Interference Avoidance and Dispersive Channels: A New Look at Multicarrier Modulation*

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Abstract

The availability of software radios and application of interference avoidance may offer a new perspective on communication over dispersive channels. By both lengthening the transmission interval so that ISI is almost inconsequential and by using combinations of channel eigenfunctions to convey the bits, communications over dispersive channels becomes (not surprisingly) a multiuser detection problem where each bit is assigned its own waveform. By choosing these waveforms to meet the Welch Bound with equality – through interference avoidance or some other numerical method – a uniform maximum SINR can be achieved for each bit. In addition by using Gaussian signaling and decoding with this codeword ensemble, the channel capacity can be (theoretically) met. A trivial extension of the single user case can be used for multiple users over identical channels. Multiple users with non-identical channels is more subtle and is discussed in the interference avoidance context with suggestions for future work.

1 Introduction

Wireless communication channels are generally dispersive and dispersion leads to intersymbol interference (ISI) where energy from signal bits spills over into the observation intervals of subsequent bits at the receiver. As noted in [1] there have been primarily two ways to combat ISI – equalization and coding – with large associated literatures.

In [1] and [2] an approach was proposed based on partitioning the communication channel into a set of parallel and independent subchannels each associated with its own carrier. Such channel partitioning is known in general as multicarrier modulation and has been present in the literature for over 30 years. The specific approach employed by Kasturia *et al* was dubbed vector coding.

Our approach is also based on a form of multicarrier modulation that uses the eigenfunctions of the channel autocorrelation [3, 4]. Similar to [1, 2], vectors are sent over the channel during some communications interval of duration $\mathcal{T}$ where $\mathcal{T}$ is assumed long enough so that the duration of the channel impulse response is small in comparison. However, our approach

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differs in that we assign different linear combinations of eigenfunctions to *each bit* sent during the $\mathcal{T}$ communications interval. That is, the signature for *each bit* is assumed to be of duration $\mathcal{T}$ – similar to, but different from prior work where bit signatures were allowed to have length longer than the bit interval [5].

Furthermore, the signatures are chosen to meet the Welch Bound with equality (WBE sequences) so that the *optimal* linear receiver is a bank of matched filters tuned to channel response to each signature [6, 7]. Such WBE sequences achieve the maximum attainable uniform signal to interference/noise ratio (SINR) per bit, and if used with Gaussian signaling could be (theoretically) used to meet the capacity of the channel.[1]

From another perhaps simpler perspective, by using channel eigenfunctions as modulation building blocks we transform the usual dispersive channel problem into a multiuser detection problem. Although this isomorphism is not unexpected, it proves useful in the interference avoidance context. Specifically, we start by casting the problem of sending a set of M independent bits over a channel during an interval of duration $\mathcal{T}$ as a problem in multiuser detection by using linear combinations of channel eigenfunctions of the channel autocorrelation (codewords) for transmission of each bit. Then we apply interference avoidance techniques [8–10] to determine optimal codewords for each bit. Since it can be shown that greedy interference avoidance always produces codeword ensembles which meet the Welch bound with equality [11], the resulting codeword sets maximize ensemble SINR and can also be used (theoretically with Gaussian signaling and decoding) to maximize channel capacity.

The simplest multiple user problem, where L users share an identical channel for communication is a simple extension of the single user problem. In this case, the "user capacity" of the channel is maximized in that the scheme supports the greatest number of users subject to a given SINR criterion. And as before, under a Gaussian signaling assumption, the sum capacity is achieved as well. The multiple user with non-identical channels case is more subtle and will be discussed.

2 Background: channel eigenfunctions

Suppose a channel can be characterized by a causal impulse response $h(t)$. We will now (re)derive [3, 4] a useful set of functions which will allow us to easily represent channel inputs and outputs. Specifically we desire a set of orthonormal basis functions $\{\Psi_i(t)\}$ which can be used to represent channel inputs. However, in addition, we would like the channel responses to these functions to be orthogonal as well to allow simple representation of outputs. That is, on the communications interval $(0,\mathcal{T})$ where $\mathcal{T}$ is assumed much larger than the duration of $h(t)$ we require

$$\int_0^{\mathcal{T}} \left[\int_0^{\mathcal{T}} \Psi_i(\tau)h(t-\tau)d\tau\right] \left[\int_0^{\mathcal{T}} \Psi_j(\mu)h(t-\mu)d\mu\right] dt = \lambda_i\delta_{ij} \tag{1}$$

Defining a channel correlation function as

$$R_h(\tau-\mu) = \int_0^{\mathcal{T}} h(t-\tau)h(t-\mu)dt \tag{2}$$

we obtain

$$\int_0^{\mathcal{T}}\int_0^{\mathcal{T}} \Psi_j(\mu)\Psi_i(\tau)R_h(\tau-\mu)d\tau d\mu = \lambda_i\delta_{ij} \tag{3}$$

[1]Assuming the channel impulse response duration is negligible compared to the communications interval duration $\mathcal{T}$.

Following [12], we readily see that the desired $\{\Psi_i(t)\}$ must satisfy the integral equation

$$\int_0^{\mathcal{T}} \Psi_i(\mu) R_h(\tau - \mu) d\mu = \lambda_i \Psi_i(\tau) \tag{4}$$

and furthermore since $R_h()$ is a correlation function, a complete set of orthonormal $\Psi_i(\tau)$ exists along with a corresponding set of non-negative λ_i. These $\{\Psi_i(t)\}$ *eigenfunctions* can therefore be used to represent any input in the mean square sense. Similarly, any feasible channel output can be written as a linear superposition of the set $\{\tilde{\Psi}_i(t)\}$ where we define

$$\tilde{\Psi}_i(t) = \lambda_i^{-\frac{1}{2}} \int_0^{\mathcal{T}} \Psi_i(\tau) h(t - \tau) d\tau \tag{5}$$

for non-zero λ_i. It is worth noting that each of the $\Psi_i(t)$ and $\tilde{\Psi}_i(t)$ is assumed to have unit energy ($\int_0^{\mathcal{T}} \Psi_i^2(t) dt = \int_0^{\mathcal{T}} \tilde{\Psi}_i^2(t) dt = 1$). So if we have as input to the channel

$$x(t) = \sum_i s_i \Psi_i(t) \tag{6}$$

the corresponding channel output is then simply

$$y(t) = \sum_i s_i \lambda_i^{\frac{1}{2}} \tilde{\Psi}_i(t) \tag{7}$$

Now suppose that there are only N non-negligible eigenvalues λ_i. This implies that the useful input space – that portion of the signal space whose component energies are not strongly absorbed by the channel – is finite. We note that such a condition is unavoidable for finite $\mathcal{T}$ and finite bandwidth constraints as would be introduced by any real $h(t)$. We can then closely approximate useful inputs as

$$x(t) = \mathbf{P}(t)^\top \mathbf{s} \tag{8}$$

where $\mathbf{s}$ is a column vector of N s_i's and $\mathbf{P}(t)$ is a column vector of the significant N $\Psi_i(t)$'s. Likewise we have as channel output

$$y(t) = \tilde{\mathbf{P}}(t)^\top \Lambda^{1/2} \mathbf{s} \tag{9}$$

for outputs where Λ is the $N \times N$ diagonal matrix of the significant eigenvalues.

3 Multicarrier Modulation

The usual bit-sequential scenario would have

$$x(t) = \sum_k b_k p(t - kT) \tag{10}$$

where b_k is the bit sent during interval k, T is the bit interval and $p(t)$ is the finite duration pulse used to convey the information over the channel. This signaling structure is depicted in FIGURE 1.

Multicarrier modulation offers a different approach. Let $\mathcal{T}$ be some interval of duration much longer than the channel impulse response duration. In this way we can safely ignore ISI between successive intervals of duration $\mathcal{T}$ by placing a relatively small "zero pad" between

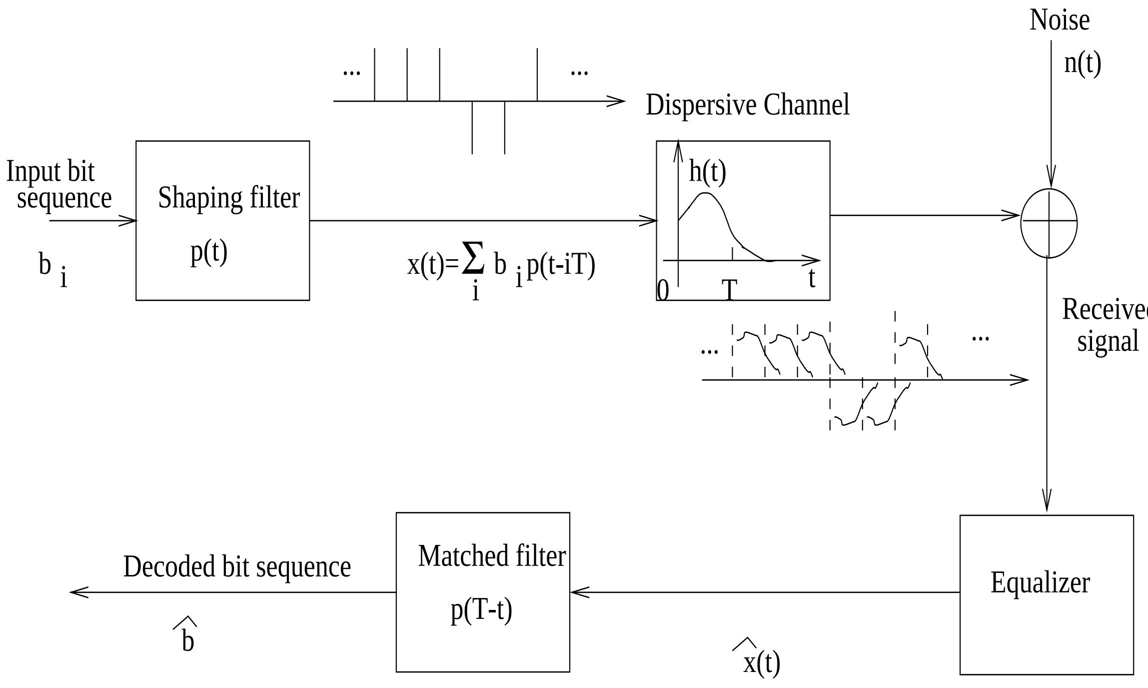

Figure 1: A depiction of sequential transmission with equalization.

them to allow settling of the channel response. We will send M bits during the interval and construct the input waveform as

$$x(t) = \sum_{k=1}^{M} b_k s_k(t) \tag{11}$$

where $s_k(t)$ is a waveform chosen specifically to convey bit i. Unlike [1,2] there is no explicit precoding. Rather, the method would be a form of CDMA as if each bit corresponded to a different user as shown in FIGURE 2.

Each of the waveforms $s_k(t)$ can be represented in terms of the spanning set of previously described orthonormal basis functions $\{\Psi_j(t)\}_{j=1}^{N}$ as

$$s_k(t) = \sum_{j=1}^{N} s_{kj}\Psi_j(t) \tag{12}$$

which implies that the input signal can be written as

$$x(t) = \sum_{k=1}^{M} b_i \sum_{j=1}^{N} s_{kj}\Psi_j(t) = \mathbf{P}(t)^T\mathbf{S}\mathbf{b} \tag{13}$$

where $\mathbf{b} = [b_1 \ldots b_M]^T$ is the vector containing the bit set to be sent, $\mathbf{P}(t) = [\Psi_1(t)\ldots\Psi_N(t)]^T$ is a vector containing the basis functions, and $\mathbf{S}$ an $N \times M$ matrix with columns $\mathbf{s}_k$ being the coordinate vectors of each waveform $\{s_k(t)\}_{k=1}^{M}$ with respect to the basis $\{\Psi_j(t)\}_{j=1}^{N}$.

The received signal is assumed to contain additive zero mean white Gaussian noise and can be represented as

$$r(t) = y(t) + n(t) \tag{14}$$

which when projected onto the orthogonal functions $\{\tilde{\Psi}_i(t)\}_{i=1}^{N}$ becomes

$$\mathbf{r} = \Lambda^{1/2}\mathbf{S}\mathbf{b} + \mathbf{n} \tag{15}$$

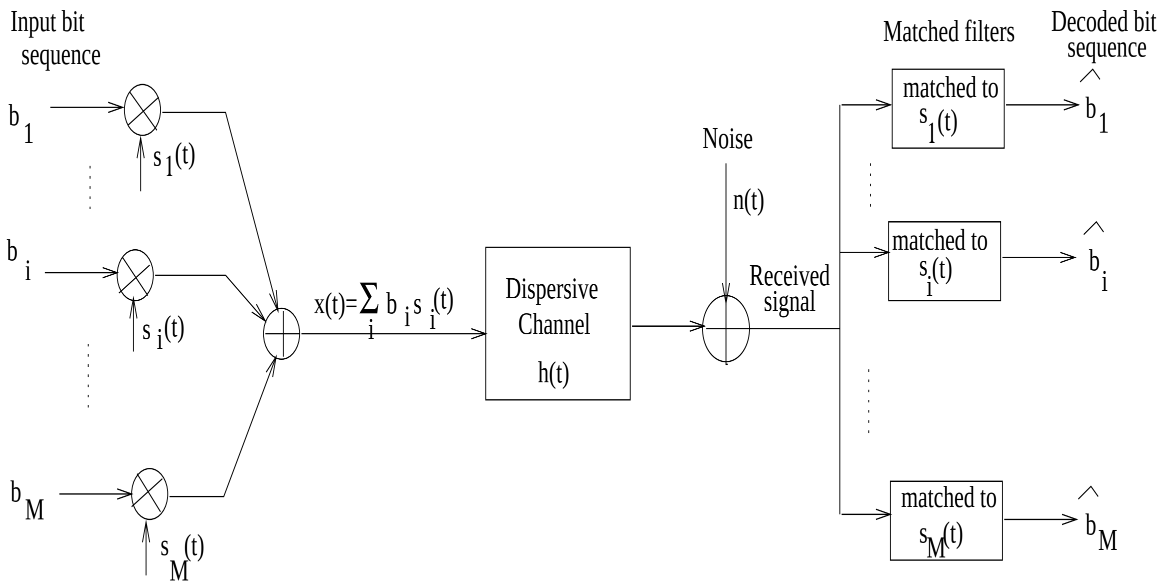

Figure 2: A depiction of our version of multicarrier modulation.

with $\mathbf{n} = [n_1 \ldots n_N]^T$ a vector containing the projections of the noise process $n(t)$ onto the basis functions – $n_j = \int_0^{\mathcal{T}} n(t)\tilde{\Psi}_j(t)dt$. Since $n(t)$ is a white Gaussian process, the projections onto orthogonal functions are mutually independent zero mean Gaussian random variables with variances $E[n_j^2] = N_0$.

Since we have limited the input signal space to those $\Psi_i(t)$ with nonzero eigenvalues, we can rewrite equation (15) to obtain

$$\tilde{\mathbf{r}} = \Lambda^{-1/2}\mathbf{r} = \mathbf{Sb} + \tilde{\mathbf{n}} \tag{16}$$

in which $\tilde{\mathbf{n}} = \Lambda^{-1/2}\mathbf{n}$ is a new vector of noise components, still uncorrelated, but no longer white – the variance of each component is now N_0/λ_i. This is a typical multiuser detection problem [13] where an ensemble of user transmissions $b_k\mathbf{s}_k$ must be jointly decoded subject to a constraint on transmitted energy for each codeword. Thus, as mentioned previously the optimal multicarrier modulation problem is (not surprisingly) isomorphic with the optimal multiuser detection problem posed in equation (16).

At this point we could turn to multiuser detection theory and seek to design the best receivers (decision feedback, MMSE etc.) for the given signatures. However, we instead pursue an alternate approach of signature optimization. This will allow us to use simple matched filters for optimal detection in the linear receiver case.

4 Interference Avoidance

We now require a means to generate such optimal signal sets. To this end a class of *interference avoidance* algorithms has been proposed [8–10] in which the signature waveform of a given user is adapted to minimize interference from other users and achieve maximum signal-to-interference ratio. This procedure in general has been shown empirically to produce optimal signal sets and has theoretical properties which suggest convergence is unavoidable [8]. Recently however [11], we have found a strict convergence proof for a subclass of interference avoidance algorithm dubbed "greedy interference avoidance." Thus, we can be assured that interference avoidance will yield an optimal signal set.

We will first introduce the greedy *eigen-algorithm* [9,10] in the current context and describe properties of the optimal signal set. Note that to simplify the argument and avoid certain special cases we hereafter assume that the number of bits to be sent M is at least as great as the number of eigenfunctions N.

Greedy interference avoidance for minimizing interference from other bit transmissions can be summarized as follows:

- Start with an initial set of signatures $\{s_k(t)\}_{k=1}^M$, with corresponding signature vectors $\{\mathbf{s}_k\}_{k=1}^M$ with respect to the orthonormal basis defined by the channel eigenfunctions.
- For each bit k, the interference generated by the other $M-1$ independent bits is described by the N-dimensional autocorrelation matrix $\mathbf{R}_k = \mathbf{S}\mathbf{S}^\top - \mathbf{s}_k\mathbf{s}_k^\top + N_0\Lambda^{-1}$.
- The minimum eigenvalue μ_k^* of $\mathbf{R}_k$ and its associated eigenvector $\mathbf{x}_k^*$ are found, and then $\mathbf{s}_k$ is replaced by $\mathbf{x}_k^*$.
- This process is repeated iteratively for each bit signature until no further improvement can be obtained. At that point, all the codewords are minimum eigenvalue eigenvectors of $\mathbf{R}_k$.
- If a suboptimal minimum is reached, employ "class warfare" methods to escape the local minimum. A finite number of minima guarantees convergence [11].

It can be shown that this procedure monotonically decreases an intuitively pleasing quantity called the total square correlation (TSC) [8–10], defined as

$$\mathrm{TSC} = \mathrm{Trace}[(\mathbf{S}\mathbf{S}^\top + N_0\Lambda^{-1})^2] \tag{17}$$

Furthermore, it can be shown by simple extension of majorization [14] results in [7] or by direct methods using Lagrange multipliers and the Kuhn-Tucker condition [11] that minimization of TSC results in a codeword set which maximizes sum capacity and user capacity. Remember that user capacity is defined as the maximum number of users (codewords) which meet a specified SINR [6,7].

We now seek the minimum TSC. Let us write the matrix $\mathbf{S}\mathbf{S}^\top + N_0\Lambda^{-1}$ in terms of canonical eigenvectors $\mathbf{e}_i = [0,\ldots,0,1,0,\ldots,0]^\top$ (zero except for 1 in the i^{th} position) of $\Re^N$. Then we have

$$\begin{aligned}\mathbf{S}\mathbf{S}^\top + N_0\Lambda^{-1} &= \sum_{i=1}^N\sum_{j=1}^N\sum_{k=1}^M s_{ik}s_{jk}\mathbf{e}_i\mathbf{e}_j^\top + \sum_{i=1}^N\sum_{j=1}^N \frac{N_0}{\lambda_j}\mathbf{e}_i\mathbf{e}_j^\top \\ &= \sum_{i=1}^N\sum_{j=1}^N\left(\sum_{k=1}^M s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)\mathbf{e}_i\mathbf{e}_j^\top\end{aligned} \tag{18}$$

We can then write

$$\begin{aligned}(\mathbf{S}\mathbf{S}^\top + N_0\Lambda^{-1})^2 &= \left[\sum_{i=1}^N\sum_{j=1}^N\left(\sum_{k=1}^M s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)\mathbf{e}_i\mathbf{e}_j^\top\right] \\ &\quad\times\left[\sum_{m=1}^N\sum_{n=1}^N\left(\sum_{l=1}^M s_{ml}s_{nl} + \frac{N_0}{\lambda_l}\right)\mathbf{e}_m\mathbf{e}_n^\top\right] \\ &= \sum_{i=1}^N\sum_{j=1}^N\sum_{n=1}^N\sum_{k=1}^M\left(s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)\sum_{l=1}^M\left(s_{jl}s_{nl} + \frac{N_0}{\lambda_j}\right)\mathbf{e}_i\mathbf{e}_n^\top\end{aligned} \tag{19}$$

Since $\mathrm{Trace}[\mathbf{e}_i\mathbf{e}_n^\top] = \mathbf{e}_n^\top\mathbf{e}_i = \delta_{in}$, we then have

$$\mathrm{Trace}[(\mathbf{SS}^\top + N_0\Lambda^{-1})^2] = \sum_{i=1}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)^2 \tag{20}$$

By defining the total signal energy projection along the i^{th} eigenfunction as $P_i = \sum_{k=1}^{M} s_{ik}^2$ we can rewrite (20) as

$$\mathrm{Trace}[(\mathbf{SS}^\top + N_0\Lambda^{-1})^2] = \sum_{i=1, i\neq j}^{N}\sum_{j=1}^{N}\left(\sum_{k=1}^{M} s_{ik}s_{jk} + \frac{N_0}{\lambda_j}\right)^2 + \sum_{j=1}^{N}\left(P_j + \frac{N_0}{\lambda_j}\right)^2 \tag{21}$$

Since all terms in the summations in i and j are non-negative we can lower bound the TSC by

$$\mathrm{Trace}[(\mathbf{SS}^\top + N_0\Lambda^{-1})^2] \geq \sum_{j=1}^{N}\left(P_j + \frac{N_0}{\lambda_j}\right)^2 \tag{22}$$

where the right hand side term is a convex function in P_j. Minimization of this term with the requirements that $P_j \geq 0,\ \forall j$ and $\sum_{j=1}^{N} P_j = M$ (total power sent – M bits each with unit energy) leads to the "water filling" distribution of powers [3, 6, 7, 15]

$$P_i = \left(c^* - \frac{N_0}{\lambda_i}\right)^+, \qquad \text{where } (x)^+ = \begin{cases} x \text{ if } x \geq 0 \\ 0 \text{ if } x < 0 \end{cases} \tag{23}$$

where c^* is the water level mark and can be calculated from the total power constraint. It will later be useful to note that the above derivation also applies to correlated noise. That is, for a non-diagonal noise covariance matrix, the noise covariance diagonalizing similarity transform could first be applied to obtain the uncorrelated noise form similar to equation (19).

We then remember [16] that if the eigenvalues of a matrix $\mathbf{A}$ are μ_i then the eigenvalues of $\mathbf{A}^2$ are μ_i^2. Likewise we remember that $\mathrm{Trace}[\mathbf{A}] = \sum_i \mu_i$. Thus,

$$\mathrm{Trace}[(\mathbf{SS}^\top + N_0\Lambda^{-1})^2] = \sum_{i=1}^{N}\beta_i^2 \tag{24}$$

where the $\{\beta_i\}$ are the eigenvalues of $\mathbf{SS}^\top + N_0\Lambda^{-1}$. If these β_i are chosen to be the $P_i + N_0/\lambda_i$ where the P_i satisfy equation (23), then the bound of equation (22) is met with equality. Finally, the existence of such codeword sets $\{\mathbf{s}_k\}$ is guaranteed by the provable convergence to the minimum TSC of greedy interference avoidance with class warfare [11].

So in summary, once the single-user intersymbol interference problem is recast as a multiuser detection problem, codeword adaptation can be brought to bear and an optimal codeword found for each bit to be transmitted by the user. Assuming simple matched filters for each bit's codeword, the bits each attain the maximum achievable uniform SINR. Furthermore, from a user capacity perspective, simple matched filter receivers *are optimal* [6, 7].

5 Extension to Multiple Users

5.1 Identical Channels

An extension to multiple users sharing the same dispersive channel is trivial. Letting each of the L users have their own codeword ensemble matrix $\mathbf{S}_i$ with M codeword columns $\mathbf{s}_{ik}$ $i =$

$1,2,\cdots,M$ and bit vectors $\mathbf{b}_i$ of dimension M, the received signal is

$$\mathbf{r} = \mathbf{n} + \Lambda^{1/2}\sum_{i=1}^{L}\mathbf{S}_i\mathbf{b}_i = \mathbf{n} + \Lambda^{1/2}\mathbf{S}'\mathbf{b}' \quad (25)$$

where

$$\mathbf{S}' = [\mathbf{s}_{11}\cdots\mathbf{s}_{1M}|\mathbf{s}_{21}\cdots\cdots\mathbf{s}_{LM}] \quad (26)$$

and

$$\mathbf{b}' = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_L \end{bmatrix} \quad (27)$$

Equation (25) is identical in form to equation (15) so the same single user results pertain.

5.2 Non-Identical Channels: a roadmap

Now consider the case where the users see channels with possibly different impulse responses $h_i(t)$ and therefore different associated eigenfunctions and eigenvalues. Since it is assumed that the communications interval is large relative the durations of all the channel impulse responses, the channel eigenfunctions will all be approximately sinusoidal "tonebursts".[2]

Under this assumption we then have

$$\mathbf{r} = \sum_{i=1}^{L}\Lambda_i^{1/2}\mathbf{S}_i\mathbf{b}_i + \mathbf{n} \quad (28)$$

which from the perspective of user k is

$$\mathbf{r} = \Lambda_k^{1/2}\mathbf{S}_k\mathbf{b}_k + \sum_{i=1, i\neq k}^{L}\Lambda_i^{1/2}\mathbf{S}_i\mathbf{b}_i + \mathbf{n} \quad (29)$$

For simplicity, assume all the Λ_i are invertible[3] The autocovariance of the received signal is then

$$E[\mathbf{r}\mathbf{r}^\top] = \sum_{i=1}^{L}\Lambda_i^{1/2}\mathbf{S}_i\mathbf{S}_i^\top\Lambda_i^{1/2} + N_0\mathbf{I} \quad (30)$$

From the perspective of an individual user, the problem becomes how to select input waveforms for its bits in the presence of colored (although in this case correlated) "noise" consisting of both additive white Gaussian noise and interference from other users. Greedy interference avoidance performed by a single user will again produce WBE sequences for a fixed set of signatures employed by other users. Specifically, the signatures comprising $\mathbf{S}_k$ will be unit energy WBE sequences for the multiuser problem

$$\tilde{\mathbf{r}}_k = \Lambda_k^{-1/2}\mathbf{r} = \mathbf{S}_k\mathbf{b}_k + \sum_{j\neq k}\Lambda_k^{-1/2}\Lambda_j^{1/2}\mathbf{S}_j\mathbf{b}_j + \Lambda_k^{-1/2}\mathbf{n} \quad (31)$$

[2] Special thanks are due to M. Honig for this perspective.

[3] Since in any event, no transmission energy will be placed in a signal dimension with sufficiently high attenuation, requiring the eigenvalue matrices to be invertible is for convenience only. For zero or near zero channel eigenvalues, the corresponding signal dimensions would simply be expurgated in the mathematical description.

This procedure could be iteratively applied by each user until some fixed point is (or is not) reached. We are currently investigating the relationship of such fixed points and sum capacity in this multiuser/multichannel problem. Our interest is particularly keen since formulations of this type figure prominently in other problems to which interference avoidance might be applied — most notably, multiple antenna systems.

6 Summary and Conclusion

Starting from a general signal space model of dispersive channels and using a sufficiently long transmission interval so that intersymbol interference is essentially eliminated, we have shown that multiuser detection and channel equalization are (not surprisingly) isomorphic. Specifically, when M bits are to be transmitted by the user, we assign each bit a "CDMA" codeword where the ensemble meets the Welch bound with equality. Welch bound equality ensembles maximize user capacity and theoretically if used with Gaussian signaling, achieve channel capacity [6, 7].

The optimal codewords may be obtained through application of interference avoidance [8, 10, 11] or any other suitable numerical algorithm [6, 7]. Matched filter receivers for each bit are the optimal linear receiver and maximize the uniform SINR for each bit. We then extended (trivially) the method to the multiple user, identical channel case. Finally, we discussed the multiple user multiple channel case where different users preferentially occupy different portions of (possibly non-overlapping) signal spaces. Variants of this problem also arise when considering multiple antenna systems, or any configuration where users are constrained to different portions of the signal space, but where those spaces may partially overlap.

Notably absent from this work is a treatment of how the channel eigenfunctions and eigenvalues can be adaptively estimated, how covariance or codeword information is relayed from the receiver to the user(s), convergence speed, complexity issues and in general how such a theoretical concept can be made practical. To these concerns we respond *nolo contendre* at this point. Nevertheless, we feel that in light of increasingly versatile transmitters and receivers [17], interference avoidance offers an interesting window on multicarrier modulation and in general, transmission over dispersive channels.

References

[1] S. Kasturia, J. T. Aslanis, and J. M. Cioffi. Vector Coding For Partial Response Channels. *IEEE Tran. on Info. Th.*, 36(4):741–762, July 1990.

[2] J. W. Lechleider. The Optimum Combination of Block Codes and Receivers for Arbitrary Channels. *IEEE Tran. on Comm.*, 38(3):615–621, May 1990.

[3] R.G. Gallager. *Information Theory and Reliable Communication*. Wiley, 1968.

[4] J. L. Holsinger. Digital Communication Over Fixed-Time Dispersive Channels. *First IEEE Ann. Comm. Conv.*, pages 731–735, June 1965.

[5] G. W. Wornell. Spread-Signature CDMA: Efficient Multiuser Communication in the Presence of Fading. *IEEE Transactions on Information Theory*, 41(5), September 1995.

[6] P. Viswanath, V. Anantharam, and D. Tse. Optimal Sequences, Power Control and Capacity of Spread Spectrum Systems with Multiuser Linear Receivers. *IEEE Transactions on Information Theory*, June 1998. Submitted, revised. http://degas.eecs.berkeley.edu/∼dtse/pub.html.

[7] P. Viswanath and V. Anantharam. Optimal sequences and sum capacity of synchronous cdma systems. College of Engineering UCB/ERL Memorandum 98/10, U.C. Berkeley, July 1998.

[8] S. Ulukus and R. D. Yates. Iterative signature adaptation for capacity maximization of cdma systems. In *Allerton Conf. on Comm., Control and Computing*, September 1998. See http://www.winlab.rutgers.edu/ ryates/papers/allerton98.ps.

[9] C. Rose, S. Ulukus, and R. Yates. Interference Avoidance in Wireless Systems. Winlab-TR 175, WINLAB, Rutgers University, 1999.

[10] C. Rose, S. Ulukus, and R. Yates. Interference Avoidance in Wireless Systems. *IEEE JSAC*, 1999. submitted 7/99, http://steph.rutgers.edu/ crose/papers/avoid11.ps.

[11] C. Rose. Convergence of Greedy Interference Avoidance. Winlab-TR 188, WINLAB, Rutgers University, 1999.

[12] H.L. Van Trees. *Detection, Estimation, and Modulation Theory, Part I*. Wiley, New York, 1968.

[13] Sergio Verdu. *Multiuser Detection*. Cambridge University Press, 1998.

[14] G.H. Hardy and J.E. Littlewood. *Inequalities: Theory of Majorization and its Applications*. Acadmic Press, 1979.

[15] T.M. Cover and J.A. Thomas. *Elements of Information Theory*. Wiley-Interscience, 1991.

[16] Gilbert Strang. *Linear Algebra and Its Applications*. Academic Press, second edition, 1992.

[17] Special issue on software radio. *IEEE Personal Communications Magazine*, 6(4), August 1999. Editors: K-C. Chen and R. Prasad and H.V. Poor.