

INTERFERENCE AVOIDANCE FOR WIRELESS SYSTEMS

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1. INTRODUCTION

Interference from natural sources and from other users of the medium have always been factors in the design of reliable communication systems. While for wired or optical systems the amount of interference may be limited through hardware means that restrict access to the medium and/or reduce relative noise energy, the wireless medium is shared by all users. Usually, restrictions on use are legislative in nature and imposed by government regulating agencies such as the Federal Commission for Communications (FCC). Specifically, the oldest method of interference mitigation for wireless systems is spectrum licensing and the implied exclusive spectrum use. Unfortunately, licensing can indirectly stanch creative wireless applications owing to high licensing fees and the concomitant need for stable returns on large investments.

In an effort to promote creative wireless applications, the FCC has released 300 MHz of unlicensed spectrum in the 5 GHz range [1] with few restrictions other than absolute power levels—a “Wild West” environment of sorts where the only restriction is the size of the weapon! Thus, no central control is assumed. Needless to say, such a scenario seems ripe for chaos. Specifically, self-interest by individual users often results in unstable behavior. Such behavior has been seen anecdotally in distributed channelized systems such as early cordless telephones in apartment buildings where the number of collocated phones exceeded the number of channels available. Phones incessantly changed channels in an attempt to find a usable one. Thus, some means of assuring efficient use in such distributed wireless environments is needed.

Reaction to mutual interference is the heart of the shared-spectrum problem, and traditional approaches to combating wireless interference start with channel measurement and/or prediction, followed by an appropriate selection of modulation methods and signal processing algorithms for reliable transmission—possibly coupled to exclusive use contracts (licensing). Interestingly, since the time of the first radio transmissions, the methods used to deal with interference can be loosely grouped into three categories:

- Build a fence (licensing)
- Use only what you need (efficient modulation, power control)
- Grin and bear it (signal processing at the receiver)

Examples of the first item are legion, while examples of the last two items include single-sideband amplitude modulation, frequency modulation with preemphasis at the transmitter and deemphasis at the receiver, power control in cellular wireless systems [2], and code-division multiple access (CDMA) coupled to sophisticated signal processing algorithms for interference suppression/cancellation at the receiver [3].

However, Moore’s law advances in microelectronics have led to the emergence of new transceiver hardware that add a new weapon to the interference mitigation arsenal. Specifically, a class of radios that can be programmed to transmit almost arbitrary waveforms and can act as almost arbitrary receiver types is emerging—the so-called software radios [4–7]. So, as opposed to traditional radios, which owing to complex transceiver hardware are difficult to modify once a modulation method has been chosen, one can now imagine programming transceivers to use more effective modulation methods. Thus, wireless systems of the near future will be able to choose modulation methods that *avoid* ambient interference as opposed to precluding it via sole-use licenses, overpowering it with increased transmission power, or mitigating it with receiver signal processing.

Interference avoidance is the term used for adaptive modulation methods where individual users—simply put—employ their signal energy in “places” where interference is weak. Such methods have been shown to optimize shared use. More precisely, iterative interference avoidance algorithms yield optimal waveforms that maximize the signal-to-interference plus noise-ratio (SINR) for all users while maximizing the sum of rates at which all users can reliably transmit information (sum capacity). In other words, interference avoidance methods, through the self-interested action of each user, lead to a socially optimum¹ equilibrium (Pareto efficient [9,10]) in various mutual interference “games.”

Interference avoidance was originally introduced in the context of “chip-based” DS-CDMA systems [11] and minimum mean-square error (MMSE) receivers, but was subsequently developed in a general signal space [12–14] framework [15,16] which makes them applicable to a wide variety of communication scenarios. Related methods for transmitter and receiver adaptation have also been used in the CDMA context for asynchronous systems [17] and systems affected by multipath [18].

The relationship between codeword assignment in a CDMA system and sum capacity has been studied in several papers [19,20]; the paper by Viswanath and Anantharam [20] provides an algorithm to obtain sum capacity optimal codeword ensembles in a finite number of steps—and perhaps more importantly, also shows that the optimal linear receiver for such ensembles is

¹ Maximum sum capacity or *user capacity* [8] in a single-receiver multiuser system.

a *matched filter* for each codeword. Interference avoidance algorithms also yield optimal codeword ensembles but seem conceptually simpler and suitable for distributed adaptive implementation in multiuser systems.

It is worth expanding upon this last point. Interference avoidance is envisioned as a distributed method for unlicensed bands as opposed to a centralized procedure done by an omniscient receiver. Of course, we will see that the mathematics of the algorithm also lends itself to central application, so the distinction is really only important for practical application. However, throughout we will assume distributed application which implicitly suggests that each user knows its associated channel and in addition that each user has access to the whole system covariance through a side-channel beacon. The receiver can adaptively track codeword variation in a manner reminiscent of adaptive equalization. Since communication is two-way and physical channels are reciprocal, it is not unreasonable to assume that both the user and the system can know the channel. More important is the rate at which the channel varies. We will assume that channel variation is slow relative the frame rate [21,22], or if the channel variation rate is rapid that the average channel varies slowly enough for interference avoidance to be applied [23].

2. THE EIGEN-ALGORITHM FOR INTERFERENCE AVOIDANCE

We consider the uplink of a synchronous CDMA communication system with L users having signature waveforms $\{S_\ell(t)\}_{\ell=1}^L$ of finite duration T , with equal received power at the base station and ideal channels. The received signal is

$$R(t) = \sum_{\ell=1}^L b_\ell S_\ell(t) + n(t) \quad (1)$$

where b_ℓ represents the information symbol sent by user ℓ with signature $S_\ell(t)$, and $n(t)$ is an additive Gaussian noise process. We assume that all signals are representable in an arbitrary N -dimensional signal space. Hence, each user's signature waveform $S_\ell(t)$ is equivalent to an N -dimensional vector \mathbf{s}_ℓ and the noise process $n(t)$ is equivalent to a noise vector \mathbf{n} . The equivalent received signal vector \mathbf{r} at the base station is

$$\mathbf{r} = \sum_{\ell=1}^L b_\ell \mathbf{s}_\ell + \mathbf{n} \quad (2)$$

By defining the $N \times L$ matrix \mathbf{S} having as columns the user codewords \mathbf{s}_ℓ

$$\mathbf{D} = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1 & \mathbf{s}_2 & \dots & \mathbf{s}_L \\ | & | & & | \end{bmatrix} \quad (3)$$

the received signal can be rewritten in vector matrix form as

$$\mathbf{r} = \mathbf{S}\mathbf{b} + \mathbf{n} \quad (4)$$

where $\mathbf{b} = [b_1 \dots b_L]^T$ is the vector containing the symbols sent by users.

Assuming simple matched filters at the receiver for all users and unit energy codewords \mathbf{s}_k , the SINR for user k is

$$\gamma_k = \frac{(\mathbf{s}_k^T \mathbf{s}_k)^2}{\sum_{j=1, j \neq k}^L (\mathbf{s}_k^T \mathbf{s}_j)^2 + E[(\mathbf{s}_k^T \mathbf{n})^2]} = \frac{1}{\mathbf{s}_k^T \mathbf{R}_k \mathbf{s}_k} \quad (5)$$

where \mathbf{R}_k is the correlation matrix of the interference plus noise seen by user k having the expression

$$\mathbf{R}_k = \mathbf{S}\mathbf{S}^T - \mathbf{s}_k \mathbf{s}_k^T - \mathbf{W} \quad (6)$$

where $\mathbf{W} = E[\mathbf{n}\mathbf{n}^T]$ is the correlation matrix of the additive Gaussian noise.

Interference avoidance algorithms maximize the SINR through adaptation of user codewords. This is also equivalent to minimizing the inverse SINR defined as

$$\beta_k = \frac{1}{\gamma_k} = \mathbf{s}_k^T \mathbf{R}_k \mathbf{s}_k \quad (7)$$

Note that for unit power codewords, Eq. (7) represents the Rayleigh quotient for matrix \mathbf{R}_k , and recall from linear algebra [21, p. 348] that equation (7) is minimized by the eigenvector corresponding to the minimum eigenvalue of the given matrix \mathbf{R}_k . Therefore, the SINR for user k can be maximized by replacing codeword \mathbf{s}_k with the minimum eigenvector of the correlation matrix \mathbf{R}_k . That is, user k *avoids interference* by seeking a place in the signal space where interference is least. Sequential application by all users of this greedy procedure defines the minimum eigenvector algorithm for interference avoidance, or the *eigen-algorithm* [15], formally stated below:

1. Start with a randomly chosen codeword ensemble specified by the codeword matrix \mathbf{S} .
2. For each user $\ell = 1 \dots L$, replace user ℓ 's codeword \mathbf{s}_ℓ with the minimum eigenvector of the correlation matrix \mathbf{R}_k of the corresponding interference-plus-noise process.
3. Repeat step 2 until a fixed point is reached for which further modification of codewords will bring no improvement.

It has been shown [22] that in a colored noise background a variant of this algorithm, in which step 3 is augmented with a procedure to escape suboptimal fixed points, converges to the optimal fixed point where the resulting codeword ensemble "waterfills" over the background noise energy and maximizes sum capacity. If the background noise is white and the system is not overloaded (fewer users than signal space dimensions $L \leq N$), the algorithm yields a set of orthonormal codewords that corresponds to an ideal situation when users are orthogonal and therefore noninterfering. In the case of overloaded systems ($L > N$) in white noise, the resulting codeword ensembles form Welch bound equality (WBE) sets [19], which also minimize total squared correlation [15], a measure of the total interference in the system. In both underloaded and overloaded cases, the absolute minimum attainable total

squared correlation is often used as a stopping criterion for the eigen-algorithm.

Finally, we note that signal space “waterfilling” and the implied maximization of sum capacity are *emergent* properties of interference avoidance algorithms. Thus, individual users do not attempt maximization of sum capacity via an individual or ensemble waterfilling scheme, but rather, they greedily maximize the SINR of their own codeword. In fact, individual waterfilling schemes over the whole signal space are impossible in this framework since each user’s transmit covariance matrix $\mathbf{X}_\ell = \mathbf{s}_\ell \mathbf{s}_\ell^T$ is of rank one and cannot possibly span an N -dimensional signal space. So, emergent waterfilling and sum capacity maximization is a pleasantly surprising property of interference avoidance algorithms.

3. GENERALIZING THE EIGENALGORITHM

In order to extend application of the eigen-algorithm to more general scenarios, we consider the general multiaccess vector channel defined by [23]

$$\mathbf{r} = \sum_{\ell=1}^L \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} \quad (8)$$

where \mathbf{x}_ℓ of dimension N_ℓ is the input vector corresponding to user ℓ ($\ell = 1, \dots, L$), \mathbf{r} of dimension N is the received vector at the common receiver corrupted by additive noise vector \mathbf{n} of the same dimension, and \mathbf{H}_ℓ is the $N \times N_\ell$ channel matrix corresponding to user ℓ . It is assumed that $N \geq N_\ell, \forall \ell = 1, \dots, L$. This is a general approach to a multiuser communication system in which different users reside in different signal subspaces, with possibly different dimensions and potential overlap between them, but all of which are subspaces of the receiver signal space. We note that each user’s signal space as well as the receiver signal space are of finite dimension — implied by a finite transmission frame \mathcal{T} , finite bandwidths W_ℓ for each user ℓ , respectively, and by a finite receiver bandwidth W (which includes all W_ℓ values corresponding to all users) [24]. We also note that for memoryless channels the channel matrix \mathbf{H}_ℓ merely relates the bases of user ℓ ’s signal space and receiver signal space, but a similar model applies to channels with memory, in which case the channel matrix \mathbf{H}_ℓ also incorporates channel attenuation and multipath [16,18,23,25]. Figure 1 provides a graphical illustration of such a signal space configuration for two users residing in 2-dimensional subspaces with a three-dimensional receiver signal space.

In this signal space setting we assume that in a finite time interval of duration \mathcal{T} , each user ℓ sends a “frame” of data using a multicode CDMA approach wherein each symbol is transmitted using a distinct signature waveform that spans the frame. This scenario is depicted in Fig. 2. In other words, the sequence of information symbols $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^T$ is transmitted as a linear superposition of distinct, unit energy waveforms $\mathbf{s}_m^{(\ell)}(t)$

$$\mathbf{x}_\ell(t) = \sum_{m=1}^{M_\ell} b_m^{(\ell)} \mathbf{s}_m^{(\ell)}(t) \quad (9)$$

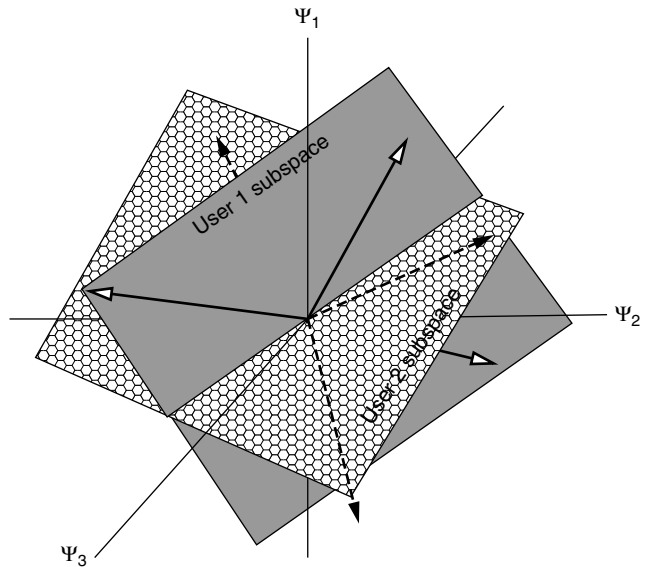


Figure 1. Three-dimensional receiver signal space with two users residing in two-dimensional subspaces. Vectors represent particular signals in user 1 (continuous line), respectively, user 2 (dashed line) signal spaces.

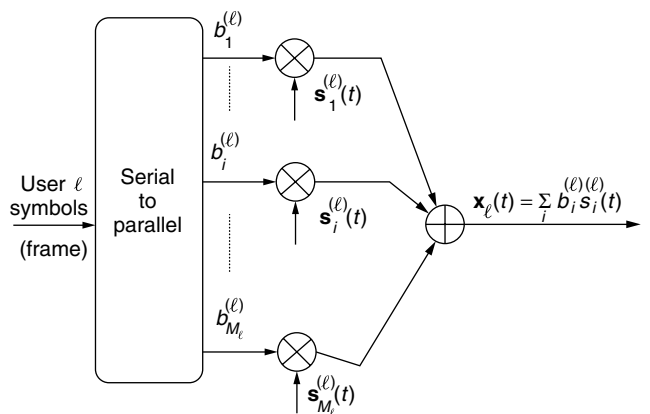


Figure 2. Multicode CDMA approach for sending frames of information. Each symbol in user ℓ ’s frame is assigned a distinct signature waveform, and the transmitted signal is a superposition of all signatures scaled by their corresponding information symbols.

as if each symbol in the frame corresponded to a distinct virtual user.

In the N_ℓ -dimensional signal space corresponding to user ℓ , each waveform can be represented as an N_ℓ -dimensional vector, thus the input vector \mathbf{x}_ℓ corresponding to user ℓ is equivalent to a linear superposition of unit norm codeword column vectors $\mathbf{s}_m^{(\ell)}$ scaled by the corresponding $b_m^{(\ell)}$. Therefore, each user uses an $N_\ell \times M_\ell$ codeword matrix \mathbf{S}_ℓ

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & \dots & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \dots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \quad (10)$$

so that

$$\mathbf{x}_\ell = \mathbf{S}_\ell \mathbf{b}_\ell \quad (11)$$

Therefore, the received signal can be rewritten as

$$\mathbf{r} = \sum_{\ell=1}^L \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \quad (12)$$

Note that under the assumption that $M_\ell \geq N_\ell$, the $N_\ell \times N_\ell$ transmit covariance matrix of user ℓ , $\mathbf{X}_\ell = E[\mathbf{x}_\ell \mathbf{x}_\ell^T] = \mathbf{S}_\ell \mathbf{S}_\ell^T$, has full rank and spans user ℓ 's signal space.

Extension of the eigen-algorithm to this general multiaccess vector channel setting is presented elsewhere in the literature [16,26]. The procedure starts by separating the interference-plus-noise seen by a given user k and rewriting the received signal in equation (12) from the perspective of user k as

$$\mathbf{r} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \underbrace{\sum_{\ell=1, \ell \neq k}^L \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell}_{\mathbf{z}_k = \text{interference} + \text{noise}} + \mathbf{n} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \mathbf{z}_k \quad (13)$$

The covariance matrix of the interference-plus-noise seen by user k

$$\mathbf{Z}_k = E[\mathbf{z}_k \mathbf{z}_k^T] = \sum_{\ell=1, \ell \neq k}^L \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^T \mathbf{H}_\ell^T + \mathbf{W} \quad (14)$$

is then used to define a whitening transformation \mathbf{T}_k of the interference-plus-noise seen by user k . The equivalent problem in which user k sees white interference-plus-noise is then projected onto the user k signal space using the singular value decomposition (SVD) [21, p. 442] of user k 's transformed channel matrix. This reduces the problem to an equivalent one given by an equation identical in form to Eq. (4) and therefore allows straightforward application of the eigen-algorithm for optimization of user k 's codewords.

One possible generalized eigen-algorithm is formally stated below:

1. Start with a randomly chosen codeword ensemble specified by user codeword matrices $\{\mathbf{S}_k\}_{k=1}^L$.
2. For each user $k = 1 \cdots L$
 - a. Compute the transformation matrix \mathbf{T}_k that whitens the interference-plus-noise seen by user k .
 - b. Change coordinates and compute the transformed user k channel matrix $\tilde{\mathbf{H}}_k = \mathbf{T}_k \mathbf{H}_k$.
 - c. Apply SVD for $\tilde{\mathbf{H}}_k$ and project the problem onto user k 's signal space.
 - d. Adjust user k 's codewords sequentially using the greedy procedure of the basic eigen-algorithm; the codeword corresponding to symbol m of user k is replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process.
 - e. Iterate the previous step until convergence (making use of escape methods [22] if the procedure stops in suboptimal points).
3. Repeat step 2 until a fixed point is reached.

We note that in steps 2d,e of the algorithm, user k waterfills its signal space by application of the basic eigen-algorithm [22] for interference avoidance by regarding all other user's signals as noise. Thus, the generalized eigen-algorithm *iteratively waterfills* each user's signal space. It has been shown [23] that an *iterative waterfilling algorithm* converges to a fixed point where the sum capacity of the vector multiaccess channel is maximized, which implies that the generalized eigen-algorithm will always yield a codeword ensemble that maximizes sum capacity.

4. THE GENERALIZED EIGEN-ALGORITHM: A VERSATILE TOOL FOR CODEWORD OPTIMIZATION

The generalized eigen-algorithm is a powerful tool that enables application of interference avoidance methods to various communication problems in which the underlying model is a multiaccess vector channel. Among these we mention codeword optimization in the uplink of a CDMA system with nonideal (dispersive) channels, multiuser systems with multiple inputs and outputs (MIMO), and asynchronous CDMA systems, for which an appropriate selection of signal space basis functions leads to particular cases of the general multiaccess vector channel model for which application of the generalized eigen-algorithm to codeword optimization becomes straightforward.

For the uplink of a CDMA system with dispersive channels considered in earlier studies [16,25], the spanning set of the signal space consists of a set of real sinusoids (sine and cosine functions) that are approximately eigenfunctions for all uplink channels corresponding to all users. Introduced in 1964 [27], channel eigenfunctions form an orthonormal spanning set with the property that their corresponding channel responses are also orthogonal, thus allowing convenient representation of channel outputs as scaled versions of the input vectors. In this case, the channel matrix of a given user is a diagonal matrix in which diagonal elements correspond to channel gain factors for the frequencies that define the signal space, and the modulation scheme turns out to be a form of multicarrier CDMA [28]. We note that even though interference avoidance is applied in this multicarrier modulation framework [25], the method is completely general and applicable to various scenarios with appropriate selection of signal space basis functions. For example using sinc functions the method is applicable to time-domain representations in which the vector channel model is obtained from Nyquist-sampled waveforms (see, e.g., Ref. 29). Using "time chips" as basis functions a vector model for DSCDMA systems with multipath is obtained [16,18].

Application of the generalized eigen-algorithm to multiuser MIMO systems is also possible [16,30]. The same multicarrier modulation framework with multiple antennas at the transmitter and receiver imply a MIMO channel matrix composed of block diagonal matrices corresponding to each transmit/receive antenna pair. We mention again that other MIMO channel models—for example, the spatiotemporal MIMO channel model [31], in which the MIMO channel matrix is composed of convolution matrices corresponding to each

transmit/receive antenna pair—are perfectly valid and the generalized eigen-algorithm can be used for codeword optimization in conjunction with such models as well.

A similar signal space approach is used to apply the generalized eigen-algorithm for codeword optimization in very general asynchronous CDMA system models [16,32]. We note that for particular cases less general interference avoidance algorithms can be used [33].

5. CONCLUSION

Motivated by the emergence of software radios and the desire to foster creative uses of wireless spectrum in unlicensed bands, interference avoidance methods have been developed for wireless systems. The underlying idea of interference avoidance is for each user to greedily optimize spectrum use (SINR or capacity) through appropriate signal placement in response to interferers. Interference avoidance is applicable to a wide range of communications scenarios including dispersive channels, multiple antenna systems, and asynchronous systems.

The utility of interference avoidance methods in real wireless systems with multiple receivers—such as is found in a cellular environment—although currently under study, is still an open question in general. Specifically, theoretical results have been established for application of interference avoidance methods in a *collaborative* scenario wherein information from all receivers is pooled and used to decode all users in the system [34,35]. Since all information is available for use in decoding users, the collaborative scenario constitutes a best case of sorts. Unfortunately, real systems may not be collaborative or even cooperative.

Early experiments with geographically dispersed users assigned to different bases showed unstable behavior under direct application of the eigen-algorithm, mirroring the anecdotally reported behavior of early cordless phones (see Section 1, above). However, by allowing each user to send and adapt *multiple codewords*—a multicode approach similar to that used for dispersive channels—the algorithm became stable [36] since each user could then waterfill their signal energy over the entire signal space when necessary as opposed to choosing exactly one channel. Of course, such convergence though welcome is a bit chimeric since the implied multiple receiver system model is an instance of the interference channel [37] for which very little is known in general. Regardless, the fact that interference avoidance can attain any equilibrium [9,10] in the implied “game” of mutually interfering wireless access is interesting, and can perhaps illuminate paths toward greater understanding of efficient and peaceful coexistence in unlicensed wireless systems.

BIOGRAPHIES

Dimitrie C. Popescu received the Engineering Diploma and M.S. degrees in 1991 from the Polytechnic Institute of Bucharest, Romania, and the Ph.D. degree from Rutgers University in 2002, all in Electrical Engineering. He is currently an Assistant Professor in the Department

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Dr. Christopher Rose received the B.S. (1979), M.S. (1981), and Ph.D. (1985) degrees all from the Massachusetts Institute of Technology in Cambridge, Massachusetts. Dr. Rose joined AT&T Bell Laboratories in Holmdel, New Jersey as a member of the Network Systems Research Department in 1985 and in 1990 moved to Rutgers University, where he is currently an Associate Professor of Electrical and Computer Engineering and Associate Director of the Wireless Networks Laboratory. He is Editor for the *Wireless Networks* (ACM), *Computer Networks* (Elsevier) and *Transactions on Vehicular Technology* (IEEE) journals and has served on many conference technical program committees. Dr. Rose was technical program Co-Chair for MobiCom'97 and Co-Chair of the WINLAB Focus'98 on the U-NII, the WINLAB Berkeley Focus'99 on Radio Networks for Everything and the Berkeley WINLAB Focus 2000 on Picoradio Networks. Dr. Rose, a past member of the ACM SIGMobile Executive Committee, is currently a member of the ACM MobiCom Steering Committee and has also served as General Chair of ACM SIGMobile MobiCom 2001 (Rome, July 2001). In December 1999 he served on an international panel to evaluate engineering teaching and research in Portugal.

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