of the interference-plus-noise

   $$\mathbf{R}_k = \mathbf{S}\mathbf{P}\mathbf{S}^\top + W - p_k \mathbf{s}_k \mathbf{s}_k^\top \tag{7.9}$$

   and determine minimum eigenvalue λ_k^* and corresponding eigenvector $\mathbf{x}_k$

   (b) Minimize effective interference for user k by replacing its current codeword $\mathbf{s}_k$ with the minimum eigenvector $\mathbf{x}_k$ of $\mathbf{R}_k$

   (c) Adjust power to meet the target SINR for user k

   $$p_k^* = \gamma_k^* \lambda_k^* \tag{7.10}$$

4. Repeat step 3 until a fixed point is reached.

Even though this algorithm is simple and easy to apply, simulations have shown that it does not always reach a fixed point. Two distinct situations have been observed from simulations:

- User codewords keep changing and powers keep increasing in their attempt to meet target SINRs. This situation is similar to that occuring for regular power control algorithms when targets are unfeasible and, without a maximum power limit set user powers would probably increase indefinitely.

- The algorithm gets trapped in limit cycles where codewords keep changing to minimize effective interference while powers cycle through distinct values but do not increase indefinitely.

A "lagged" version of this algorithm can be defined by modifying step 3(b) and replacing the current codeword of user k with a linear combination of the old codeword $\mathbf{s}_k$ and the minimum eigenvector $\mathbf{x}_k$ normalized to unit norm

$$\mathbf{v} = \frac{\alpha \mathbf{s}_k + (1 - \alpha \mathbf{x}_k)}{\|\alpha \mathbf{s}_k + (1 - \alpha \mathbf{x}_k)\|} \tag{7.11}$$

With an appropriately chosen lag constant $\alpha \in (0, 1)$ simulations have shown that the lagged version of the combined interference avoidance and power control algorithm usually reaches a fixed point which is defined by the requirement that user codewords be minimum eigenvectors of their respective $\mathbf{R}_k$ matrices.

Although we have not been able to prove this result theoretically, simulations have shown that at the fixed point reached by the algorithm user coderwords and powers have properties identical to those in [84], thus leading to an optimal situation in which users water fill the signal space, with eventual oversized users that have private channels for communication [63, 84]. We note that in our case oversized users are implied by high target SINRs relative to the target SINRs of the other users. More precisely, in our case users with high target SINRs relative to the rest of the users end up using those signal space dimensions with lowest background noise energy thus ensuring that minimum power is used to achieve the target SINR.

We also note that a more detailed theoretical analysis of the fixed point for the combined interference avoidance and power control algorithm and an eventual convergence proof must take into account the fact that the total received power at the basestation is not constant throughout the algorithm. Therefore the use of global criteria like TSC or sum capacity as in [3,62] is no longer possible, thus making the convergence analysis more difficult.

7.2 Transients of the Eigen-Algorithm

Consider the transient phase of the greedy interference avoidance algorithm before the codewords have settled. We ask the following question: *What if the SINR of one or several users drops below a certain level and the connection with the basestation is lost*? We present empirical evidence which suggests that the minimum SINR is usually not decreased during application of the eigen-algorithm for interference avoidance. As a consequence, this ensures that if the user with the worst SINR had

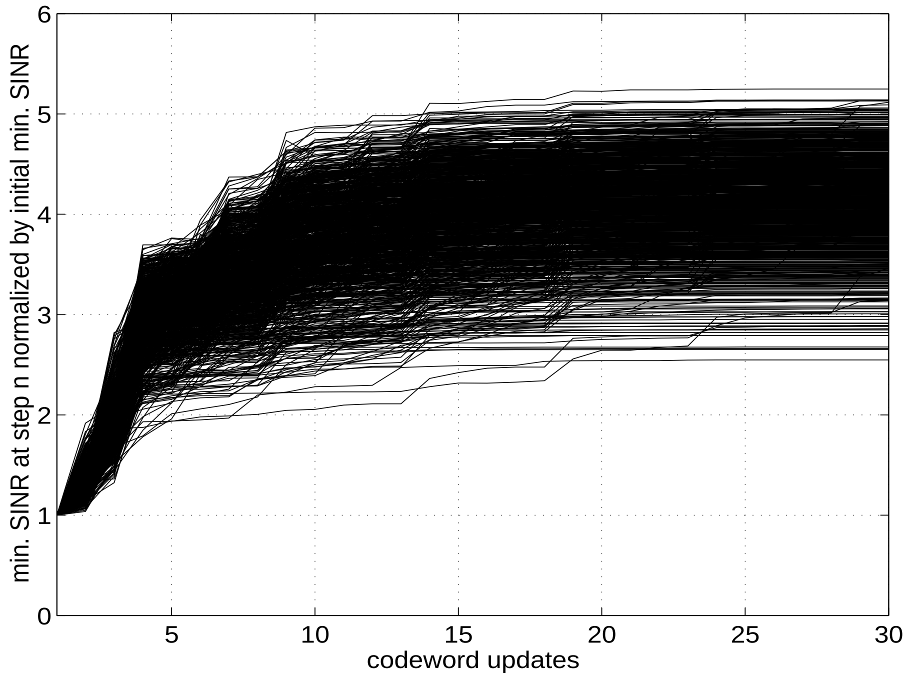

Figure 7.1: Variation of the minimum SINR normalized by the initial minimum SINR represented vs. the number of codeword updates for 1,000 random codeword ensembles for $M = 5$ users in a signal space with dimension $N = 3$ and white noise with variance $N_0 = 0.1$. We note that after 6 iterations the eigen-algorithm settles down.

an acceptable connection with the basestation, then usually no one's connection will be any worse than this as codewords are updated.

We have performed experiments for various values of signal space dimensions N, and number of users M. For each value of M and N, the eigen-algorithm was applied to 1,000 random codeword ensembles and the minimum SINR value was recorded before and after a codeword update was performed. We note that convergence to near optimal codeword ensembles occurs after several iterations of the eigen-algorithm [63], and therefore each user updates its codeword several times. The minimum SINR after each codeword update was normalized by the initial minimum SINR and this value was plotted versus the number of codeword updates. From Figure 7.1, which is typical for randomly chosen initial codeword ensembles, we note that the value of the normalized minimum SINR is always larger than 1 which implies that at any step of the algorithm the minimum SINR does not go below the initial minimum SINR. Furthermore, we note that with each codeword update the minimum SINR is increasing and after several iterations (6 for the experiment in Figure 7.1), once the the eigen-algorithm settles down, the minimum SINR becomes equal to the optimal value. As we have already mentioned, this empirical evidence suggests that if the user with the worst SINR has an acceptable connection with the basestation when codeword update begins, connections of all users with the basestation will not be lost during the codeword adaptation process.

Of course there exist special cases of codeword ensembles for which the codeword update of a given user has a negative impact on the user with worst SINR, thus contradicting the empirical results that have been obtained. We provide such an example for $M = 4$ users in a signal space of dimension $N = 3$ and additive white Gaussian noise with zero mean and variance $N_0 = 0.1$. The matrix containing initial user codewords is

$$\mathbf{S} = \begin{bmatrix} 0 & 0.1 & -\sqrt{0.99} & \sqrt{0.99} \\ 0.1 & \sqrt{0.98} & 0 & 0.1 \\ \sqrt{0.99} & 0.1 & 0.1 & 0 \end{bmatrix} \tag{7.12}$$

with initial SINR values equal to

$$\gamma_1 = 6.6935 \quad \gamma_2 = 5.3530 \quad \gamma_3 = 0.9107 \quad \gamma_4 = 0.8932 \tag{7.13}$$

When user 1 changes codeword according to the eigen-algorithm the new codeword matrix becomes

$$\mathbf{S} = \begin{bmatrix} 0.0558 & 0.1 & -\sqrt{0.99} & \sqrt{0.99} \\ -0.1108 & \sqrt{0.98} & 0 & 0.1 \\ 0.9923 & 0.1 & 0.1 & 0 \end{bmatrix} \tag{7.14}$$

and the new SINR values are

$$\gamma_1 = 9.6238 \quad \gamma_2 = 6.7827 \quad \gamma_3 = 0.9174 \quad \gamma_4 = 0.8917 \tag{7.15}$$

As it can be seen, the resulting minimum SINR after codeword replacement by user 1 is smaller than the original minimum SINR.

However, since the order in which codewords are updated is not fixed this situation can be easily avoided by changing the codeword corresponding to the user with worst SINR first. For the particular example above changing user 4 codeword according to the eigen-algorithm results in the new codeword matrix

$$\mathbf{S} = \begin{bmatrix} 0 & 0.1 & -\sqrt{0.99} & -0.4487 \\ 0.1 & \sqrt{0.98} & 0 & 0.6499 \\ \sqrt{0.99} & 0.1 & 0.1 & -0.6135 \end{bmatrix} \tag{7.16}$$

with the corresponding SINR values

$$\gamma_1 = 2.2383 \quad \gamma_2 = 2.2941 \quad \gamma_3 = 3.7562 \quad \gamma_4 = 1.1987 \tag{7.17}$$

with the minimum SINR after codeword update being larger than before.

An intuitive explanation of this nice behaviour of the eigen-algorithm can be given in terms of codeword migration in signal space. More precisely, the user with worst SINR dwells in a crowded region of the signal space, receiving a lot of interference from other users that share the same part of the signal space, and which have also bad SINRs. By applying the eigen-algorithm for interference avoidance the user with worst SINR goes away from the "noisy crowd", thus offering some relief in terms of energy in the crowded region. Consequently, the SINRs can not decrease below the initial value.

7.3 Codeword Quantization

For interference avoidance methods, user codewords and corresponding receiver filters are adapted iteratively in order to improve performance. For practical implementation one must consider the feedback channel between the receiver, which calculates the codeword adjustments, and the transmitter which uses them. In particular, compact representation of codewords is extremely important for systems which employ interference avoidance since as opposed to current CDMA systems where uniform-amplitude codeword chips are used, interference avoidance employs real-valued "chips"– real-valued coefficients for a set of orthonormal basis functions of the signal space used by the transmitter and receiver.

Individual user waveforms are represented as linear superpositions of orthonormal basis functions which span a signal space. For a single waveform, the real-valued superposition coefficients comprise the *codeword*. These codewords are adjusted iteratively in response to interference conditions and fed back to the transmitter. Since in a real system real values cannot be specified with infinite precision, if interference avoidance is to move from theory to practice, then the necessary precision of codeword representation must be considered.

The problem of quantizing the optimal codewords generated by the interference avoidance algorithms is investigated in order to determine its influence on system performance. Quantization effects may be seen when comparing the SIR obtained with optimal codewords vs. the SIR obtained with quantized optimal codewords. Also, the value of the TSC which was minimized by the interference avoidance algorithms is usually increased by quantizing optimal codewords.

Scalar quantization of each codeword component is considered, with the final goal of encoding the resulting levels into a binary sequence to be transmitted over a control channel. The amount of information sent over this control channel can be regarded also as a measure for the efficiency of codeword feedback for interference avoidance.

In scalar quantization [56] the set of real numbers $\mathbb{R}$ is partitioned into L disjoint subsets $\{\mathcal{R}_k\}_{k=1}^{L}$ and a representation point is chosen for each subset. The quantization function is

$$Q(x) = \hat{x}_k \quad \forall x \in \mathcal{R}_k \tag{7.18}$$

and is nonlinear and noninvertible. For L quantization levels a number $B = \log_2 L$ of bits are enough to encode them into a binary sequence[1]. Uniform and non-uniform quantization schemes are considered and their effects on the SIR and TSC will be investigated.

Uniform quantizers are the simplest type of quantizers in which regions $\{\mathcal{R}_i\}_{i=2}^{L} - 1$ are equal with some Δ, called quantization width. The design of a uniform quantizer for random variables is done to minimize the squared error distortion

$$D = E[(X - Q(X))^2] \tag{7.19}$$

and is done mainly by numerical techniques.

In the case of non-uniform quantizers, the condition that quantized regions be equal is not imposed which implies fewer constraints in the minimization of distortion. The quantizer is designed based on the necessary conditions for optimality, known as Lloyd-Max conditions [56].

A set of quantization experiments has been performed using codewords obtained in 1000 trials of the eigen-algorithm. The resulting optimal codewords have been quantized using both uniform and non-uniform quantizers, with up to 32 quantization levels (which implies up to 5 bits in the representation of quantized codewords), and both the SIR and the TSC for quantized optimal codewords have been computed. The SIR is the same for all users and equal to $N/(M-N)$ [63] for

[1] L is chosen in general to be a power of 2.

the optimal codeword set, but this is no longer true for the quantized codeword set. Different users have different SIRs with a distribution which is approximately Gaussian as it can be seen from figures 7.2 and 7.3. The mean of the distribution gets closer to the optimal SIR and the variance decreases as the number of quantization levels increases.

Similar remarks can be made about the TSC value, which is equal to M^2/N for the optimal codeword set, whose resulting distributions for different number of quantization levels are plotted in figures 7.4 and 7.5.

Mean values of the SIR and TSC distributions presented in the previous plots are presented in figures 7.6 and 7.7.

We can conclude that quantization degrades the performance of interference avoidance algorithms, by decreasing the mean value of the SIR and increasing the mean value TSC. Furthermore, coarse quantization leads to reasonably large non-uniformity in performance across a given codeword ensemble. Each effect constitutes a degradation in overall performance. With non-uniform quantization 16 to 32 quantization levels (corresponding to $4-5$ bits) seem to be enough to keep the SIR distribution and overall TSC close to optimal values. The average number of bits required for codeword representation might be further decreased by doing entropy coding [55]. This is done by assigning codewords of variable lengths to the possible outcomes of the quantizer, such that highly probable outcomes are assigned shorter codewords, and outcomes with lower probability are assigned longer codewords.

7.4 Complexity Issues

7.4.1 Operational Complexity

At each step, the eigen-algorithm for interference avoidance performs an eigen-decomposition of the correlation matrix of the interference plus noise seen by a given user after which the given user's codeword is replaced by the minimum eigenvector computed. For a system with M users operating in an N-dimensional signal space, the eigen-decomposition has to be performed M times – each on an $N \times N$ matrix – for one iteration of the algorithm, with convergence occuring usually within 3 – 6 iterations. Note that we assume the case of overloaded systems with $M > N$.

For symmetric matrices computation of eigenvalues and eigenvectors can be done using various

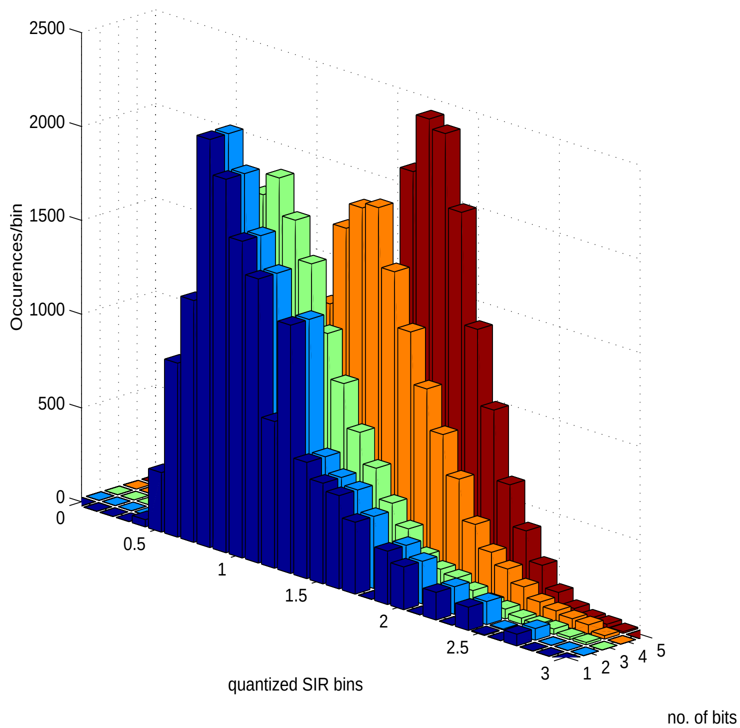

Figure 7.2: SIR distribution with uniformly quantized codewords for 1000 interference avoidance algorithm trials, $M = 15$ users with $N = 10$ dimensions.

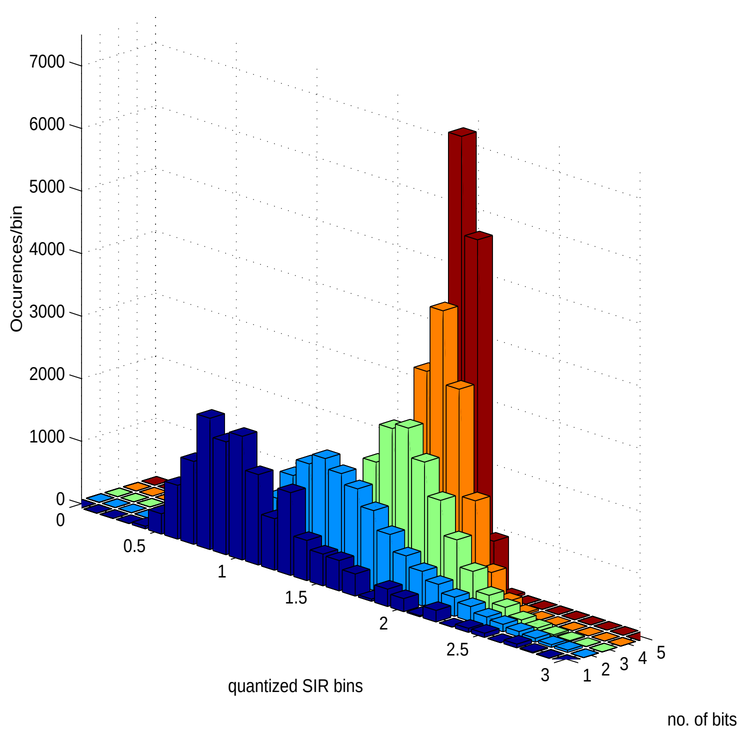

Figure 7.3: SIR distribution with non-uniformly quantized codewords for 1000 interference avoidance algorithm trials, $M = 15$ users with $N = 10$ dimensions.

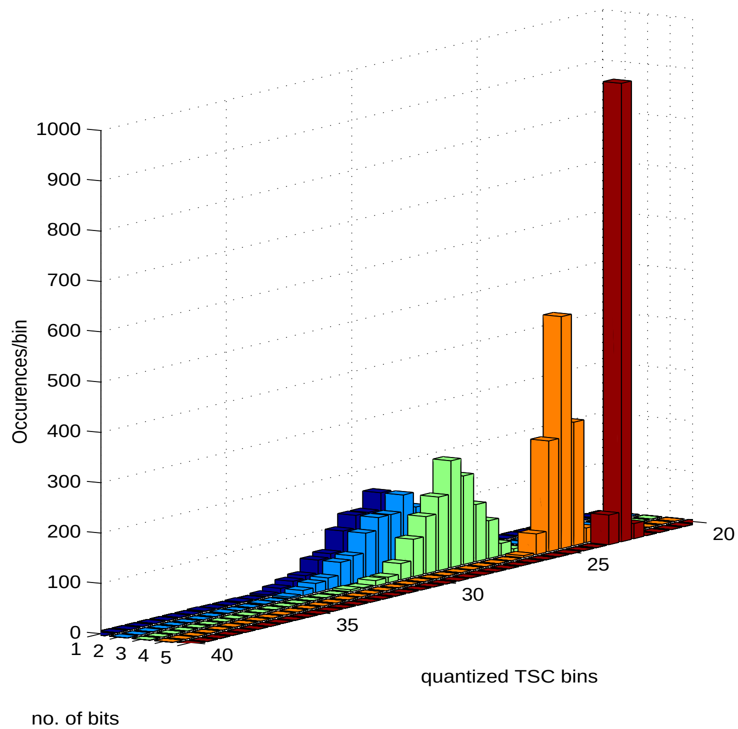

Figure 7.4: TSC distribution with uniformly quantized codewords for 1000 interference avoidance algorithm trials, $M = 15$ users with $N = 10$ dimensions.

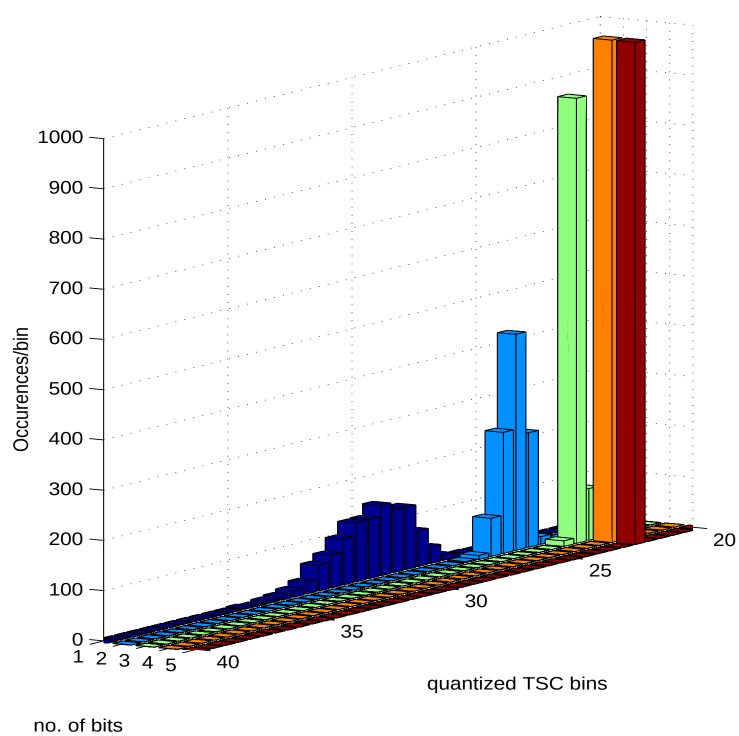

Figure 7.5: TSC distribution with non-uniformly quantized codewords for 1000 interference avoidance algorithm trials, $M = 15$ users with $N = 10$ dimensions.

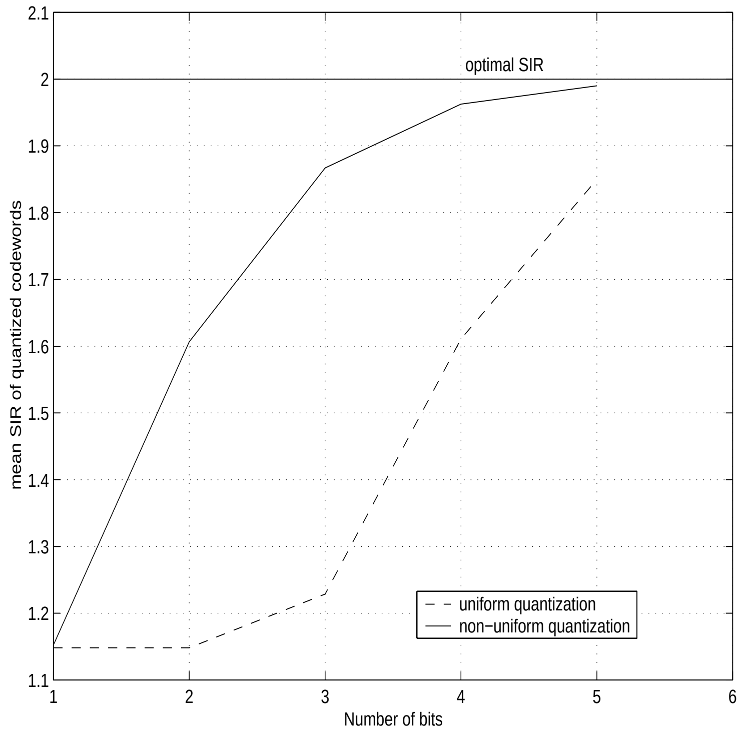

Figure 7.6: Mean values of SIR after uniform and non-uniform quantization of codewords. 1000 interference avoidance algorithm trials, $M = 15$ users, $N = 10$ dimensions.

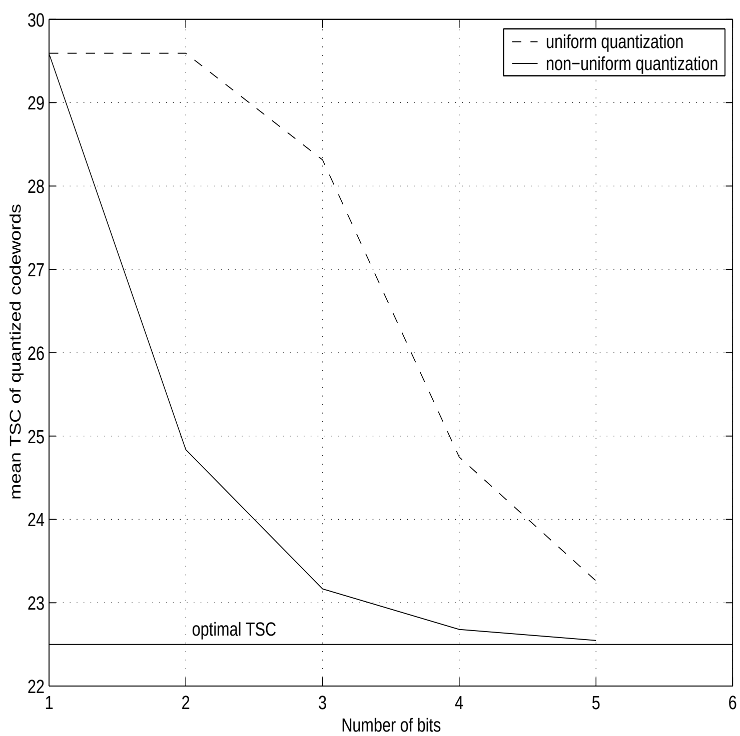

Figure 7.7: Mean values of TSC after uniform and non-uniform quantization of codewords. 1000 interference avoidance algorithm trials, $M = 15$ users, $N = 10$ dimensions.

numerical techniques among which the Jacobi and Hauseholder methods [54, Ch. 11] are the most widely used.

The Jacobi method consists of a sequence of orthogonal similarity transformations and its computational complexity is $O(N^3)$ (usually $18N^3$ to $30N^3$) operations when eigenvectors as well as eigenvalues are desired.

The Hauseholder method is based on reduction of the symmetric matrix to a tri-diagonal form followed by a QL algorithm (consisting of a sequence of factorizations $\mathbf{Q}\cdot\mathbf{L}$ in which $\mathbf{Q}$ is orthogonal and $\mathbf{L}$ is lower triangular). The computational complexity of this method is also $O(N^3)$, but the constants are lower than in the case of Jacobi method (it requires $(4/3)N^3$ operations for the Hauseholder reduction to a tri-diagonal form and $3N^3$ for the QL algorithm when both eigenvalues and eigenvectors are desired).

When only a few eigenvectors[2] are desired instead of the full eigenvector set, computational complexity can be reduced by using the inverse iteration method to determine the eigenvectors. Eigenvalues will still be obtained by the Hauseholder method whose computational complexity is reduced to $(2/3)N^3 + 30N^2$ in this case. The inverse iteration method requires $O(N^2)$ operations to determine the eigenvector associated with a given eigenvalue.

Since all these methods require $O(N^3)$ operations, their use in conjunction with the eigen-algorithm for interference avoidance will imply a computational complexity of $O(MN^3)$ for the eigen-algorithm. For comparison we look at the algorithm proposed in [84], which yields codeword ensembles with the same properties as the eigen-algorithm (WBE sequence sets). This algorithm is based on a mathematical procedure of constructing a symmetric matrix with given eigenvalues and diagonal elements and requires M multiplications of $M \times M$ matrices. Although in general this requires $O(M^4)$ operations, we note that the matrices involved are rotation matrices [14] and have simpler form, thus the actual complexity of the algorithm being $O(M^3)$.

From a different perspective however, the eigen-algorithm is suited for distributed implementation, while the alternative algorithm in [84] is not. We note that the distributed nature of the eigen-algorithm makes it more attractive for implementation in a real system. In addition, the adaptive algorithms for estimating the eigenstructure of covariance matrices [71] or for tracking

[2]Usually less than 25% of the eigenvectors

only a few eigenvectors [13] developed in the context of signal processing applications might be successfully used for further reducing the computational complexity of the eigen-algorithm as well as for more efficient implementations.

7.4.2 Receiver Complexity

In Chapter 2 we have seen that under reasonably loose assumptions on the channel gain matrices, two given users can overlap in at most one frequency. And when taken together with the optimality of matched filter receivers, single frequency overlap can drastically reduce the receiver complexity. Specifically, suppose that our signal space is spanned by N frequencies. In the real-valued formulation of Chapter 2 this implies a signal space of dimension 2N in which each user requires in general $2N$ matched filters with $2N$ coefficients each for an implied $4N^2$ multiply operations per frame per user. However, if users can only overlap in at most one frequency (2 signal space dimensions in our formulation) one might expect, depending on the actual gain matrices that each of k users will occupy on the order of $2N/k$ signal dimensions. This implies that only $2N/k$ codewords are necessary and for each codeword only $2N/k$ coefficients will be nonzero. Thus, complexity could be reduced by a factor on the order of k^2 per user receiver and k overall.

To illustrate with a more concrete example, consider $2N$ dimensions and k users. Assume that the user gain matrices are randomly perturbed identity matrices (pairwise perturbation since each sinusoid frequency must have the same gain) as given by equation (7.20) where $\epsilon_i^{(\ell)}$ is uniform random number with $|\epsilon| \leq 0.1$. Uniform white background noise is also assumed.

$$\mathbf{\Lambda}_\ell = \begin{bmatrix} 1+\epsilon_1^{(\ell)} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 1+\epsilon_1^{(\ell)} & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & 0 & \ddots & 0 & \cdots & \cdots & \vdots \\ \vdots & \cdots & 0 & \ddots & \ddots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \ddots & 1+\epsilon_N^{(\ell)} & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1+\epsilon_N^{(\ell)} \end{bmatrix} \tag{7.20}$$

For $N = 10$ and $k = 2,3,4,5,6$ we have applied interference avoidance to a number of such randomly chosen systems and a plot of the average number of dimensions per user is provided in

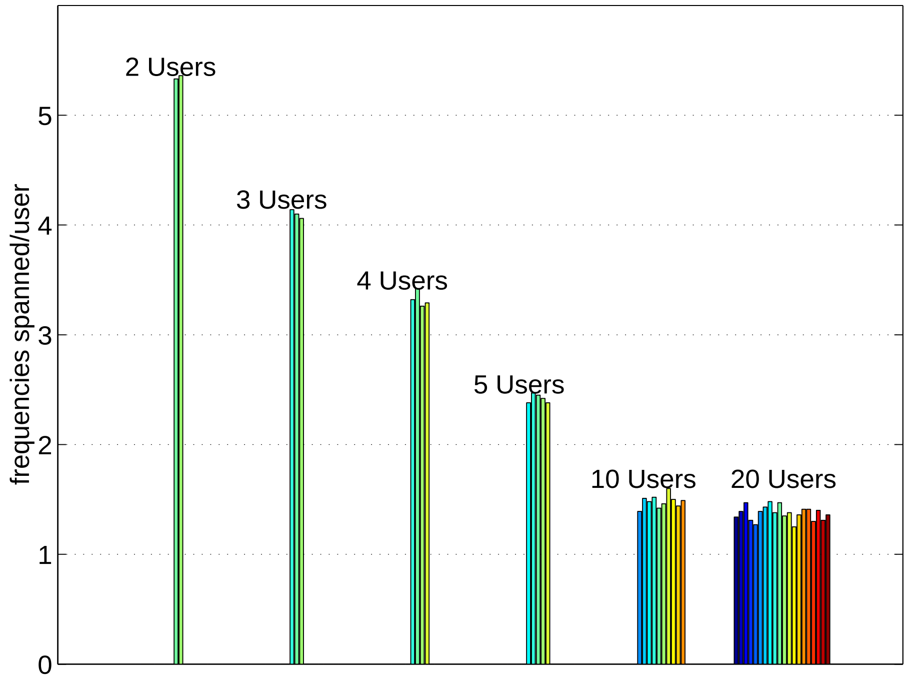

Figure 7.8: Average number of frequencies occupied per user for increasing numbers of users. The signal space is spanned by $N = 10$ frequencies and each user's gain matrix was selected according to equation (7.20).

Figure 7.8. As a consequence of the fact that users overlap in only one frequency we note that the more users are present in the system, the fewer frequencies are spanned by each user – with the implied decrease in user receiver complexity with k.

7.5 Chapter Summary

In this chapter, some issues related to the practical implementation of interference avoidance algorithms have been examined. An algorithm that combines power control and interference avoidance was introduced where users adjust both codewords and power to achieve target SINRs with minimum power. Although we have not been able to prove theoretically convergence of the proposed algorithm to a fixed point, simulations suggest that the algorithm usually reaches an optimal fixed point with properties similar to those described in [63, 84].

An analysis of the transient phase of the eigen-algorithm was also performed. Empirical evidence shows that when starting with random codeword ensembles, the minimum SINR after any codeword update is always larger than the initial minimum SINR. Pathological cases in which the codeword update for a given user has a negative impact on the user with worst (smallest) SINR can usually be avoided by updating codewords for users with worst SINR first.

Quantization of optimal codewords generated by interference avoidance algorithms has also been considered. Quantization degrades the performance of interference avoidance algorithms, by decreasing the mean value of the SIR and increasing the mean value TSC. Furthermore, coarse quantization leads to reasonably large non-uniformity in performance across a given codeword ensemble. Each effect constitutes a degradation in overall performance. With non-uniform quantization 16 to 32 quantization levels (corresponding to 4 - 5 bits) seem to be enough to keep the SIR distribution and overall TSC close to optimal values.

Finally, complexity issues are analyzed. The operational complexity of the eigen-algorithm is evaluated and compared to that of the algorithm proposed in [84] which yields codeword ensembles with the same properties (WBE sequence sets). The reduction in receiver complexity implied by the channelization produced by interference avoidance for dispersive channels was also analyzed. The potentially large reductions in receiver complexity coupled with the fact that matched filters are optimal linear receivers providing uniform SINR for all symbols imply a uniform receiver structure which may be attractive for integration purposes.

Chapter 8

Conclusion and Future Work

8.1 Thesis Summary

In this thesis we have provided a comprehensive theoretical analysis of interference avoidance methods for the uplink of wireless systems. Interference avoidance algorithms, by which individual users in a multiuser system adjust transmitted waveforms and corresponding receiver filters, assume *waveform agile* transmitters and receivers as would be the case with software radios [1, 42, 68, 69]. The algorithms are developed in a general signal space framework which makes them applicable to a wide variety of communication scenarios with multiple users accessing the same communication resources (bandwidth) through a CDMA approach in which codewords rather than waveforms are optimized.

The core for all presented algorithms is a greedy interference avoidance procedure based on SINR maximization in which the codeword of a given symbol/user is replaced by the minimum eigenvector of the corresponding interference-plus-noise covariance matrix. Using results from majorization theory it is shown that application of this greedy procedure which defines the eigen-algorithm, does not decrease sum capacity.

Application of the eigen-algorithm is presented first for codeword optimization in the uplink of a CDMA system in which the dispersive channels between each user and the basestation are assumed known. This is extended to general multiaccess vector channels for which it is shown that sequential application of the eigen-algorithm for interference avoidance by all users in a multiuser system is equivalent to *iterative water filling* and always yields codeword ensembles that maximize sum capacity. This powerful result enables straightforward application of interference avoidance to codeword optimization in more complex problems like systems with multiple inputs and multiple outputs (MIMO) and asynchronous CDMA systems.

For MIMO systems, usually associated with the uplink of wireless systems in which the transmitters and the receiver use multiple antennas for transmission/reception, information is sent using a multicode CDMA approach in which the colums of the precoding matrix are used to "spread" symbols over the available signal space dimensions. Optimal precoding matrices that maximize sum capacity are then obtained by application of interference avoidance.

Asynchronous systems are modeled as frame-synchronous with symbols transmitted using a similar multicode CDMA scheme and frames of extended duration. This approach relaxes synchronization requirements at the common receiver and allows also the presence of users with different data rates.

It is worth noting that the use of optimal codeword ensembles which maximize sum capacity has potential advantages. Most notable is the fact that they provide each user a uniform SINR for all its symbols and that the optimal linear multiuser detector for each symbol is a matched filter. Such uniform identical receiver structures seem good candidates for integration. From this perspective, interference avoidance provides a class of simple and effective algorithms for obtaining such optimal codeword ensembles. Furthermore, when multiple users are present, loose assumptions on the channels seen by each user cause natural segregation of users and potentially large reductions in receiver complexity.

Additional aspects of interference avoidance algorithms are also investigated in the thesis. These address issues of practical importance like incorporating power control with interference avoidance, an analysis of the transient phase of interference avoidance algorithms, and codeword quantization for interference avoidance.

Most of the research results in this thesis have been presented previously at various conferences, see [45–51].

8.2 Future Directions

In order for interference avoidance to become a useful tool in the design of future wireless communication systems, additional theoretical issues should be addressed.

For single base systems these include a more rigurous theoretical analysis of the combined interference avoidance and power control algorithm introduced in Chapter 7, as well as downlink related issues. For the former problem, a more thorough analysis of fixed-point properties is necessary with eventual convergence results established. For the latter problem, recent work on codeword optimization for downlink CDMA systems [4] can be used as a starting reference.

To determine the utility for real systems, application of interference avoidance methods in a cellular system must be investigated. Cellular systems consist of a collection of basestations and associated users in which all users interfere with one another to some extent, and experimental results [63] have shown unstable behaviour of interference avoidance algorithms when applied directly. More recent results [52, 53] have established application of interference avoidance in a collaborative scenario in which information received at all bases is centrally processed. However, the general problem of decoding one base while interfering with reception at other bases is an instance of a still mostly open information theory problem – the interference channel [2, 8–10, 25, 65–67, 70]. We think that a better understanding of the interference channel along with results on interference avoidance and itertive water filling for multiaccess vector channels may provide a starting point towards establishing application of interference avoidance in cellular systems.

Finally, for implementation purposes on a real software radio, the aspects in Chapter 7 and [73] must be complemented with more specific details and a software radio architecture for interference avoidance should be defined, as it has been done for related applications [68, 69].

We are confident, owing to the simplicity of interference avoidance concepts and because of advances in hardware capabilities, interference avoidance methods will develop into practical, affordable, and efficient methods of wireless systems resource allocation.

References

[1] Special issue on software radio. *IEEE Personal Communications Magazine*, 6(4), August 1999. Editors: K-C. Chen and R. Prasad and H.V. Poor.

[2] R. Ahlswede. The Capacity Region of a Channel with Two Senders and Two Receivers. *Annals of Probability*, 2(5):805–814, October 1974.

[3] P. Anigstein and V. Anantharam. On the Convergence of the MMSE Algorithm for Interference Avoidance. In *Proceedings* 38^{th} *Allerton Conference on Communication, Control, and Computing*, volume I, pages 307–316, October 2000.

[4] H. Bi and M. L. Honig. Power and Signature Optimization for Downlink CDMA. In *Proceedings 2002 IEEE International Conference on Communications – ICC'02*, volume 3, pages 1758–1762, May 2002.

[5] E. Biglieri, J. G. Proakis, and S. Shamai. Fading Channels: Information Theoretic and Communication Aspects. *IEEE Transactions on Information Theory*, 44(6):2619 – 2692, October 1998.

[6] J. A. C. Bingham. Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come. *IEEE Communications Magazine*, 28(5):5–14, May 1990.

[7] R. E. Blahut. *Principles and Practice of Information Theory.* Addison-Wesley, Reading, MA, 1987.

[8] A. B. Carleial. A Case Where Interference Does Not Reduce Capacity. *IEEE Transactions on Information Theory*, 21(6):569–570, September 1975.

[9] A. B. Carleial. Interference Channels. *IEEE Transactions on Information Theory*, 24(1):60–70, January 1978.

[10] A. B. Carleial. Outer Bounds on the Capacity of Interference Channels. *IEEE Transactions on Information Theory*, 29(4):602–60, July 1983.

[11] R. S. Cheng and S. Verdú. Gaussian Multiacess Channels with ISI: Capacity Region and Multiuser Water-Filling. *IEEE Transactions on Information Theory*, 39(3):773–785, May 1993.

[12] Federal Communications Commission. FCC Report and Order 97-5: Amendment of the commission's rules to provide for operation of unlicensed NII devices in the 5 GHz frequency range. ET Docket No. 96-102, 1997.

[13] P. Comon and G. H. Golub. Tracking a Few Extreme Singular Values and Vectors in Signal Processing. *Proceedings of the IEEE*, 78(8):1327 – 1343, August 1990.

[14] P. Cotae. An Algorithm for Obtaining Welch Bound Equality Sequences for S-CDMA Systems. *AEÜ, International Journal of Electronics and Communincations*, 55(2):95–99, 2001.

[15] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley-Interscience, New York, NY, 1991.

[16] S. N. Diggavi. Multiuser DMT: A Multiple Access Modulation Scheme. In *Proceedings 1996 IEEE Global Telecommunications Conference - GLOBECOM '96*, pages 1566 – 1570.

[17] S. N. Diggavi. Properties of Sum-Capacity Achieving Solutions for Multiuser DMT. Private communication, October 2000.

[18] S. N. Diggavi. On Achievable Performance of Spatial Diversity Fading Channels. *IEEE Transactions on Information Theory*, 47(1):308 – 325, January 2001.

[19] G. J. Foschini and M. J. Gans. On Limits of Wireless Communications in a Fading Environment Using Multiple Antennas. *Wireless Personal Communications*, 6(3):311 – 335, March 1998.

[20] R. G. Gallager. *Information Theory and Reliable Communication*. Wiley, New York, NY, 1968.

[21] A. J. Goldsmith and P. Varaiya. Capacity for Fading Channels with Channel Side Information. *IEEE Transactions on Information Theory*, 43(6):1986–1992, November 1997.

[22] T. Guess. Jointly Optimal Sequence Sets and Power-Control Policies for Synchronous CDMA with Multiuser Decision-Feedback Receivers and QoS Constraints. *IEEE Transactions on Information Theory*. Accepted for publication, July 2002. To appear.

[23] T. Guess. Combined CDMA Signal Design and Power Control for Decision-Feedback Receivers Subject to a Quality-of-Service Constraint. In *Proceedings* 38^{th} *Allerton Conference on Communication, Control, and Computing*, volume II, pages 806–815, October 2000.

[24] T. Guess. Joint Signal Design and Power Control for CDMA with Decision-Feedback Receivers and Asymmetric QoS Constraints. In *Proceedings* 35^{th} *Conference on Information Sciences and Systems – CISS'01*, volume II, pages 549–554, Baltimore, MD, March 2001.

[25] T. S. Han and K. Kobayashi. A New Achievable Rate Region for the Interference Channel. *IEEE Transactions on Information Theory*, 27(1):49 – 60, January 1981.

[26] H. Hashemi. The Indoor Radio Propagation Channel. *Proceedings of the IEEE*, 81(7):943 – 968, July 1993.

[27] S. Haykin. *Communication Systems*. Wiley, New York, NY, fourth edition, 2001.

[28] B. M. Hochwald and T. L. Marzetta. Unitary Space-Time Modulation for Multiple-Antenna Communications in Rayleigh Flat Fading. *IEEE Transactions on Information Theory*, 46(2):543 – 564, March 2000.

[29] J. L. Holsinger. Digital Communication Over Fixed Time-Continuous Channels With Memory - With Special Application to Telephone Channels. Technical Report 366, MIT - Lincoln Lab., 1964.

[30] M. L. Honig, K. Steiglitz, and S. A. Norman. Optimization of Signal Sets for Partial-Response Response Channels - Part I: Numerical Techniques. *IEEE Transactions on Information Theory*, 37(5):1327–1341, September 1991.

[31] R. A. Horn and C. A. Johnson. *Matrix Analysis*. Cambridge University Press, Cambridge, United Kingdom, 1985.

[32] S. Kasturia, J. T. Aslanis, and J. M. Cioffi. Vector Coding For Partial Response Channels. *IEEE Transactions on Information Theory*, 36(4):741–762, July 1990.

[33] Kiran and D. Tse. Effective Interference and Effective Bandwith of Linear Multiuser Receivers in Asynchronous CDMA Systems. *IEEE Transactions on Information Theory*, 46(4):1426 – 1447, July 2000.

[34] H. J. Landau and H. O. Pollack. Prolate Spheroidal Wave Functions, Fourier Analysis and Uncertainty – III: The Dimension of the Space of Essentially Time- and Band-Limited Signals. *The Bell System Technical Journal*, 40(1):43–64, January 1961.

[35] J. W. Lechleider. The Optimum Combination of Block Codes and Receivers for Arbitrary Channels. *IEEE Transactions on Communications*, 38(3):615–621, May 1990.

[36] T. M. Lok and T. F. Wong. Transmitter and Receiver Optimization in Multicarrier CDMA Systems. *IEEE Transactions on Communications*, 48(7):1197–1207, July 2000.

[37] U. Madhow and M. L. Honig. MMSE Interference Suppression for Direct-Sequence Spread-Spectrum CDMA. *IEEE Transactions on Communications*, 42(12):3178–3188, December 1994.

[38] A. W. Marshall and I. Olkin. *Inequalities: Theory of Majorization and its Applications*. Academic Press, Orlando, FL, 1979.

[39] T. L. Marzetta and B. M. Hochwald. Capacity of a Mobile Multiple-Antenna Communication Link in Rayleigh Flat Fading. *IEEE Transactions on Information Theory*, 45(1):139 – 157, January 1999.

[40] J. L. Massey and T. Mittelholzer. Welch's Bound and Sequence Sets for Code-Division Multiple Access Systems. In *Sequences II: Methods in Communication, Security and Computer Science*, Springer-Verlag, New York, 1991. R. Capocelli, A. De Santis, and U. Vaccaro, Editors.

[41] M. Médard. The Effect upon Channel Capacity in Wireless Communications of Perfect and Imperfect Knowledge of the Channel. *IEEE Transactions on Information Theory*, 46(3):933 – 946, May 2000.

[42] J. Mitola. The Software Radio Architecture. *IEEE Communications Magazine*, 33(5):26–38, May 1995.

[43] R. Negi and J. Cioffi. Pilot Tone Selection for Channel Estimation in a Mobile OFDM System. *IEEE Transactions on Consumer Electronics*, 44(3):1122–1128, August 1998.

[44] S. Ohno, P. Anghel, G. Giannakis, and Z. Luo. Multicarrier Multiple Access is Sum-Rate Optimal for Block Transmissions over Circulant ISI Channels. In *Proceedings 2002 IEEE International Conference on Communications – ICC'02*, volume 3, pages 1656–1660, May 2002.

[45] D. C. Popescu, O. Popescu, and C. Rose. Interference Avoidance for Multiaccess Vector Channels. In *2002 IEEE International Symposium on Information Theory - ISIT'02*, page 499, Lausanne, Switzerland, July 2002.

[46] D. C. Popescu and C. Rose. Interference Avoidance and Dispersive Channels. A New Look at Multicarrier Modulation. In *Proceedings* 37^{th} *Allerton Conference on Communication, Control, and Computing*, pages 505–514, Monticello, IL, September 1999.

[47] D. C. Popescu and C. Rose. Codeword Quantization for Interference Avoidance. In *Proceedings 2000 International Conference on Acoustics, Speech, and Signal Processing - ICASSP 2000*, volume 6, pages 3670 – 3673, Istanbul, Turkey, 2000.

[48] D. C. Popescu and C. Rose. A New Approach to Multiple Antenna Systems. In *Proceedings* 35^{th} *Conference on Information Sciences and Systems – CISS'01*, volume II, pages 868–871, Baltimore, MD, March 2001.

[49] D. C. Popescu and C. Rose. Fading Channels and Interference Avoidance. In *Proceedings* 39^{th} *Allerton Conference on Communication, Control, and Computing*, pages 1073–1074, Monticello, IL, October 2001.

[50] D. C. Popescu and C. Rose. Interference Avoidance Applied to Multiaccess Dispersive Channels. In *Proceedings* 35^{th} *Annual Asilomar Conference on Signals, Systems, and Computers*, volume II, pages 1200–1204, Pacific Grove, CA, November 2001.

[51] D. C. Popescu and C. Rose. Codeword Optimization for Asynchronous CDMA Systems Through Interference Avoidance. In *Proceedings* 36^{th} *Conference on Information Sciences and Systems - CISS 2002*, Princeton, NJ, March 2002.

[52] O. Popescu and C. Rose. Interference Avoidance and Sum Capacity for Multibase Systems. In *Proceedings* 39^{th} *Allerton Conference on Communication, Control, and Computing*, pages 1036–1045, Monticello, IL, October 2001.

[53] O. Popescu and C. Rose. Minimizing Total Squared Correlation with Multiple Receivers. In *Proceedings* 39^{th} *Allerton Conference on Communication, Control, and Computing*, pages 1063–1072, Monticello, IL, October 2001.

[54] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling. *Numerical Recipes. The Art of Sceintific Computing.* Cambridge University Press, Cambridge, United Kingdom, first edition, 1988.

[55] J. G. Proakis. *Digital Communications.* McGraw Hill, Boston, MA, fourth edition, 2000.

[56] J. G. Proakis and M. Salehi. *Communication Systems Engineering.* Prentice Hall, Englewood Cliffs, NJ, 1994.

[57] G. S. Rajappan and M. L. Honig. Spreading Code Adaptation for DS-CDMA with Multipath. In *Proceedings 2000 IEEE Military Communications Conference – MILCOM 2000*, volume 2, pages 1164–1168, Los Angeles, CA, October 2000.

[58] G. S. Rajappan and M. L. Honig. Signature Sequence Adaptation for DS-CDMA with Multipath. *IEEE Journal on Selected Areas in Communications*, 20(2):384–395, February 2002.

[59] G. G. Raleigh and J. M. Cioffi. Spatio-Temporal Coding for Wireless Communication. *IEEE Transactions on Communications*, 46(3):357–366, March 1998.

[60] P. B. Rapajic and B. S. Vucetic. Linear Adaptive Transmitter-Receiver Structures for Asynchronous CDMA Systems. *European Transactions on Telecommunications*, 6(1):21 – 27, Jan. - Feb. 1995.

[61] C. Rose. Sum Capacity and Interference Avoidance: Convergence Via Class Warfare. In *Proceedings 2000 Conference on Information Sciences and Systems – CISS 2000*, pages WA3–11 – WA3–16, Princeton, NJ, March 2000.

[62] C. Rose. CDMA Codeword Optimization: Interference Avoidance and Convergence Via Class Warfare. *IEEE Transactions on Information Theory*, 47(6):2368–2382, September 2001.

[63] C. Rose, S. Ulukus, and R. Yates. Wireless Systems and Interference Avoidance. *IEEE Transactions on Wireless Communications*, 1(3):415–428, July 2002.

[64] M. Rupf and J.L. Massey. Optimum Sequence Multisets for Synchronous Code-Division Multiple-Access Channels. *IEEE Transactions on Information Theory*, 40(4):1226–1266, July 1994.

[65] H. Sato. Two-User Communication Channels. *IEEE Transactions on Information Theory*, 23(3):295–304, May 1977.

[66] H. Sato. On the Capacity Region of a Discrete Two-User Channel for Strong Interference. *IEEE Transactions on Information Theory*, 24(3):377–779, May 1978.

[67] H. Sato. The Capacity of the Gaussian Interference Channel Under Strong Interference. *IEEE Transactions on Information Theory*, 27(6):786–788, November 1981.

[68] I. Seskar and N. Mandayam. A Software Radio Architecture for Linear Multiuser Detection. *IEEE Journal on Selected Areas in Communications*, 17(5):814 – 823, May 1999.

[69] I. Seskar and N. Mandayam. Software Defined Radio Architectures for Interference Cancellation in DS-CDMA Systems. *IEEE Personal Communications Magazine*, 6(4):26 – 34, August 1999.

[70] C. E. Shannon. Two-way Communication Channels. In *Proceedings* 4^{th} *Berkeley Symposium on Mathematics, Statistics, and Probabilities*, pages 611–644. University of California Press, Berkeley, 1961.

[71] K. C. Sharman. Adaptive Algorithms for Estimating the Complete Covariance Eigenstructure. In *Proceedings 1986 International Conference on Acoustics, Speech, and Signal Processing - ICASSP '86*, pages 27.15.1 – 27.15.4, Tokyo, Japan, 1986.

[72] Gilbert Strang. *Linear Algebra and Its Applications*. Harcourt Brace Jovanovich College Publishers, San Diego, CA, third edition, 1988.

[73] D. Tabora. An Analysis of Covariance Estimation, Codeword Feedback, and Multiple Base Performance of Interference Avoidance. Master's thesis, Rutgers University, Department of Electrical and Computer Engineering, 2001. Thesis Director: Prof. C. Rose.

[74] D. Tse and S. Hanly. Multi-access Fading Channels. Part I: Polymatroid Structure, Optimal Resource Allocation and Throughput Capacities. *IEEE Transactions on Information Theory*, 44(7):2796 – 2815, November 1998.

[75] S. Ulukus. *Power Control, Multiuser Detection and Interference Avoidance in CDMA Systems*. PhD thesis, Rutgers University, Department of Electrical and Computer Engineering, 1998. Thesis Director: Prof. R. D. Yates.

[76] S. Ulukus and R. Yates. Iterative Signature Adaptation for Capacity Maximization of CDMA Systems. In *Proceedings* 36^{th} *Allerton Conference on Communication, Control, and Computing*, September 1998.

[77] S. Ulukus and R. Yates. Optimum Signature Sequence Sets for Asynchronous CDMA Systems. In *Proceedings* 38^{th} *Allerton Conference on Communication, Control, and Computing*, volume I, pages 307–316, October 2000.

[78] S. Ulukus and R. Yates. Iterative Construction of Optimum Signature Sequence Sets in Synchronous CDMA Systems. *IEEE Transactions on Information Theory*, 47(5):1989–1998, July 2001.

[79] H. L. Van Trees. *Detection, Estimation, and Modulation Theory, Part I*. Wiley, New York, NY, 1968.

[80] S. Verdú. Capacity Region of Gaussian CDMA Channels. The Symbol-Synchronous Case. In *Proceedings* 24^{th} *Allerton Conference on Communication, Control, and Computing*, pages 1025–1034, October 1986.

[81] S. Verdú. Multiple-Access Channels with Memory with and without Frame Synchronism. *IEEE Transactions on Information Theory*, 35(3):605–619, May 1989.

[82] S. Verdú. The Capacity Region of the Symbol-Asynchronous Gaussian Multiple-Access Channel. *IEEE Transactions on Information Theory*, 35(4):733–751, July 1989.

[83] S. Verdú. *Multiuser Detection*. Cambridge University Press, Cambridge, United Kingdom, 1998.

[84] P. Viswanath and V. Anantharam. Optimal Sequences and Sum Capacity of Synchronous CDMA Systems. *IEEE Transactions on Information Theory*, 45(6):1984–1991, September 1999.

[85] P. Viswanath, V. Anantharam, and D. Tse. Optimal Sequences, Power Control and Capacity of Spread Spectrum Systems with Multiuser Linear Receivers. *IEEE Transactions on Information Theory*, 45(6):1968–1983, September 1999.

[86] P. Viswanath, D. Tse, and V. Anantharam. Asymptotically Optimal Waterfilling in Vector Multiple Access Channels. *IEEE Transactions on Information Theory*, 47(1):241 – 267, January 2001.

[87] P. Viswanath and Anantharam V. Total Capacity of Multiaccess Vector Channels. Technical Memorandum M99/47, Electronics Research Laboratory, University of California, Berkeley, 1999.

[88] L. R. Welch. Lower Bounds on the Maximum Cross Correlation of Signals. *IEEE Transactions on Information Theory*, IT-20(3):397–399, May 1974.

[89] T. F. Wong and T. M. Lok. Transmitter Adaptation in Multicode DS-CDMA Systems. *IEEE Journal on Selected Areas in Communications*, 19(1):69–82, January 2001.

[90] G. W. Wornell. Spread-Signature CDMA: Efficient Multiuser Communication in the Presence of Fading. *IEEE Transactions on Information Theory*, 41(5):1418–1438, September 1995.

[91] R. Yates. A Framework for Uplink Power Control in Cellular Radio Systems. *IEEE Journal on Selelected Areas in Communincations*, 13(7):1341–1348, September 1995.

[92] N. Yee and J. P. Linnartz. Multi-Carrier CDMA in an Indoor Wireless Radio Channel. Technical Memorandum M94/6, Electronics Research Laboratory, University of California, Berkeley, 1994.

[93] W. Yu. Interference Avoidance and Iterative Water Filling: A Connection. Private communication, May 2001.

[94] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi. Iterative Water-Filling for Gaussian Vector Multiple Access Channels. In *Proceedings 2001 IEEE International Symposium on Information Theory - ISIT'01*, page 322, Washington, DC, June 2001. Submitted for journal publication.

[95] W. Yu, W. Rhee, and J. M. Cioffi. Optimal Power Control in Multiple Access Fading Channels with Multiple Antennas. In *Proceedings 2001 IEEE International Conference on Communications – ICC'01*, volume 2, pages 575–579, May 2001.

Vita

Dimitrie C. Popescu

1991 Diploma in Electrical and Computer Engineering and M.S. with specialization in Control Engineering and Computers, Polytechnic Institute of Bucharest, Romania.

1992-1996 Assistant Lecturer, Department of Control Engineering and Computers, Politehnica University of Bucharest, Romania.

1994-1996 Pre-doctoral studies in Systems and Control, Politehnica University of Bucharest, Romania.

1996-1998 Teaching Assistant, Department of Electrical and Computer Engineering, Rutgers University, Piscataway, New Jersey.

1997 Member of Technical Staff, AT&T Laboratories, Florham Park, New Jersey.

1998-2002 Graduate Research Assistant, Wireless Information Network Laboratory – WINLAB, Rutgers University, Piscataway, New Jersey.

1999 Dimitrie C. Popescu and C. Rose. "Interference Avoidance and Dispersive Channels. A New Look at Multicarrier Modulation." in *Proceedings* 37^{th} *Allerton Conference on Communication, Control, and Computing*, pages 505-514, Monticello, Illinois, September 1999.

Second Place in the *AT&T Student Research Symposium* for the work on interference avoidance and dispersive channels, Shannon Laboratory, October 1999.

2000 Member of Technical Staff, Telcordia Technologies, Red Bank, New Jersey.

Dimitrie C. Popescu and C. Rose. "Codeword Quantization for Interference Avoidance", in *Proceedings 2000 International Conference on Acoustics, Speech, and Signal Processing – ICASSP 2000*, vol. 6, pages 3670-3673, Istanbul, Turkey, June 2000.

2001 Dimitrie C. Popescu and C. Rose. "A New Approach to Multiple Antenna Systems", in *Proceedings* 35^{th} *Conference on Information Sciences and Systems – CISS'01*, vol. II, pages 868-871, Baltimore, Maryland, March 2001.

Dimitrie C. Popescu and C. Rose. "Fading Channels and Interference Avoidance" in *Proceedings* 39^{th} *Allerton Conference on Communication, Control, and Computing*, pages 1073-1074, Monticello, Illinois, October 2001.

Dimitrie C. Popescu and C. Rose. "Interference Avoidance Applied to Multiaccess Dispersive Channels", in *Proceedings* 35^{th} *Annual Asilomar Conference on Signals, Systems, and Computers*, volume II, pages 1200-1204, Pacific Grove, California, November 2001.

2002 Dimitrie C. Popescu, Christopher Rose, "Codeword Optimization for Asynchronous CDMA Systems Through Interference Avoidance", in *Proceedings* 36^{th} *Conf. on Information Sciences and Systems* - CISS'02, March 2002, Princeton, New Jersey.

Dimitrie C. Popescu, Otilia Popescu, Christopher Rose, "Interference Avoidance and Multiaccess Vector Channels", in *Proceedings 2002 IEEE International Symposium on Information Theory* - ISIT 2002, page 499, July 2002, Lausanne, Switzerland.

Ph.D. in Electrical and Computer Engineering, Rutgers University, Piscataway, New Jersey.