Codeword Optimization for Asynchronous CDMA Systems Through Interference Avoidance

Dimitrie C. Popescu and Christopher Rose
WINLAB, Department of Electrical and Computer Engineering
Rutgers, The State University of New Jersey
94 Brett Road, Piscataway, NJ 08854-8058 , USA
e-mail: {cripop,crose}@winlab.rutgers.edu

Abstract — **The paper investigates application of interference avoidance to asynchronous CDMA systems for which symbol intervals corresponding to different users may not be necessarily synchronized at the receiver. The asynchronous system is modeled as frame-synchronous with symbols sent in parallel using a multicode CDMA approach. Each symbol in the frame is transmitted using a distinct signature waveform of extended duration. This approach relaxes synchronization requirements at the common receiver and allows also the presence of users with different data rates. A vector multiple access channel model is derived for which application of interference avoidance to codeword optimization becomes straightforward.**

I. Introduction

In an asynchronous CDMA system users are received at the common receiver with non-coincident symbol intervals. This can be a consequence of lack of synchronization for systems with users having the same data rate, or can be generated by differences in data rates for systems where distinct users are allowed to transmit at different data rates. The latter case is especially interesting for future wireless systems which may have to support users with different data rates. Relaxing synchronization constraints simplifies system design by relaxing timing control.

The capacity region of symbol-asynchronous Gaussian multiple access channels has been derived in [9] using an equivalent multiple access channel model with memory and frame synchronism [8]. Recently, for chip-based DS-CDMA systems user capacity of the asynchronous system has been analyzed and compared to that of the synchronous system [1] and a class of optimum signature sequences has been identified [7] for which there is no loss in user capacity due to asynchrony. However, the analysis is restricted to symbol-asynchronous but chip-synchronous DS-CDMA systems.

In this paper, application of interference avoidance methods to codeword optimization in an asynchronous CDMA system is presented. Similar to [9] users in the system are considered frame synchronous and a multicode CDMA transmission scheme is used. Symbols of a frame are sent in parallel using distinct signature waveforms of extended duration. This approach relaxes synchronization requirements for the system since the frame duration will be larger than the duration of individual symbol intervals would have been if sequential transmission of symbols in a frame had been used.

An equivalent discrete time vector channel model is obtained for which application of interference avoidance to codeword optimization is straightforward and results in codeword (or equivalently waveform) ensembles that maximize sum capacity.

II. Problem Statement

We start by considering an asynchronous CDMA system with users transmitting at the same data rate $1/T$ for which symbol intervals corresponding to different users are not synchronized at the common receiver. Following the methodology described in [10, p. 21] one needs to introduce offsets that model the lack of alignment of symbol intervals at the receiver. As opposed to a synchronous system for which the received signal can be written by taking only one-shot of the model over the symbol interval $[0, T]$ as

$$R(t) = \sum_{\ell=1}^{L} b_\ell S_\ell(t) + n(t) \tag{1}$$

for the asynchronous case one needs to include the offsets $\tau_\ell \in [0, T)$, $\ell = 1, \dots, L$, and consider the fact that users send frames with many symbols $\mathbf{b}_\ell = [b_1^{(\ell)}, \dots, b_M^{(\ell)}]$ of duration $\mathcal{T}$ as shown in figure 1. This implies that frame-synchronism rather than symbol-synchronism is assumed, and users start and finish their transmissions within T units of each other. The received signal is then written as

$$R(t) = \sum_{\ell=1}^{L} \sum_{m=1}^{M} b_m^{(\ell)} S_\ell(t - mT - \tau_\ell) + n(t) \tag{2}$$

Similar to [9] we assume that the offsets τ_ℓ, $\ell = 1, \dots, L$, are known to the receiver, but unknown to the transmitters so that they cannot advance or retard their transmissions. As noted in [9] the assumption of frame synchronism can be made at any rate with some sort of channel feedback which is an underlying assumption for systems using interference avoidance methods [5].

Since each symbol transmitted by a given user overlaps also with past/future symbols transmitted by the other users the symbol asynchronous multiple access channel has memory and can be modeled through an equivalent multiple access channel with memory [8, 10]. Thus, one can think of transmitting information in the asynchronous CDMA system using an approach similar to that used in a CDMA system where symbols are subject to ISI (dispersive channels [2]): multicode CDMA transmission of symbols using distinct signature waveforms of extended duration to convey each symbol in the frame. While in the case of dispersive channels this approach makes ISI inconsequential [2], for asynchronous systems this relaxes the synchronization requirement, making it easier to be obtained in practice. Furthermore, this approach can be easily generalized to users with different data rates by considering a

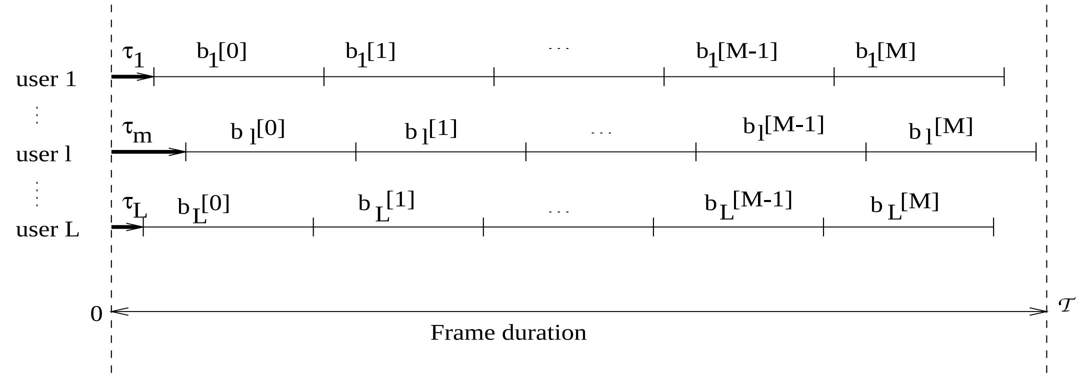

Figure 1: Symbol-asynchronous users with the same data rate $1/T$ modeled as frame synchronous. Within the frame duration each user sends M symbols.

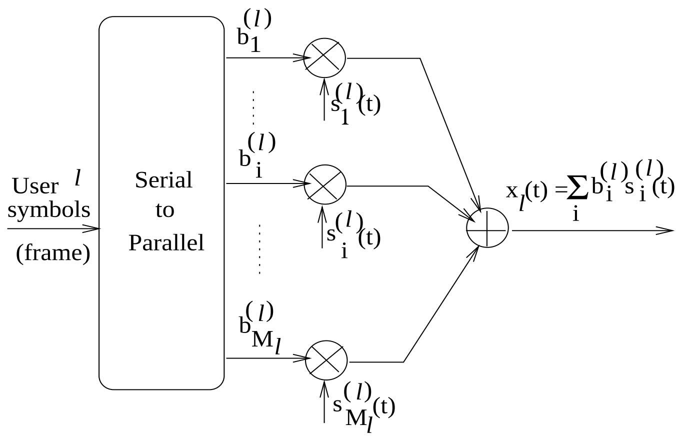

Figure 2: Multicode CDMA for sending frames of information. Each symbol in user ℓ frame is assigned a distinct signature waveform of extended duration and the transmitted signal is a superposition of signature waveforms scaled by their corresponding information symbols.

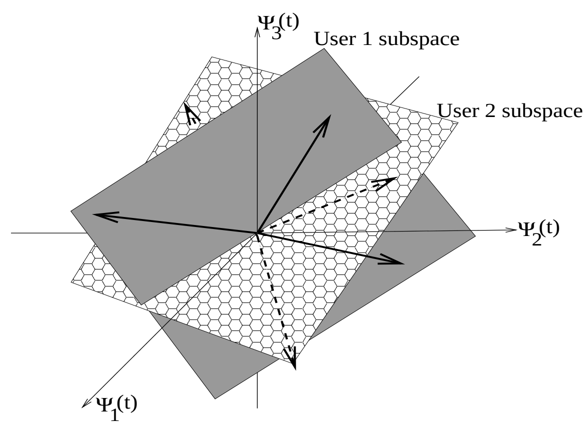

Figure 3: 3-dimensional receiver signal space with 2 users residing in 2-dimensional subspaces. Both users have 3 signature waveforms to transmit symbols.

different number of symbols M_ℓ in a frame for distict user ℓ. The transmitted signal corresponding to user ℓ is written as (see figure 2)

$$x_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t) \tag{3}$$

with $s_m^{(\ell)}(t)$ being the signature waveform corresponding to symbol m of user ℓ of duration $\mathcal{T}$. We note here that the multicode CDMA approach is a generalized form of CDMA as if each symbol in the frame corresponded to a different virtual user, and that a similar approach has been used in [10, Ch. 4] in the analysis of multiuser detectors for asynchronous CDMA systems. The received signal at the basestation is the sum of signals transmitted by all users plus additive Gaussian noise and is written as

$$R(t) = \sum_{\ell=1}^{L} x_\ell(t-\tau_\ell)+n(t) = \sum_{\ell=1}^{L}\sum_{m=1}^{M_\ell} b_m^{(\ell)} s_m^{(\ell)}(t-\tau_\ell)+n(t) \tag{4}$$

Our goal is then to find an optimal ensemble of waveforms $s_m^{(\ell)}(t)$, $\ell = 1, \ldots, L$, $m = 1, \ldots, M_\ell$, for which the sum capacity of the multiple access channel is maximized.

III. The Equivalent Vector Multiple Access Channel

The multicode CDMA scheme outlined in the previous section implies a vector multiple access channel representation for which application of interference avoidance is straightforward [3]. In order to derive the equivalent vector channel model, we assume that each user ℓ resides in a signal space of finite dimension N_ℓ implied by the frame duration $\mathcal{T}$ and finite bandwidth W_ℓ, and spanned by the vector of functions $\mathbf{\Psi}^{(\ell)}(t) = [\Psi_1^{(\ell)}(t)\ldots\Psi_{N_\ell}^{(\ell)}(t)]^\top$. Furthermore, we assume that the receiver signal space of dimension N is spanned by $\mathbf{\Psi}(t) = [\Psi_1(t)\ldots\Psi_N(t)]^\top$ and is implied by bandwidth W that includes all W_ℓ's corresponding to all users and observation interval $\mathcal{T}+\tau$ with $\tau = \max_\ell\{\tau_\ell\}$ being the largest user delay measured with respect to the begining of the observation interval at the receiver (see figure 4). Figure 3 provides a graphical illustration of such a signal space configuration for 2 users residing in 2-dimensional subspaces with a 3-dimensional receiver signal space.

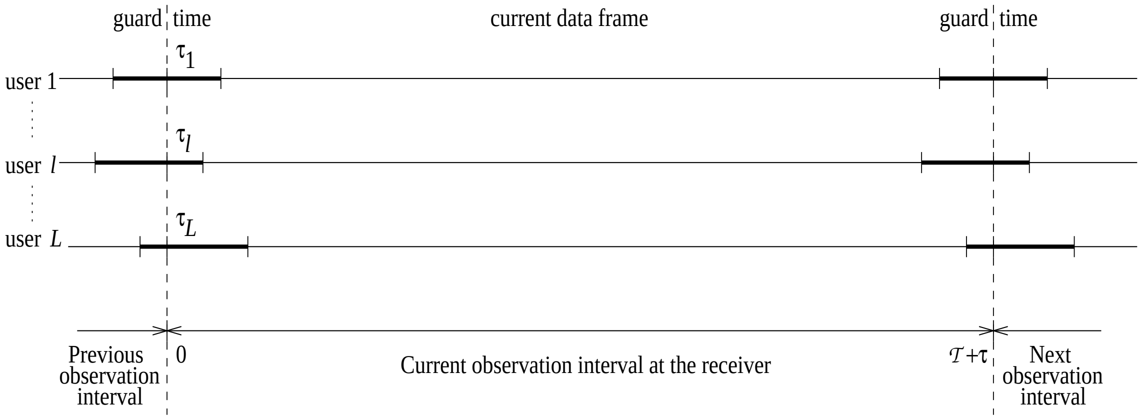

Figure 4: Users sending data frames of duration $\mathcal{T}$ with guard intervals.

We note that, by adding the maximum delay τ to the frame duration we ensure that all waveforms corresponding to the current frame are completely observed during the observation interval at the receiver. We also note that, in order to avoid overlap between succesive frames at the receiver, time guard intervals are inserted at the begining and end of each transmitted frame as it can be seen in figure 4.

Each user ℓ transmits the signal $x_\ell(t)$ given by equation (3) which is written in terms of user ℓ basis functions as

$$x_\ell(t) = \mathbf{\Psi}^{(\ell)}(t)^\top \mathbf{x}_\ell = \mathbf{\Psi}^{(\ell)}(t)^\top \mathbf{S}_\ell \mathbf{b}_\ell \tag{5}$$

with

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \dots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \qquad \ell = 1, \dots, L$$

the codeword matrix of user ℓ. The received signal at the common receiver contains signals of all users with corresponding delays plus additive noise

$$r(t) = \sum_{\ell=1}^{L} x_\ell(t - \tau_\ell) + n(t) \tag{6}$$

and is observed over the interval $[0, \mathcal{T} + \tau]$. By projecting the received signal onto the basis functions of the receiver signal space an equivalent vector channel model is obtained:

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{7}$$

with the $N \times N_\ell$ matrix $\mathbf{H}_\ell$ relating the user ℓ signal space and receiver signal space defined by

$$\mathbf{H}_\ell = \int_0^{\mathcal{T}+\tau} \mathbf{\Psi}(t) \mathbf{\Psi}^{(\ell)}(t - \tau_\ell)^\top dt \tag{8}$$

Note that equation (7) allows now application of interference avoidance to determine optimal codeword ensembles that maximize sum capacity. Following [3] we rewrite the received signal in equation (7) from the perspective of user k

$$\mathbf{r} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \sum_{\ell=1, \ell \neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{9}$$

and apply the singular value decomposition (SVD) [6] to the channel matrix corresponding to user k.

$$\mathbf{H}_k = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^\top \tag{10}$$

where matrix $\mathbf{U}_k$ of dimension $N \times N$ has as columns the eigenvectors of $\mathbf{H}_k \mathbf{H}_k^\top$, matrix $\mathbf{V}_k$ of dimension $N_k \times N_k$ has as columns the eigenvectors of $\mathbf{H}_k^\top \mathbf{H}_k$, and matrix $\mathbf{D}_k$ of dimension $N \times N_k$ contains the singular values of $\mathbf{H}_k$ on the main diagonal and zero elsewhere. Without loss of generality we assume that $\mathbf{H}_k$ has full rank[1] N_k. Thus, the singular value matrix $\mathbf{D}_k$ can be partitioned as

$$\mathbf{D}_k = \begin{bmatrix} \tilde{\mathbf{D}}_k \\ \mathbf{0} \end{bmatrix} \tag{11}$$

with $\tilde{\mathbf{D}}_k$ an $N_k \times N_k$ diagonal matrix containing the non-zero singular values along the diagonal and zeros in rest. The left inverse of $\mathbf{D}_k$ is defined as

$$\mathbf{D}_k^\dagger = \begin{bmatrix} \tilde{\mathbf{D}}_k^{-1} & \mathbf{0} \end{bmatrix} \tag{12}$$

and it is obvious that

$$\mathbf{D}_k^\dagger \mathbf{D}_k = \mathbf{I}_{N_k} \tag{13}$$

Returning to equation (9) in which the SVD for channel matrix $\mathbf{H}_k$ has been applied we obtain

$$\mathbf{r} = \mathbf{U}_k \mathbf{D}_k \mathbf{V}_k^\top \mathbf{S}_k \mathbf{b}_k + \sum_{\ell=1, \ell \neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{14}$$

We pre-multiply by $\mathbf{U}_k^\top$

$$\mathbf{r}_k = \mathbf{U}_k^\top \mathbf{r} = \mathbf{D}_k \mathbf{V}_k^\top \mathbf{S}_k \mathbf{b}_k + \mathbf{U}_k^\top \left(\sum_{\ell=1, \ell \neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \right) \tag{15}$$

and define

$$\tilde{\mathbf{S}}_k = \mathbf{V}_k^\top \mathbf{S}_k$$

[1] We note that this is not a restriction since if $\mathbf{H}_k$ is not full rank then some dimensions of the user k signal space will have zero projection on the output space. Therefore we can redefine a reduced codeword matrix $\mathbf{S}_k$ which uses only dimensions with nonzero projections on the output space.

and

$$\mathbf{z}_k = \mathbf{U}_k^\top \left(\sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \right)$$

We have then

$$\mathbf{r}_k = \mathbf{D}_k \tilde{\mathbf{S}}_k \mathbf{b}_k + \mathbf{z}_k \qquad (16)$$

and furthermore we pre-multiply by the left inverse of $\mathbf{D}_k$ obtaining

$$\tilde{\mathbf{r}}_k = \mathbf{D}_k^\dagger \mathbf{r}_k = \tilde{\mathbf{S}}_k \mathbf{b}_k + \tilde{\mathbf{z}}_k \qquad (17)$$

We note that equation (17) allows straightforward application of the eigen-algorithm for interference avoidance. The "noise" term $\tilde{\mathbf{z}}_k$ in equation (17) represents the interference-plus-noise from the rest of the system that is present in user k signal space and has covariance matrix

$$\begin{aligned} \mathbf{Z}_k &= E[\mathbf{z}_k \mathbf{z}_k^\top] \\ &= \mathbf{D}_k^\dagger \mathbf{U}_k^\top \left(\sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \right) \mathbf{U}_k \mathbf{D}_k^{\dagger\top} \end{aligned} \qquad (18)$$

Also, the transformed codeword matrix $\tilde{\mathbf{S}}_k$ is completely equivalent with the original codeword matrix $\mathbf{S}_k$ since they are related through an orthogonal transformation $\mathbf{V}_k^\top$.

The eigen-algorithm for interference avoidance for the vector multiaccess channel obtained for asynchronous systems is formally stated below [3]:

1. Start with a randomly chosen codeword ensemble specified by the codeword matrices $\{\mathbf{S}_k\}_{k=1}^L$
2. For each user $k = 1 \dots L$
   (a) project the problem onto the signal space of user k applying SVD to the user k channel matrix $\mathbf{H}_k$ and obtain the signal $\tilde{\mathbf{r}}_k$ in equation (17)
   (b) adjust user k codewords sequentially: the codeword corresponding to b_m of user k is replaced by the minimum eigenvalue eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process in clear space

   $$\mathbf{R}_m^{(k)} = \tilde{\mathbf{S}}_k \tilde{\mathbf{S}}_k^\top - \tilde{\mathbf{s}}_m^{(k)} \tilde{\mathbf{s}}_m^{(k)\top} + \mathbf{Z}_k \qquad (19)$$

   (c) After all N_k codewords have been changed the new codeword matrix corresponding to user k is $\mathbf{S}_k = \mathbf{V}_k \tilde{\mathbf{S}}_k$
3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.
4. If a suboptimal point is reached in step 2b use escape methods [4] and continue iterations.

It has been shown [3] that this algorithm performs iterative water filling [13] of each user's signal space and converges to a fixed point [13] where the sum capacity of the vector multiaccess channel is maximized.

So, in summary, by using parallel transmission of symbols in a frame with signature waveforms of extended duration the asynchronous CDMA system can be modeled as a multiaccess vector channel for which application of the eigen-algorithm for interference avoidance to codeword optimization is straightforward. Using the resulting optimal codeword ensembles implies also a very simple structure at the receiver consisting of matched filters which are optimal linear receivers [11, 12].

IV. Conclusions

Application of interference avoidance for asynchronous systems was presented. The asynchronous system is modeled as a frame-synchronous one and parallel transmission of symbols in a frame through signature waveforms of extended duration is assumed. Using a vector multiple access channel model interference avoidance methods apply in a straightforward way and yield optimal codeword ensembles that maximize the sum capacity of the multiple access channel.

References

[1] Kiran and D. Tse. Effective Interference and Effective Bandwith of Linear Multiuser Receivers in Asynchronous CDMA Systems. *IEEE Transactions on Information Theory*, 46(4):1426 – 1447, July 2000.

[2] D. C. Popescu and C. Rose. CDMA Codeword Optimzation for Uplink Dispersive Channels Through Interference Avoidance. *IEEE Transactions on Information Theory*. submitted 12/2000, revised 10/2001, preprint available at http://www.winlab.rutgers.edu/∼cripop/papers.

[3] D. C. Popescu and C. Rose. Interference Avoidance for Multiaccess Vector Channels. In *2002 IEEE International Symposium on Information Theory - ISIT'02*, Lausanne, Switzerland, July 2002. accepted, preprint available at http://www.winlab.rutgers.edu/∼cripop/papers.

[4] C. Rose. CDMA Codeword Optimization: Interference Avoidance and Convergence Via Class Warfare. *IEEE Transactions on Information Theory*, 47(6):2368–2382, September 2001.

[5] C. Rose, S. Ulukus, and R. Yates. Wireless Systems and Interference Avoidance. *IEEE Journal on Selected Areas in Communications*, July 2002. to appear; available at http://steph.rutgers.edu/∼crose/papers/avoid17.ps.

[6] Gilbert Strang. *Linear Algebra and Its Applications*. Harcourt Brace Jovanovich College Publishers, third edition, 1988.

[7] S. Ulukus and R. Yates. Optimum Signature Sequence Sets for Asynchronous CDMA Systems. In *Proc. 38th Allerton Conf. on Communication, Control, and Computing*, October 2000.

[8] S. Verdu. Multiple-Access Channels with Memory with and without Frame Synchronism. *IEEE Transactions on Information Theory*, 35(3):605–619, May 1989.

[9] S. Verdu. The Capacity Region of the Symbol-Asynchronous Gaussian Multiple-Access Channel. *IEEE Transactions on Information Theory*, 35(4):733–751, July 1989.

[10] S. Verdu. *Multiuser Detection*. Cambridge University Press, 1998.

[11] P. Viswanath and V. Anantharam. Optimal Sequences and Sum Capacity of Synchronous CDMA Systems. *IEEE Transactions on Information Theory*, 45(6):1984–1991, September 1999.

[12] P. Viswanath, V. Anantharam, and D. Tse. Optimal Sequences, Power Control and Capacity of Spread Spectrum Systems with Multiuser Linear Receivers. *IEEE Transactions on Information Theory*, 45(6):1968–1983, September 1999.

[13] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi. Iterative Water-Filling for Gaussian Vector Multiple Access Channels. In *2001 IEEE International Symposium on Information Theory - ISIT'01*, Washington, DC, June 2001. submitted for journal publication.