

# Interference Avoidance for Multiaccess Vector Channels

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**Abstract** — Using a general signal space approach we show that sequential application of the eigen-algorithm for interference avoidance by all users in a CDMA system is equivalent to iterative water filling and always yields codeword ensembles that maximize the sum capacity of multiaccess vector channels.

## I. INTRODUCTION

We consider a general multiaccess vector channel model along with a Code Division Multiple Access (CDMA) scheme for transmission of information and present an iterative algorithm for codeword adaptation which yields optimal codeword ensembles that maximize sum capacity. The algorithm, which is an extension of the eigen-algorithm for interference avoidance [1, 2] to multiaccess vector channels, is equivalent to an iterative water filling procedure for which convergence to maximum sum capacity has been established in [3].

## II. SYSTEM DESCRIPTION

A multiaccess vector channel with  $L$  users communicating with a common receiver (basestation) is defined by the equation

$$\mathbf{r} = \sum_{\ell=1}^L \mathbf{H}_{\ell} \mathbf{x}_{\ell} + \mathbf{n} \quad (1)$$

where  $\mathbf{x}_{\ell}$  of dimension  $N_{\ell}$  is the input vector corresponding to user  $\ell$ ,  $\ell = 1, \dots, L$ ,  $\mathbf{r}$  of dimension  $N$  is the received vector at the common receiver corrupted by additive noise vector  $\mathbf{n}$  of the same dimension, and  $\mathbf{H}_{\ell}$  is the  $N \times N_{\ell}$  channel matrix corresponding to user  $\ell$ . It is assumed that  $N \geq N_{\ell}, \forall \ell = 1, \dots, L$ . This is a general approach to a multiuser communication system in which different users reside in different signal spaces, with different dimensions and potential overlap between them, and all being subspaces of the receiver signal space.

In this signal space setting we assume that in a given frame each user  $\ell$  transmits a sequence of  $M_{\ell}$  independent symbols  $b_m^{(\ell)}$  using a multicode CDMA approach. Thus, the transmitted signal corresponding to user  $\ell$  is given by

$$\mathbf{x}_{\ell} = \mathbf{S}_{\ell} \mathbf{b}_{\ell} \quad (2)$$

with  $\mathbf{S}_{\ell}$  being the  $N \times M_{\ell}$  codeword matrix corresponding to user  $\ell$ . The received signal at the common receiver can be rewritten as

$$\mathbf{r} = \sum_{\ell=1}^L \mathbf{H}_{\ell} \mathbf{S}_{\ell} \mathbf{b}_{\ell} + \mathbf{n} \quad (3)$$

and our goal is to find an optimal ensemble of codewords given by matrices  $\mathbf{S}_{\ell}$ ,  $\ell = 1, \dots, L$ , that maximize the sum capacity of the multiaccess channel. We note that under the assumption that  $M_{\ell} \geq N_{\ell}$  the  $N_{\ell} \times N_{\ell}$  covariance matrix of  $\mathbf{x}_{\ell}$  defined as  $\mathbf{X}_{\ell} = E[\mathbf{x}_{\ell} \mathbf{x}_{\ell}^T] = \mathbf{S}_{\ell} \mathbf{S}_{\ell}^T$  has full rank and spans user  $\ell$  signal space.

## III. THE GENERALIZED EIGEN-ALGORITHM FOR MULTIACCESS VECTOR CHANNELS

1. Start with a randomly chosen codeword ensemble specified by the codeword matrices  $\{\mathbf{S}_k\}_{k=1}^L$
2. For each user  $k = 1 \dots L$ 
  - (a) Compute the transformation matrix  $\mathbf{T}_k$  that whitens the interference-plus-noise seen by user  $k$
  - (b) Change coordinates and compute transformed user  $k$  channel matrix  $\tilde{\mathbf{H}}_k = \mathbf{T}_k \mathbf{H}_k$
  - (c) Apply SVD for  $\tilde{\mathbf{H}}_k$  and project the problem onto user  $k$  signal space
  - (d) adjust user  $k$  codewords sequentially: the codeword corresponding to symbol  $m$  of user  $k$  is replaced by the minimum eigenvector of the autocorrelation matrix of the corresponding interference-plus-noise process
  - (e) Iterate previous step until convergence (making use of escape methods [1] if the procedure stops in suboptimal points)
3. Repeat step 2 until a fixed point is reached.

We note that in steps 2(d)–(e) of the algorithm, user  $k$  water fills its signal space through an interference avoidance procedure [1] regarding all other user's signals as noise. Thus, the generalized eigen-algorithm *iteratively water fills* each user's signal space. Since it has been shown that an *iterative water filling algorithm* converges to a fixed point [3] where the sum capacity of the vector multiaccess channel is maximized, we conclude that the generalized eigen-algorithm will always yield a codeword ensemble that maximizes sum capacity.

## IV. CONCLUSIONS

Sequential application of the eigen-algorithm for interference avoidance by all users in CDMA system is equivalent to iterative water filling in which each user water fills its corresponding signal space considering other user's signals as noise. The result is powerful and clears the way for application of interference avoidance methods to a variety of problems including asynchronous multiuser systems and multiuser multiple antenna systems.

## REFERENCES

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