

# CODEWORD QUANTIZATION FOR INTERFERENCE AVOIDANCE

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## ABSTRACT

Programmable radios offer a new perspective on wireless communications since the modulation method is no longer fixed. Adaptive methods where user signatures and corresponding receiver filters are iteratively adapted can be used to improve performance. However, since codeword adjustments must be fed back to the transmitter, compact representation of codewords is extremely important. This issue is important for systems which employ interference avoidance since as opposed to current CDMA systems where uniform-amplitude codeword chips are used, interference avoidance employs real-valued “chips”—real-valued coefficients for a set of orthonormal basis functions of the signal space used by the transmitter and receiver. The paper represents a simple investigation of how codeword quantization affects the performance of interference avoidance algorithms. Results indicate that using 4 – 5 bits per chip for codeword representation is sufficient to maintain performance close to optimal values.

## 1. INTRODUCTION

The study of interference avoidance methods is motivated by the ever-present need in wireless communications to deliver increased bit rates under fixed bandwidth and power requirements. Since radios are becoming more sophisticated [1] it is now not unthinkable to adjust transmission/reception methods to suit the communications environment and in this way achieve better performance.

Under such an *agile transmitter/receiver* scenario, interference avoidance [2, 3] provides a way to maximize the signal-to-interference ratio (SIR) in the presence of interference. Individual user waveforms are represented as linear superpositions of orthonormal basis functions which span a signal space. For a single waveform, the real-valued superposition coefficients comprise the *signature vector* or *codeword*. These codewords are adjusted iteratively in response to interference conditions and fed back to the transmitter. Since in a real system, real values cannot specified with infinite precision, if interference avoidance is to move from theory to practice, then the necessary precision of codeword representation must be considered. This paper presents empirical evidence that with suitable quantization methods, 4 or 5 bits per chip yields minimal degradation in performance.

## 2. INTERFERENCE AVOIDANCE ALGORITHMS

We consider a multiuser system in which the signal received at a common receiver (basestation) is composed of signals sent by the

$M$  users plus additive white Gaussian noise (AWGN) and can be represented in terms of the chosen basis function set as

$$\mathbf{r} = \sum_{i=1}^M b_i \mathbf{s}_i + \mathbf{n} \quad (1)$$

where  $\mathbf{s}_i$  is the codeword corresponding to user  $i$  ( $N$ -dimensional vector containing projections of signature waveform of user  $i$  onto the orthonormal basis functions  $\{\Psi_n(t)\}_{n=1}^N$ ),  $\mathbf{n}$  contains noise projections onto the basis functions, and  $b_i$  is the bit sent by user  $i$ . Note that we have assumed unit received power for all users at the common basestation. Let us also denote by  $\mathbf{S}$  the  $N \times M$  matrix having as columns the codewords of all users

$$\mathbf{S} = \begin{bmatrix} | & | & \dots & | \\ \mathbf{s}_1 & \mathbf{s}_2 & \dots & \mathbf{s}_M \\ | & | & \dots & | \end{bmatrix} \quad (2)$$

From the perspective of user  $k$  the received signal in 1 can be rewritten as

$$\mathbf{r} = b_k \mathbf{s}_k + \sum_{i=1, i \neq k}^M b_i \mathbf{s}_i + \mathbf{n} \quad (3)$$

and we can also define the correlation matrix of interference (plus noise) seen by user  $k$  from the other users as

$$\mathbf{R}(k) = \sum_{i=1, i \neq k}^M \mathbf{s}_i \mathbf{s}_i^T = \mathbf{S} \mathbf{S}^T - \mathbf{s}_k \mathbf{s}_k^T \quad (4)$$

The idea behind interference avoidance algorithms is to maximize the shared SIR through greedy adaptation of codewords. The SIR for user  $k$  is defined as the ratio of user  $k$  energy  $p_k$  to interference energy  $p_k^I$

$$\gamma_k = \frac{p_k}{p_k^I} = \frac{\mathbf{s}_k^T \mathbf{s}_k}{\sum_{j=1, j \neq k}^M (\mathbf{s}_k^T \mathbf{s}_j)^2} \quad (5)$$

and we want to maximize it. One can also define the inverse SIR  $\beta_k = 1/\gamma_k$  which must be minimized. The interference energy can be rewritten as

$$\begin{aligned} p_k^I &= \sum_{j=1, j \neq k}^M (\mathbf{s}_k^T \mathbf{s}_j)(\mathbf{s}_j^T \mathbf{s}_k) \\ &= \mathbf{s}_k^T \left( \sum_{j=1, j \neq k}^M \mathbf{s}_j \mathbf{s}_j^T \right) \mathbf{s}_k = \mathbf{s}_k^T \mathbf{R}(k) \mathbf{s}_k \end{aligned} \quad (6)$$

which leads to the following expression for the inverse SIR

$$\beta_k = \frac{\mathbf{s}_k^T \mathbf{R}(\mathbf{k}) \mathbf{s}_k}{\mathbf{s}_k^T \mathbf{s}_k} \quad (7)$$

which represents the Rayleigh quotient of the matrix  $\mathbf{R}(\mathbf{k})$ . Since it is known that the Rayleigh quotient is minimized by the eigenvector corresponding to the minimum eigenvalue [4], we can conclude that the SIR for user  $k$  is maximized when its codeword  $\mathbf{s}_k$  is replaced by the minimum eigenvalue eigenvector of the correlation matrix  $\mathbf{R}(\mathbf{k})$ . This replacement defines the minimum eigenvalue eigenvector algorithm for interference avoidance<sup>1</sup> which can be summarized as follows:

- The receiver for each user  $k$  computes the  $N$ -dimensional correlation matrix  $\mathbf{R}(\mathbf{k})$  of the interference generated by other users.
- The minimum eigenvalue  $\lambda^*(k)$  is found, and the current codeword  $\mathbf{s}_k$  is replaced by the corresponding eigenvector. If  $\mathbf{s}_k$  is already a suitable eigenvector, then it is not changed.
- The procedure is repeated for each codeword iteratively until no improvement occurs.

When the algorithm settles down, all matrices  $\mathbf{R}(\mathbf{k})$ ,  $\mathbf{k} = 1, \dots, M$  have an identical minimum eigenvalue,  $\lambda^*$  and the SIR is  $\gamma^* = 1/\lambda^*$  for all users.

Note that by considering the AWGN vector  $\mathbf{n}$  one can define the signal-to-interference plus noise-ratio (SINR) as the ratio of user energy  $p_k$  to interference energy  $p_k^I$  plus noise energy

$$\gamma_k' = \frac{p_k}{p_k^I} = \frac{\mathbf{s}_k^T \mathbf{s}_k}{\sum_{j=1, j \neq k}^M (\mathbf{s}_k^T \mathbf{s}_j)^2 + \mathbf{E}[(\mathbf{s}_k^T \mathbf{n})^2]} \quad (8)$$

and at equilibrium we get

$$\gamma^{*'} = \frac{\mathbf{s}_k^T \mathbf{s}_k}{\lambda^* \mathbf{s}_k^T \mathbf{s}_k + \mathbf{s}_k^T \mathbf{E}[\mathbf{nn}^T] \mathbf{s}_k} = \frac{1}{\lambda^* + N_0}$$

Properties of optimal and sub-optimal ensembles of codeword sets generated by interference avoidance algorithms have been explored in [2, 5]. Optimal codeword sets have minimum total squared correlation<sup>2</sup> (TSC) defined as

$$\text{TSC} = \text{Trace} [(\mathbf{S}\mathbf{S}^T)^2] = \sum_{i=1}^M \sum_{j=1}^M (\mathbf{s}_i^T \mathbf{s}_j)^2 \quad (9)$$

The lower bound for TSC has been derived in [9] for unit energy sequences and this is equal to  $M^2/N$ .

As for convergence of the interference avoidance algorithms, numerical studies have shown that suboptimal minima have never been reached with randomly chosen initial signature sequences. Even more, in [5] a convergence proof for greedy interference avoidance algorithms is provided, based on a procedure dubbed "class warfare" which is able to escape suboptimal fixed points.

<sup>1</sup>Also called the "eigen-algorithm" [2]

<sup>2</sup>Codeword sets with minimum TSC can be obtained also through some other methods [6, 7, 8].

### 3. CODEWORD QUANTIZATION

The problem of quantizing the optimal codewords generated by the interference avoidance algorithms is investigated in order to determine its influence on system performance. Quantization effects may be seen when comparing the SIR obtained with optimal codewords vs. the SIR obtained with quantized optimal codewords. Also, the value of the TSC which was minimized by the interference avoidance algorithms may be increased by the quantization of optimal codewords.

Scalar quantization of each codeword component is considered, with the final goal of encoding the resulting levels into a binary sequence to be transmitted over a control channel. The amount of information sent over this control channel can be regarded also as a measure for the efficiency of interference avoidance.

In scalar quantization [10] the set of real numbers  $\mathcal{R}$  is partitioned into  $L$  disjoint subsets  $\{\mathcal{R}_k\}_{k=1}^L$  and a representation point is chosen for each subset. The quantization function is

$$Q(x) = \hat{x}_k \quad \forall x \in \mathcal{R}_k \quad (10)$$

and is nonlinear and non-invertible. For  $L$  quantization levels a number  $B = \log_2 L$  of bits are enough to encode them into a binary sequence<sup>3</sup>. Uniform and non-uniform quantization schemes are considered and their effects on the SIR and TSC will be investigated.

Uniform quantizers are the simplest type of quantizers in which regions  $\{\mathcal{R}_i\}_{i=2}^L - 1$  are equal with some  $\Delta$ , called quantization width. The design of a uniform quantizer for random variables is done to minimize the squared error distortion

$$D = E[(X - Q(X))^2] \quad (11)$$

and is done mainly by numerical techniques.

In the case of non-uniform quantizers, the condition that quantized regions be equal is not imposed which implies fewer constraints in the minimization of distortion. The quantizer is designed based on the necessary conditions for optimality, known as Lloyd-Max conditions [10].

### 4. QUANTIZATION EXPERIMENTS

A set of quantization experiments has been performed using codewords obtained in 1000 trials of the eigen-algorithm. The resulting optimal codewords have been quantized as described in the previous section, using both uniform and non-uniform quantizers, with up to 32 quantization levels (which implies up to 5 bits in the representation of quantized codewords), and both the SIR and the TSC for quantized optimal codewords have been computed.

The SIR is the same for all users and equal to  $N/(M - N)$  [2] for the optimal codeword set, but this is no longer true for the quantized codeword set. Different users have different SIRs with a distribution which is approximately Gaussian as it can be seen from figures 1 and 2. The mean of the distribution gets closer to the optimal SIR and the variance decreases as the number of quantization levels increases.

Similar remarks can be made about the TSC value, which is equal to  $M^2/N$  for the optimal codeword set, whose resulting distributions for different number of quantization levels are plotted in figures 3 and 4.

<sup>3</sup> $L$  is chosen in general to be a power of 2.

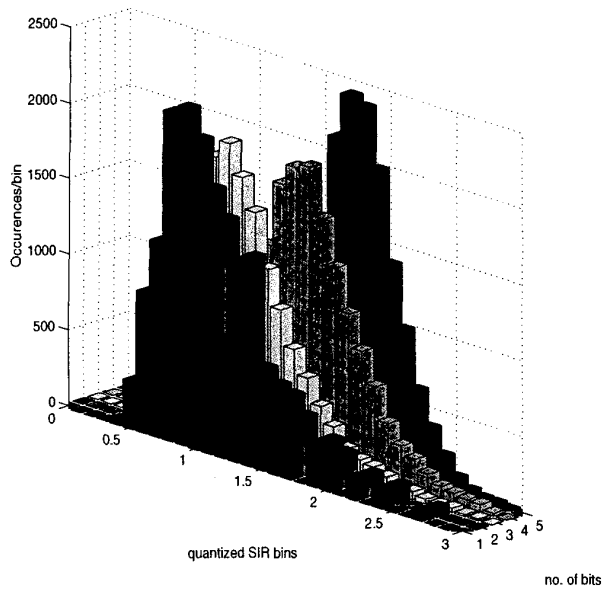


Figure 1: Histograms containing the distribution of SIR for quantized codewords after uniform quantization. 1000 interference avoidance algorithm trials,  $M = 15$  users with  $N = 10$  “chips”.

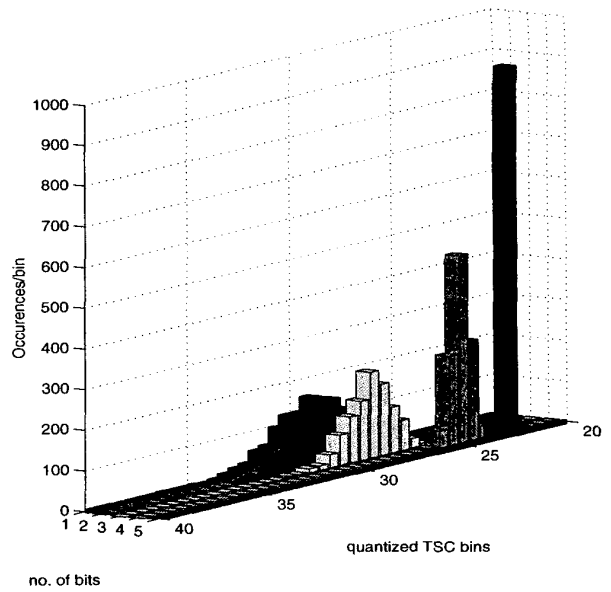


Figure 3: Histograms containing the resulting TSC values after uniform quantization of codewords. 1000 interference avoidance algorithm trials,  $M = 15$  users,  $N = 10$ .

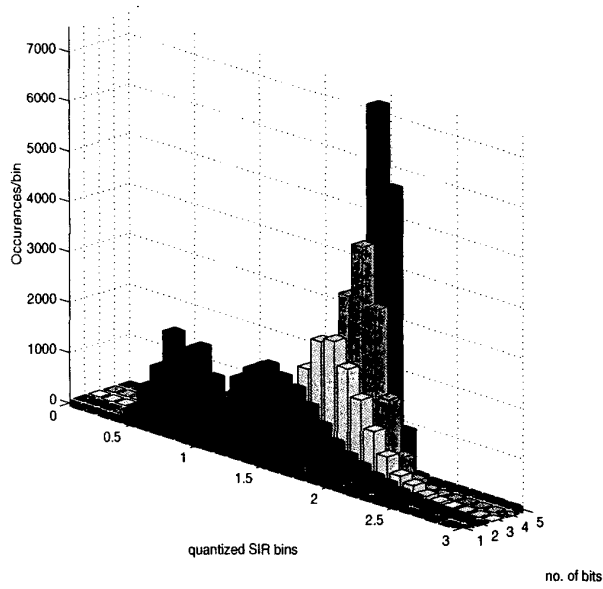


Figure 2: Histograms containing the distribution of SIR for quantized codewords after non-uniform quantization. 1000 interference avoidance algorithm trials,  $M = 15$  users,  $N = 10$ .

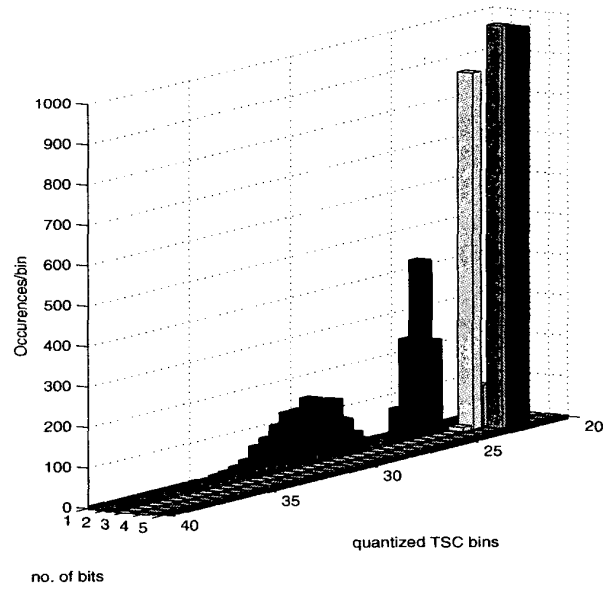


Figure 4: Histograms containing the resulting TSC values after non-uniform quantization of codewords. 1000 interference avoidance algorithm trials,  $M = 15$ ,  $N = 10$ .

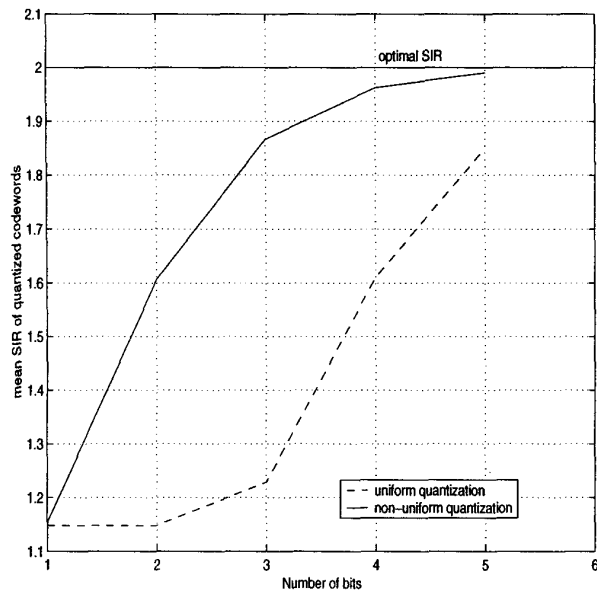


Figure 5: Mean values of SIR after uniform and non-uniform quantization of codewords. 1000 interference avoidance algorithm trials,  $M = 15$  users,  $N = 10$ .

Mean values of the SIR and TSC distributions presented in the previous plots are presented in figures 5 and 6. We can conclude that non-uniform quantizers yield better results than uniform ones, and that the number of bits required to keep SIR and TSC close to optimal values is 4 – 5 bits.

## 5. CONCLUSIONS

Quantization of optimal codewords generated by interference avoidance algorithms has been considered. Quantization degrades the performance of interference avoidance algorithms, by decreasing the mean value of the SIR and increasing the mean value TSC. Furthermore, coarse quantization leads to reasonably large non-uniformity in performance across a given codeword ensemble. Each effect constitutes a degradation in overall performance. With non-uniform quantization 16 to 32 quantization levels (corresponding to 4 – 5 bits) seem to be enough to keep the SIR distribution and overall TSC close to optimal values.

As future work, entropy coding [11] can be explored in connection with codeword quantization, as a means of further decreasing the average number of bits needed for codeword representation. It should also be noted that under energy constraints on the codewords, the chip values are not independent and perhaps this too can be exploited in future work.

## 6. REFERENCES

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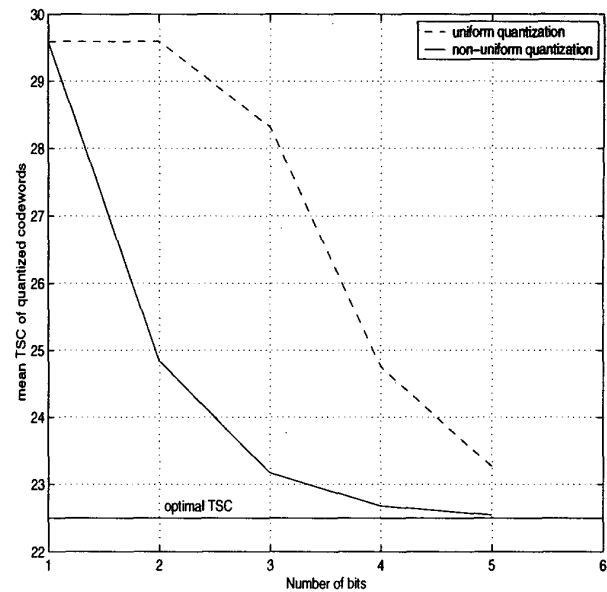


Figure 6: Mean values of TSC after uniform and non-uniform quantization of codewords. 1000 interference avoidance algorithm trials,  $M = 15$  users,  $N = 10$

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