EMITTER LOCALIZATION IN A MULTIPATH ENVIRONMENT USING EXTENDED KALMAN FILTER

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ABSTRACT

The paper presents the use of Kalman filter for emitter localization in a multipath environment where line of sight propagation cannot be assumed. Differences between the times of arrival of direct and reflected signals are used to compute the position estimate. Because of the nonlinear relationship between position and times of arrival extended Kalman filter must be used.

1. INTRODUCTION

Emitter localization is a standard military problem where a radio source must be located. Cellular 911 service where mobile users must be localized is similar. Other potential applications of emitter localization in wireless communications include: location-sensitive billing, fraud detection, cellular system design and resource management, and fleet management and intelligent transportation systems.

For current radio location systems based on signal strength, angle of arrival, or time of arrival measurements line of sight is essential for highly accurate location estimates. However, in urban environments multipath propagation is the primary reason for inaccuracies observed in the angle of arrival and signal strength measurement systems. Multipath also affects the time-based location systems, causing errors in the timing estimates. Therefore, new localization techniques must be developed for multipath environments.

Methods have been developed to mitigate the effects of multipath as for example delay estimators using least mean squares techniques [7] and extended Kalman filter [5]. Other methods make distinction between line of sight and non-line of sight propagation [8].

There are however, potential benefits from using multipath rather than combating it, as for example the need for fewer listening posts (potentially one as opposed to a minimum of three with standard triangulation) and a commensurate decrease in the need for inter-post communication and synchronization.

The paper deals with direct analysis of the problem which assumes that reflector positions as well as the time difference of arrival (TDOA) identities are known at the listening post.

2. PROBLEM STATEMENT

Consider the localization problem of an emitter E situated at unknown coordinates (x, y) in a multipath environment as described in figure 1. For simplicity we assume that the listening post L is placed at the origin. There are N reflectors placed at coordinates (x_k, y_k), $k = 1, \dots, N$, that generate additional signals at the listening post. The k-th TDOA is calculated by measuring the difference between the time of arrival (TOA) of the direct signal (t_0) and the TOA of its k-th reflection (t_k), and is proportional to the difference between the length of the reflected path and the direct path of the signal. Therefore, we get N measurements

$$s_k = ct_k - ct_0 = \sqrt{(x - x_k)^2 + (y - y_k)^2} + \tag{1}$$

$$+\sqrt{x_k^2 + y_k^2} - \sqrt{x^2 + y^2}, \quad k = 1, \dots, N$$

Using only these measurements we must get an estimate of the unknown emitter coordinates (x, y).

In order to use the Kalman filter for estimation, the above problem must be cast in the state-space form. Be-

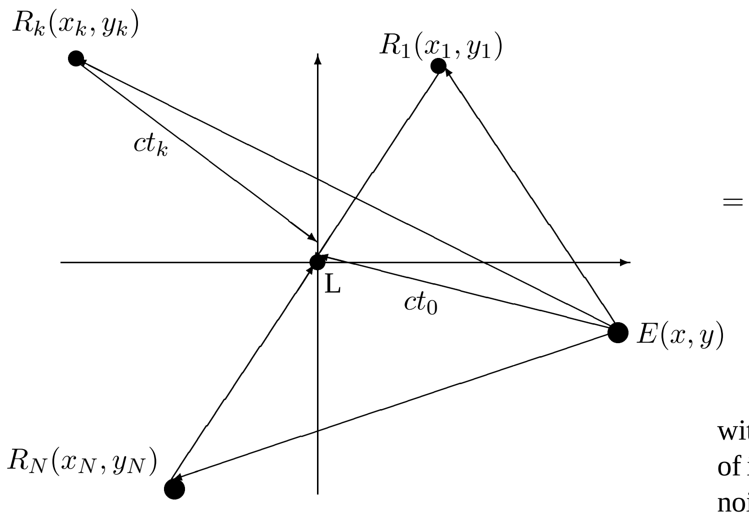

Figure 1: Multipath Environment

cause it is assumed that the emitter is not moving[1] the problem is similar to that of bias estimation where a constant, but unknown vector is estimated from a set of measurements. This problem has been solved in the control literature [1], [3], and has also been extended to estimating a set of slowly varying bias-like variables [4].

Let us denote by $\mathbf{x} = [x\ y]^T$ the vector containing the unknown coordinates of the emitter. The dynamic behavior of $\mathbf{x}$ is modeled by the state equation

$$\mathbf{x}(n) = \mathbf{x}(n-1) + w(n) \tag{2}$$

with $w(n)$ being zero-mean Gaussian random sequence of intensity $E[w(j)w(n)] = Q(n)\delta_{jn}$. The model defined in (2) for $\mathbf{x}$ constitutes a random-walk process for the coordinates of the emitter, and can be seen as either a slow variation in time of the emitter position, or as uncertainty in the location of emitter's position.

The measurement equation attached to (2) is nonlinear and contains all the TDOA measurements (1)

$$\mathbf{y}(n) = \begin{bmatrix} s_1(n) \\ \vdots \\ s_k(n) \\ \vdots \\ s_N(n) \end{bmatrix} + v(n) = \mathbf{s}(n) + v(n) =$$

$$= \begin{bmatrix} \sqrt{(x(n)-x_1)^2 + (y(n)-y_1)^2} + \\ + \sqrt{x_1^2 + y_1^2} - \sqrt{x(n)^2 + y(n)^2} \\ \vdots \\ \sqrt{(x(n)-x_k)^2 + (y(n)-y_k)^2} + \\ + \sqrt{x_k^2 + y_k^2} - \sqrt{x(n)^2 + y(n)^2} \\ \vdots \\ \sqrt{(x(n)-x_N)^2 + (y(n)-y_N)^2} + \\ + \sqrt{x_N^2 + y_N^2} - \sqrt{x(n)^2 + y(n)^2} \end{bmatrix} + v(n) \tag{3}$$

with $v(n)$ being zero-mean Gaussian random sequence of intensity $E[v(j)v(n)] = R(n)\delta_{jn}$ and represents the noise that corrupts TDOA measurements. Because (3) is nonlinear the extended Kalman filter must be used for estimation.

3. THE EXTENDED KALMAN FILTER

For a general nonlinear system described by the equations

$$\mathbf{x}(n) = f(\mathbf{x}(n-1), n-1) + w(n-1) \tag{4}$$

$$\mathbf{y}(n) = h(\mathbf{x}(n), n) + v(n) \tag{5}$$

with $w(n),\ v(n)$ as defined in the previous section, the equations of the extended Kalman filter [2] are

- predicted state estimate (a priori estimate)

$$\hat{\mathbf{x}}(n)^- = f(\hat{\mathbf{x}}(n-1)^+, n-1) \tag{6}$$

- predicted measurement

$$\hat{\mathbf{y}}(n) = h(\hat{\mathbf{x}}(n)^-, n) \tag{7}$$

- error covariance extrapolation

$$P(n)^- =$$

$$= F(n-1)P(n-1)^+ F(n-1)^T + Q(n-1) \tag{8}$$

- the Kalman gain matrix

$$K(n) = P(n)^- H(n)^T \times$$

$$\times [H(n)P(n)^- H(n)^T + R(n)]^{-1} \tag{9}$$

- error covariance update

$$P(n)^+ = [I - K(n)H(n)^T]^{-1} P(n)^- \tag{10}$$

[1] This is not a restriction since it will be seen that the emitter position will be allowed to have some limited random variation.

- state estimate update (a posteriori estimate)

$$\hat{\mathbf{x}}(n)^+ = \hat{\mathbf{x}}(n)^- + K(n)[\mathbf{y}(n) - \hat{\mathbf{y}}(n)] \quad (11)$$

with F and H being the linear approximations of f and g, i.e.

$$F(n-1) \simeq \left.\frac{\partial f(\mathbf{x}, n-1)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(n-1)^-} \quad (12)$$

$$H(n) \simeq \left.\frac{\partial h(\mathbf{x}, n)}{\partial \mathbf{x}}\right|_{\mathbf{x}=\hat{\mathbf{x}}(n)^-} \quad (13)$$

For our problem the state equation is linear with state matrix equal to the identity matrix. The nonlinear function that appears in the measurement equation

$$h(\mathbf{x}) = \begin{bmatrix} h_1(\mathbf{x}) \\ \vdots \\ h_N(\mathbf{x}) \end{bmatrix} = \quad (14)$$

$$= \begin{bmatrix} \sqrt{(x-x_1)^2 + (y-y_1)^2} + \\ +\sqrt{x_1^2 + y_1^2} - \sqrt{x^2+y^2} \\ \vdots \\ \sqrt{(x-x_N)^2 + (y-y_N)^2} + \\ +\sqrt{x_N^2 + y_N^2} - \sqrt{x^2+y^2} \end{bmatrix}$$

has the corresponding linear approximation

$$H(n) = \begin{bmatrix} \frac{\partial h_1}{\partial x} & \frac{\partial h_1}{\partial y} \\ \vdots & \vdots \\ \frac{\partial h_N}{\partial x} & \frac{\partial h_N}{\partial y} \end{bmatrix} =$$

$$= \left[\begin{matrix} \frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} - \frac{x}{\sqrt{x^2+y^2}} \\ \vdots \\ \frac{x-x_N}{\sqrt{(x-x_N)^2+(y-y_N)^2}} - \frac{x}{\sqrt{x^2+y^2}} \end{matrix} \right.$$

$$\left. \begin{matrix} \frac{y-y_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}} - \frac{y}{\sqrt{x^2+y^2}} \\ \vdots \\ \frac{y-y_N}{\sqrt{(x-x_N)^2+(y-y_N)^2}} - \frac{y}{\sqrt{x^2+y^2}} \end{matrix} \right]_{|\mathbf{x}=\hat{\mathbf{x}}(n)} \quad (15)$$

Hence, the extended Kalman filtering equations become[2]

[2] $\hat{\mathbf{x}}_n^- = \hat{\mathbf{x}}_n^+ = \hat{\mathbf{x}}_n$ in this case since the state matrix of the system is the identity matrix.

$$\begin{aligned} \hat{\mathbf{y}}(n) &= h(\hat{\mathbf{x}}(n-1)) \\ P(n)^- &= P(n-1)^+ + Q(n-1) \\ K(n) &= P(n)^- H(n)^T [H(n)P(n)^- H(n)^T + R(n)]^{-1} \\ P(n)^+ &= [I - K(n)H(n)^T]P(n)^- \quad (16) \\ \hat{\mathbf{x}}(n) &= \hat{\mathbf{x}}(n-1) + K(n)[\mathbf{y}(n) - \hat{\mathbf{y}}(n)] \end{aligned}$$

4. PRACTICAL ASPECTS AND EXPERIMENTAL RESULTS

The above filtering algorithm has been implemented in MATLAB and has been used with good results to track the position of the emitter in a lot of simulated situations involving random configuration of reflectors.

We would like to note as a practical issue that if the emitter is at rest, the state equation (2) becomes

$$\mathbf{x}(n) = \mathbf{x}(n-1) \quad (17)$$

instead of the random-walk process. This means that we can use only one noisy TDOA measurement $\mathbf{y} = \mathbf{s} + \mathrm{n}oise$ in the estimate update equation since theoretically we should get the same value all the time. This will modify the the Kalman filtering equations (16) as follows[3]

$$\begin{aligned} \hat{\mathbf{y}}(n) &= h(\hat{\mathbf{x}}(n)) \\ K(n) &= P(n)H(n)^T [H(n)P(n)H(n)^T + R(n)]^{-1} \\ P(n+1) &= [I - K(n)H(n)^T]P(n) \quad (18) \\ \hat{\mathbf{x}}(n+1) &= \hat{\mathbf{x}}(n) + K(n)[\mathbf{y} - \hat{\mathbf{y}}(n)] \end{aligned}$$

Experimental results are consistent with theoretical expectations and show an increase in the accuracy of the estimate when more reflectors are present as well as when noise intensities are smaller, as it can be seen from figure 2.

5. POSSIBLE EXTENSIONS: REFLECTOR POSITION ESTIMATION

The extended Kalman filtering algorithm can be used for emitter localization in a multipath environment provided the reflector positions are known. However, if

[3] $P(n)^- = P(n-1)^+ = P(n)$ since $Q(n) = 0$ in this case.

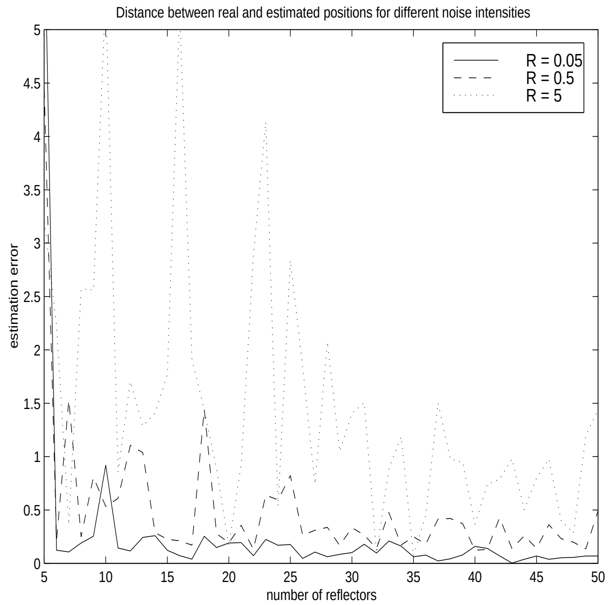

Figure 2: Estimation error vs. number of reflectors for different noise levels.

they are unknown, one can estimate them using a similar technique from a series of field measurements of TDOAs.

Let us denote by $\mathbf{x}_k$ the unknown coordinate vector of the k-th reflector. Being at rest, its dynamic equation is

$$\mathbf{x}_k(n) = \mathbf{x}_k(n-1) \quad (19)$$

The corresponding measurement equation is given by the k-th TDOA

$$y_k = \sqrt{(x_m - x_k)^2 + (y_m - y_k)^2} + \quad (20)$$

$$+\sqrt{x_k^2 + y_k^2} - \sqrt{x_m^2 + y_m^2} + \text{noise}$$

measured at M points (of known coordinates) inside the area of interest where the reflectors are placed.

The same Kalman filtering algorithm (16) is now used to estimate the position of the reflector. The only difference is that in the emitter localization case we had one N-dimensional measurement that was fed into the filtering algorithm until steady-state was reached, while for the reflector localization case M scalar measurements are used to get the estimate. Therefore M must be chosen large enough to ensure that steady-state is reached by the reflector Kalman filter.

6. CONCLUSIONS

The paper presented the use of extended Kalman filter for emitter localization in a multipath environment. The problem was analyzed under the assumption that reflector positions are known. Note that if they are not known, one can estimate them using the same technique from a series of field measurements of TDOAs.

Future research can be done in this area by solving related problems as for example tracking the trajectory of a mobile emitter.

7. REFERENCES

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