Interference Avoidance Applied to Multiaccess Dispersive Channels

Dimitrie C. Popescu and Christopher Rose
WINLAB, Department of Electrical and Computer Engineering
Rutgers University
94 Brett Rd, Piscataway, NJ, 08854-8058
e-mail: {cripop, crose}@winlab.rutgers.edu

Abstract

We consider application of interference avoidance to multiaccess dispersive channels corresponding to the uplink of a CDMA system in which the channel between each user and the basestation is assumed to be known. By using a symbol transmission interval long enough a common set of signal space basis functions (sinusoids) is shared by all users at the base. Each user sends frames containing multiple symbols (as many as there are signal space dimensions) and each symbol is assigned a signature waveform. Optimal signature waveforms which achieve sum capacity are obtained through iterative application of interference avoidance. Such optimal signature waveforms provide to each user a uniform signal-to-interference plus noise-ratio (SINR) for all its symbols, and the optimal linear detector for each symbol is a matched filter.

1 Introduction

In wireless communications one has to deal with dispersive channels which introduce intersymbol interference (ISI) where energy from signal bits spills over into the observation intervals of subsequent bits at the receiver. The ISI problem in digital communication systems has been extensively studied and there is a vast literature associated with it. Multiaccess channels with ISI have been studied in [1, 19] and the capacity region has been derived. Water-filling algorithms that maximize sum capacity of the multiaccess channel have also been presented in [1, 19]. In the context of multiuser detection, methods for transmitter and receiver adaptation [11] have been used for channels with multipath [10]. In this case transmitter/receiver adaptation compensates for the distortion introduced by the channel and avoids multiaccess interference.

In our paper we consider optimization of uplink codewords for a CDMA system in which the dispersive channels between users and basestation are known. Our approach uses the eigenfunctions of the autocorrelation of the channel [4] to formulate the problem in a signal space setting with an equivalent discrete vector channel model. Similar approaches based on partitioning the communication channel into a set of parallel and independent subchannels each associated with its own carrier have been proposed previously [5, 6] and all have a common assumption that the symbol interval duration $\mathcal{T}$ is long enough so that the duration of the channel impulse responses is small in comparison, thus making ISI inconsequential. Our approach differs in that different linear combinations of eigenfunctions are assigned to each symbol sent during the $\mathcal{T}$ communications interval, that is, each symbol is conveyed by means of a signature waveform of duration $\mathcal{T}$.

Under the assumption that symbol intervals $\mathcal{T}$ are sufficiently long compared to the duration of channel impulse responses then channel eigenfunctions are approximately sinusoids [4] and the same orthonormal basis can be used for all users *at the receiver*. This implies that a multicarrier CDMA (MC-CMDA) [18] modulation scheme is employed by each user in which each symbol in a given user's frame can be regarded as belonging to a different virtual user. However, different from [18], we do not assume orthogonal codewords for different symbols/users. Multicarrier modulation has been used before in frequency dispersive multiple access channels based on discrete multi-tone (DMT) scheme [2]. Recently, it has been proved that a DMT scheme with appropriately loaded carriers is optimal with respect to maximizing sum capacity subject to a given power constraint [7].

In this context we present interference avoidance [12, 13] as a mean to to derive optimal ensembles of signature waveforms (codewords) that maximize the sum capacity of the multiaccess dispersive channel in the uplink of a CDMA system. We note that optimal signature waveforms provide to each user a uniform signal-to-interference plus noise-ratio (SINR) for all its symbols, and the optimal linear detector for each symbol is a matched filter [16, 17].

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2 Problem Statement

We consider the uplink of a synchronous CDMA system with L users communicating with a common basestation. Each user sends frames containing multiple symbols using a multicode CDMA approach in which each symbol in the frame is assigned a specific signature waveform. Thus, the signals sent by users are written as

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t) \qquad \ell = 1, \ldots, L \tag{1}$$

where $b_m^{(\ell)}$, $m = 1, \ldots, M_\ell$, denote the symbols sent by user ℓ and $s_m^{(\ell)}(t)$ is the signature waveform assigned to convey symbol m of user ℓ assumed of finite duration $\mathcal{T}$.

The channel between a given user ℓ and the basestation is assumed to be linear and time-invariant and is characterized by the causal impulse response $h_\ell(t)$ of duration T_ℓ. We assume that the frame duration is much larger than the duration of all channel impulse responses $\mathcal{T} \gg T_\ell$, $\forall \ell = 1, \ldots, L$. Thus, one can safely ignore ISI between successive frames of duration $\mathcal{T}$ by placing a relatively small "zero pad" between them to allow settling of channel responses.

The received signal at the basestation is a sum of signals transmitted by all users convolved with their corresponding impulse responses plus additive Gaussian noise $n(t)$

$$r(t) = \sum_{\ell=1}^{L} x_\ell(t) * h_\ell(t) + n(t) \tag{2}$$

In this context we present application of interference avoidance to derive optimal signature waveform ensembles $\{s_m^{(\ell)}(t)\}$, $\ell = 1, \ldots, L$, $m = 1, \ldots, M_\ell$ that maximize the sum capacity of the multiaccess channel. In order to apply interference avoidance methods the continuos time (waveform) multiaccess channel in equation (2) will be converted into an equivalent vector multiaccess channel using the channel eigenfunctions as basis for the signal space representation.

3 Channel Eigenfunctions and Vector Channels

Channel eigenfunctions [4] are a set of orthonormal functions which can be used to represent channel inputs and outputs of a dispersive channel and convert it into a discrete vector channel. They are defined by the requirement that their corresponding channel responses be orthogonal to allow simple representation of channel outputs. That is, on the communication interval $(0, \mathcal{T})$ where $\mathcal{T}$ is assumed much larger than the duration of $h(t)$ we require

$$\int_0^{\mathcal{T}} \left[\int_0^{\mathcal{T}} \Psi_i(\tau) h(t-\tau) d\tau \right] \times \tag{3}$$

$$\times \left[\int_0^{\mathcal{T}} \Psi_j(\mu) h(t-\mu) d\mu \right] dt = \lambda_i \delta_{ij}$$

By defining the autocorrelation function of the channel characterized by impulse response $h(t)$

$$R_h(\tau - \mu) = \int_0^{\mathcal{T}} h(t-\tau) h(t-\mu) dt \tag{4}$$

the requirement in equation (3) becomes

$$\int_0^{\mathcal{T}} \int_0^{\mathcal{T}} \Psi_j(\mu) \Psi_i(\tau) R_h(\tau - \mu) d\tau d\mu = \lambda_i \delta_{ij} \tag{5}$$

which leads to the integral equation

$$\int_0^{\mathcal{T}} \Psi_i(\mu) R_h(\tau - \mu) d\mu = \lambda_i \Psi_i(\tau) \tag{6}$$

that defines channel eigenfunctions, where λ_i is the eigenvalue corresponding to the i-th eigenfunction. We note here that the eigenvalues λ_i are non-negative and they rapidly approach zero [14, p. 193], and we assume that there are only N non-negligible eigenvalues λ_i. This implies that the useful input signal space – that portion of the signal space whose component energies are not strongly absorbed by the channel – is of dimension N.

By using channel eigenfunctions as basis for the signal space and projecting input and output signals, $x(t)$ and $y(t)$ respectively, an equivalent vector multiaccess channel is obtained

$$\mathbf{y} = \mathbf{\Lambda}^{1/2} \mathbf{x} + \mathbf{n} \tag{7}$$

where $\mathbf{x}$ and $\mathbf{y}$ are the input and output vectors (N-dim.) respectively, $\mathbf{n}$ is the noise vector (also N-dim.), and $\mathbf{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_N)$ is the $N \times N$ diagonal matrix containing the channel eigenvalues.

In the multiuser case where different users are received at the basestation through different channels with different impulse responses the set of channel eigenfunctions may be different for distinct users. However, since it is assumed that the frame duration $\mathcal{T}$ is large relative the durations of all the channel impulse responses, the channel eigenfunctions will all be approximately sinusoidal "tonebursts". Substitution of sinusoids for channel eigenfunctions has been studied in detail in [4] and appears practical for a wide range of channels. Also, similar to [4] we choose to use separate dimensions for sine and cosine of the same frequency rather than complex exponentials which implies pairs of eigenfunctions with the same eigenvalue.

4 The Single User Case and Interference Avoidance

For a single user transmitting over a dispersive channel the transmitted signal is composed of a superposition of waveforms

$$x(t) = \sum_{k=1}^{M} b_k s_k(t) \qquad (8)$$

or equivalently codewords, corresponding to each symbol to be sent

$$\mathbf{x} = \mathbf{S}\mathbf{b} \qquad (9)$$

and the received signal vector is

$$\mathbf{r} = \Lambda^{1/2}\mathbf{S}\mathbf{b} + \mathbf{n} \qquad (10)$$

where $\mathbf{S}$ is the $N \times M$ matrix with columns $\mathbf{s}_k$ the codewords specifying the superposition of channel eigenfunctions that yields the waveforms $\{s_k(t)\}_{k=1}^{M}$, $\mathbf{b} = [b_1 \dots b_M]^\top$ is the vector containing the symbol sequence to be sent, Λ is the channel eigenvalue matrix, and $\mathbf{n}$ is the corresponding noise vector assumed colored with uncorrelated components and diagonal covariance matrix $\mathbf{W}$.

We define an equivalent problem for the given user by pre-multiplying with the inverse of the channel eigenvalue matrix in equation (10)

$$\tilde{\mathbf{r}} = \Lambda^{-1/2}\mathbf{r} = \mathbf{S}\mathbf{b} + \tilde{\mathbf{n}} \qquad (11)$$

In the equivalent problem the user has a perfect channel with colored noise $\tilde{\mathbf{n}} = \Lambda^{-1/2}\mathbf{n}$ and equation (11) is typical multiuser detection problem [15, ch. 4] where an ensemble of user transmissions $b_k\mathbf{s}_k$ must be jointly decoded subject to a constraint on transmitted energy for each codeword. One could turn to multiuser detection theory at this point and seek to design the best receivers (decision feedback, MMSE etc.) for a given ensemble of codewords. However, our goal here is to find codeword ensembles which maximize sum capacity.

Optimal signature waveforms that maximize the SINR for all symbols are obtained through the greedy *eigen-algorithm* for interference avoidance [13, 12] formally stated below:

- Start with an initial set of signatures $\{s_k(t)\}_{k=1}^{M}$, with corresponding codewords $\{\mathbf{s}_k\}_{k=1}^{M}$ with respect to the orthonormal basis defined by the channel eigenfunctions.
- For each symbol k, the interference generated by the other $M-1$ symbols is described by the N-dimensional autocorrelation matrix $\mathbf{R}_k = \mathbf{S}\mathbf{S}^\top - \mathbf{s}_k\mathbf{s}_k^\top + N_0\Lambda^{-1}$.
- The minimum eigenvalue μ_k^* of $\mathbf{R}_k$ and its associated eigenvector $\mathbf{x}_k^*$ are found, and then $\mathbf{s}_k$ is replaced by $\mathbf{x}_k^*$.
- This process is repeated iteratively for each codeword until a fixed point is reached where no further improvement can be obtained.
- If the fixed point is suboptimal use "class warfare" methods [12] to escape it and continue iterations.

The idea behind this algorithm is to adapt the codeword for each symbol so as to minimize interference coming from the other symbols and maximize the SINR. It has been shown [12] that in a colored noise background the eigen-algorithm *water fills* those dimensions with minimum background noise energy thus implying maximization of sum capacity.

5 The Multiuser Case

An identical approach is taken in the multiuser case where multiple users are received at the basestation over distinct channels with different impulse responses $h_\ell(t)$, which is the case corresponding to the uplink scenario in a CDMA system.

We note again that channel eigenfunctions for all users are practically sinusoids and that each user in the system employs a MC-CDMA modulation scheme to transmit frames. In this case each user ℓ transmits the signal $x_\ell(t)$

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m s_m^{(\ell)}(t) \qquad \ell = 1, \dots, L \qquad (12)$$

or equivalently the vector $\mathbf{x}_\ell$

$$\mathbf{x}_\ell = \sum_{m=1}^{M_\ell} b_m \mathbf{s}_m^{(\ell)} = \mathbf{S}_\ell \mathbf{b}_\ell \qquad \ell = 1, \dots, L \qquad (13)$$

where $\mathbf{S}_\ell$ is the $N \times M_\ell$ matrix with columns $\mathbf{s}_m^{(\ell)}$ the codewords corresponding to user ℓ specifying the superposition of sinusoids that yield the waveforms $s_m^{(\ell)}(t)$ and $\mathbf{b}_\ell$ the vector containing the symbol sequence to be sent by user ℓ. The received vector at the receiver consists of the sum

$$\mathbf{r} = \sum_{\ell=1}^{L} \Lambda_\ell^{1/2}\mathbf{S}_\ell\mathbf{b}_\ell + \mathbf{n} \qquad (14)$$

with Λ_ℓ representing the channel eigenvalue matrix of user ℓ. From the perspective of user k the received signal in equation (14) can be rewritten as

$$\mathbf{r} = \Lambda_k^{1/2}\mathbf{S}_k\mathbf{b}_k + \sum_{\ell=1,\ell\neq k}^{L} \Lambda_\ell^{1/2}\mathbf{S}_\ell\mathbf{b}_\ell + \mathbf{n} \qquad (15)$$

Similar to equation (11) we define the equivalent problem for user k, pre-multiplying by the corresponding inverse channel eigenvalue matrix $\Lambda_k^{-1/2}$ in equation (15)

$$\mathbf{r}_k = \mathbf{S}_k\mathbf{b}_k + \Lambda_k^{-1/2}\left(\sum_{\ell\neq k}\Lambda_l^{1/2}\mathbf{S}_\ell\mathbf{b}_\ell + \mathbf{n}\right) \qquad (16)$$

The eigen-algorithm is then applied for user k equivalent problem by modifying its codewords sequentially; the codeword corresponding to symbol m of user k being replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference process

$$\mathbf{R}_m^{(k)} = \mathbf{S}_k\mathbf{S}_k^\top - \mathbf{s}_m^{(k)}\mathbf{s}_m^{(k)^\top} + \qquad (17)$$
$$+\Lambda_k^{-1/2}\left(\sum_{\ell\neq k}\Lambda_\ell^{1/2}\mathbf{S}_\ell\mathbf{S}_\ell^\top\Lambda_\ell^{1/2} + \mathbf{W}\right)\Lambda_k^{-1/2}$$

Applied iteratively by each user the eigen-algorithm reaches a fixed point for which the SINR of all symbols corresponding to a given user is the same and maximized. A formal statement of the eigen-algorithm for the multiuser case is given below:

1. Start with a randomly chosen codeword ensemble specified by the user codeword matrices $\{\mathbf{S}_k\}_{k=1}^L$
2. For each user $k = 1 \ldots L$
   (a) Define the equivalent problem as in equation (16)
   (b) adjust user k codewords sequentially: the codeword corresponding to symbol m of user k is replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process (17)
3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.
4. If a suboptimal point is reached in step 2*b* use "class warfare" methods [12] to escape it and continue iterations.

Provided that $M_\ell \geq N$ at any step of the algorithm each user water fills the signal space in the equivalent problem. Thus, at the fixed point the resulting codeword ensemble corresponds to a situation where all users simultaneously water fill the signal space in their corresponding equivalent problem which implies maximization of the sum capacity of the multiaccess dispersive channel [8, 19].

Also, it has been shown using properties of interference avoidance algorithms [8] that the optimal codeword ensemble implies a partitioning of the signal space in frequency such that no two users reside in subspaces that overlap in more than one frequency which comes in agreement with previous work [3, 7]. As the number of signal space dimensions $N \to \infty$ this corresponds to generalizes to distinct frequency bands for distinct users which represents Frequency Division Multiple Access (FDMA) known to maximize total capacity of multiple access channels with ISI [1].

6 Conclusions

A novel approach to communication over dispersive channels is presented in the paper. Multiple symbols are sent by users in frames of extended duration using a multicode CDMA approach. The problem is approached using a general signal space model of dispersive channels and cast as a multiuser detection problem. Optimal codeword ensembles that maximize sum capacity are obtained through successive application of interference avoidance by all users.

The use of optimal codeword ensembles which maximize sum capacity allows matched filters to be used as the optimal linear receivers [16, 17]. This implies a simple receiver structure consisting of a matched filter bank for each user ℓ and identical independent modulation of the M_ℓ symbol streams associated user ℓ. In a practical setting the outputs of the matched filters would be processed (perhaps with various multiuser detection techniques [15]). From a practical point of view receiver complexity can be further reduced using the single frequency overlap property of the optimal codeword ensemble [3, 7, 8].

Interference avoidance methods presented in the paper can be applied to other problems where multiple users are constrained to different portions of the signal space, with possible overlap between spaces. Of particular interest are MIMO channels which result when multiple inputs and outputs are used for communication and and application of interference avoidance to MIMO channels can be found in [9].

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