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Interference avoidance and multiuser MIMO systems

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SUMMARY

We apply interference avoidance methods to multiuser systems with multiple inputs and multiple outputs (MIMO) such as would be used on the uplink (or in overlapping downlink footprints) for satellite systems with multiple antennas. An arbitrary signal space can be used and the method can be applied to any MIMO system model regardless of the choice signal basis functions. Information is transmitted via multicode CDMA where symbols that comprise the data frame from a given user are 'spread' over the available dimensions using a precoding matrix. Optimal precoding matrices that maximize signal-to-interference plus noise-ratio for all symbols/users are then obtained by application of distributed greedy interference avoidance methods. In general, algorithms based on greedy interference avoidance are different from water filling schemes. However, the codeword ensembles obtained provide similar simultaneous water filling solutions and thus maximize sum capacity. Numerical simulations have been performed and the signal-to-noise ratio distribution for receiver antennas and complementary cumulative distribution functions for sum capacity with optimal precoding matrices are also presented. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: multiple antennas; MIMO channel; vector channels; CDMA; precoder optimization; uplink satellite channels

1. INTRODUCTION

Wireless communication systems with multiple inputs and multiple outputs (MIMO) in which many antennas are used for transmission and reception have received increased attention from the research community owing to the large gains in performance they can afford. Usually, multiple antennas are employed to provide spatial diversity, and improve the performance of wireless systems by mitigating the effects of multipath fading. In satellite communications, new technology and applications may require more sophisticated antenna systems that provide quasi-stationary footprints from moving satellites or in-orbit reconfiguration of antenna beams to serve different areas [1, Chapter 12]. Furthermore, satellites play an important role in the proposed Universal Mobile Telecommunication System (UMTS) by providing among other things satellite diversity in both forward and reverse links of the system [2, 3].

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Key in the satellite problem, as in almost all wireless problems, is efficient use of scarce resources (spectrum) over multiple users. Interference avoidance can aid such efficient use by providing a distributed method whereby individual transmitters greedily adjust their transmissions for maximum performance, but for which the end result is optimum shared usage of the available spectrum. Here, we will explore the use of interference avoidance for satellite systems.

Performance of wireless systems with multiple antennas has been analysed in several papers [4, 5] which have shown that these are more effective than single antenna systems in fading environments which are characteristic of wireless communications. It has also been shown that presence of multipath can improve performance with an appropriate multiple antenna structure [6]. New modulation schemes for multiple antenna systems have also been proposed and analysed [7, 8] in an attempt to bring performance close to the theoretical limits [4, 5].

In this paper, we present application of interference avoidance methods to multiuser MIMO systems such as those associated with the uplink of a wireless system in which users and the base station are equipped with multiple antennas. We note that interference avoidance methods provide distributed algorithms for codeword optimization in CDMA systems [9, 10] based on maximization of the signal-to-interference plus noise-ratio (SINR). Our approach is based on application of interference avoidance to general multiaccess vector channels presented in References [11, 12] for the particular multiaccess vector channel corresponding to a multiuser MIMO system. We note that a vector channel characterized by a channel matrix between a given transmitter and the receiver is a natural representation in the case of MIMO systems and several models can be found in the literature [4–7]. We also note that the approach is completely general and applicable to any MIMO system model regardless of the choice of signal space basis functions.

Information is transmitted over the MIMO channel using multicode CDMA where a sequence of information symbols from a given user are 'spread' over the available dimensions using a precoding matrix. More precisely, the columns of the precoding matrix are used as codewords, or spreading sequences, for the information symbols in the frame. Formulation of the MIMO channel problem in this CDMA context allows direct application of interference avoidance techniques [11, 12] to determine optimal precoding matrices which maximize the SINR for all symbols/users. We note that the codeword ensemble formed by the optimal precoding matrices also satisfies a simultaneous water filling solution which is an emergent property of interference avoidance algorithms [11, 12] and ensures also maximization of sum capacity [13]. We also note that application of interference avoidance, which is a codeword optimization procedure, is in general different than application of a water filling algorithm which optimizes the signal covariance directly. Such water filling algorithms have been proposed recently for multiuser multiple antenna systems [14].

The paper is organized as follows. We introduce the multiuser multiple antenna system with the equivalent multiuser vector channel model and the multicode CDMA approach for transmission of information in Section 2. In Section 3 we show how interference avoidance can be applied to optimize precoder matrices in both single and multiple user cases. Numerical simulations are presented in Section 4 and performance is analysed in terms of signal-to-noise ratios (SNRs) at the receiver and sum capacity. For a single-user case, we note that with random precoding matrices the SNR at receiver antennas has a bell-shaped distribution. However, after interference avoidance is performed optimal precoding matrices imply that the SNR is approximately the same for all receive antennas even though no prior assumption about equal SNRs at each antenna has been made as in Reference [4]. We also note that for the same average noise power at the receiver, doubling the number of antennas (both in the transmitter and in the receiver) translates to an approximately 3 dB increase in the SNR.

Next we look at sum capacity in the context of fading channels. Precoding matrices optimal for the average channel are obtained for all users in the system using interference avoidance algorithms and these are used to compute sum capacity for actual realizations of the fading channel models. Sum capacity is treated as a random variable in this case and complementary cumulative distribution functions (CCDFs) similar to those in Reference [4] are plotted. The plots show what capacity can be achieved with a given probability of outage when precoders optimal for the average channel are used, and are consistent with those in Reference [4], supporting the generally accepted idea that the use of multiple antennas in both the transmitter and the receiver is beneficial for system performance.

2. THE MIMO SYSTEM MODEL

We consider a multiuser system with L users communicating with a common receiver (base station) in which all users and the base station are equipped with antenna arrays for transmission/reception. We denote by T_ℓ the number of transmit antennas corresponding to user $\ell, \ell = 1, \ldots, L$ and by R the number of receive antennas at the base station. The system is described schematically in Figure 1.

Each user ℓ transmits information in frames of duration $\mathscr{T}$. The channel between user ℓ's transmit antenna i and receive antenna j at the base station is characterized by the causal impulse response $h_{ij}^{(\ell)}(t)$ of duration $T_{ij}^{(\ell)}$ assumed stable (time-invariant) over the duration $\mathscr{T}$ of the frame. The transmitted waveform $x_i^{(\ell)}(t)$ at transmit antenna i of user ℓ convolved with this impulse response yields the corresponding waveform $y_{ij}^{(\ell)}(t)$ due to user ℓ transmit antenna i at receive antenna j

$$y_{ij}^{(\ell)}(t) = x_i^{(\ell)}(t) * h_{ij}^{(\ell)}(t) \tag{1}$$

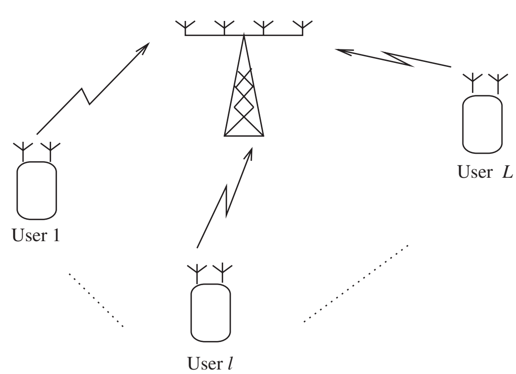

Figure 1. Multiuser MIMO system in which all users and the base station are equipped with antenna arrays for transmission/reception.

The received waveform at receive antenna j is then a superposition of all such received waveforms from all transmit antennas of all users plus additive Gaussian noise

$$r_j(t) = \sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} y_{ij}^{(\ell)}(t) + n_j(t) = \sum_{\ell=1}^{L} \sum_{i=1}^{N_\ell} x_i^{(\ell)}(t) * h_{ij}^{(\ell)}(t) + n_j(t) \tag{2}$$

If we approached the problem using the waveform representation above, the mathematics involved would become very complex and would artificially increase its difficulty. The use of a signal space approach overcomes the difficulties associated with the waveform representation above by working with equivalent signal vectors and vector channels, and making use of much simpler linear algebra methods. We note that different MIMO channel models may be obtained by using different basis functions for the signal space. For example, by using sine functions one ends up with models similar to those in References [6, 7] obtained by Nyquist sampling. Our simulation results in Section 4 are obtained for a MIMO channel model based on a multicarrier modulation scheme similar to that used in References [11, 15]. Nevertheless, once the MIMO channel matrix $\mathbf{H}_\ell$ is obtained, application of interference avoidance methods remains unchanged.

In the equivalent signal space representation, the N-dimensional received signal vector at the base station is given by [7, 4, 6]

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} \tag{3}$$

with $\mathbf{x}_\ell$ being the N_ℓ-dimensional signal vector transmitted by user ℓ, $\mathbf{H}_\ell$ the $N \times N_\ell$ MIMO channel matrix corresponding to user ℓ, and $\mathbf{n}$ the noise vector at the receiver. We note that Equation (3) is identical in form with the general multiaccess vector channel equation in Reference [12], and that the dimensions of the transmitter and receiver signal spaces will depend on the number of transmit and receive antennas employed.[§] Similar to [12] users transmit sequences of symbols consisting of zero-mean unit variance Gaussian random variables as frames, and for a given user ℓ the N_ℓ-dimensional transmitted vector $\mathbf{x}_\ell$ is obtained from the sequence of symbols to be sent $\mathbf{b}_\ell = [b_1^{(\ell)} \ldots b_{M_\ell}^{(\ell)}]^\top$ through a spreading operation specified by the $N_\ell \times M_\ell$ precoding matrix

$$\mathbf{S}_\ell = \begin{bmatrix} | & | & & | \\ \mathbf{s}_1^{(\ell)} & \mathbf{s}_2^{(\ell)} & \cdots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & | & & | \end{bmatrix} \tag{4}$$

whose columns have unit norm and determine the 'spreading' of corresponding symbols in the frame over the N_ℓ available dimensions. Therefore, the transmitted vector sent by user ℓ is written as

$$\mathbf{x}_\ell = \mathbf{S}_\ell \mathbf{b}_\ell \tag{5}$$

[§] For the MIMO channel models in References [4, 6, 7] these are actually equal to the number of antennas employed.

which implies that the received signal vector at the base station is given by

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{6}$$

Our problem is now to optimize precoder matrices $\mathbf{S}_\ell$, $\ell = 1, \ldots, L$, such that the SINR corresponding to all codewords/users is maximized. As we apply interference avoidance to find optimal codeword ensembles with maximum SINR, we would like to observe, as it has also been noted in References [11, 12] that this is different than finding the optimal transmit covariance matrices for all users such that the sum capacity of the multiple access vector channel defined in Equation (6) is maximized. While the solution to both problems turns out to satisfy a simultaneous water filling condition [11–13], the latter problem implies an iterative water filling algorithm, whereas interference avoidance is not in general a water filling procedure.

3. PRECODER OPTIMIZATION THROUGH INTERFERENCE AVOIDANCE

The fact that Equations (3) and (6) are identical to those in References [11, 12] corresponding to a general multiaccess vector channel using multicode CDMA to transmit information, suggests that the same greedy interference avoidance procedure can be used for precoder optimization. While complete details about application of the greedy interference avoidance procedure in the general multiaccess vector channel case can be found in References [11, 12] we outline the procedure for MIMO systems.

Similar to [11, 12] we start by rewriting the received signal in Equation (6) from the perspective of user k

$$\mathbf{r} = \mathbf{H}_k \mathbf{S}_k \mathbf{b}_k + \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{7}$$

and denoting by

$$\mathbf{z}_k = \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{8}$$

the interference-plus-noise seen by user k with the corresponding covariance matrix

$$\mathbf{Z}_k = E[\mathbf{z}_k \mathbf{z}_k^\top] = \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \tag{9}$$

Note that if user k channel matrix is not full-rank, some of the signal space dimensions will need to be discarded as they do not carry useful information for user k. However, this can only be done if the interference-plus-noise present in these dimensions is statistically independent from that in the remaining dimensions. This is accomplished by whitening the interference-plus-noise seen by user k through the transformation

$$\mathbf{T}_k = \mathbf{\Delta}_k^{-1/2} \mathbf{E}_k^\top \tag{10}$$

in which matrices $\mathbf{\Delta}_k$ and $\mathbf{E}_k$ are obtained from the eigenvalue decomposition of matrix $\mathbf{Z}_k = \mathbf{E}_k \mathbf{\Delta}_k \mathbf{E}_k^\top$.

In the transformed co-ordinates, Equation (7) is equivalent to

$$\tilde{\mathbf{r}} = \mathbf{T}_k\mathbf{r} = \mathbf{T}_k\mathbf{H}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{T}_k\mathbf{z}_k = \tilde{\mathbf{H}}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{w}_k \tag{11}$$

where $\tilde{\mathbf{H}}_k = \mathbf{T}_k\mathbf{H}_k$ is the MIMO channel matrix seen by user k in the new co-ordinates and $\mathbf{w}_k = \mathbf{T}_k\mathbf{z}_k$ is the equivalent 'white noise' term with covariance matrix $E[\mathbf{w}_k\mathbf{w}_k^\top] = \mathbf{T}_k\mathbf{Z}_k\mathbf{T}_k^\top = \mathbf{I}$ equal to the identity matrix. Following [11, 12] we apply the SVD and obtain

$$\tilde{\mathbf{H}}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\top \tag{12}$$

where matrix $\mathbf{U}_k$ of dimension $N \times N$ has as columns the eigenvectors of $\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^\top$, matrix $\mathbf{V}_k$ of dimension $N_k \times N_k$ has as columns the eigenvectors of $\tilde{\mathbf{H}}_k^\top\tilde{\mathbf{H}}_k$, and matrix $\mathbf{D}_k$ of dimension $N \times N_k$ contains the singular values of $\tilde{\mathbf{H}}_k$ on the main diagonal and zeros elsewhere. Any vector in the N_k-dimensional input space of user k can then be represented in terms of the orthonormal set of vectors $\{\mathbf{v}_i^{(k)}\}$ representing the columns of $\mathbf{V}_k$. Similarly, any vector in the N-dimensional receiver space is representable in terms of the orthonormal set of vectors $\{\mathbf{u}_i^{(k)}\}$ representing the columns of $\mathbf{U}_k$. Furthermore, because these sets of vectors come from the SVD decomposition (12) we have

$$\mathbf{v}_i^{(k)\top}\mathbf{v}_j^{(k)} = \delta_{ij} \Rightarrow \mathbf{v}_i^{(k)\top}\tilde{\mathbf{H}}_k^\top\tilde{\mathbf{H}}_k\mathbf{v}_j^{(k)} = d_i^{(k)2}\delta_{ij} \tag{13}$$

Therefore, user k should only put energy into those vectors $\mathbf{v}_i^{(k)}$ that correspond to non-zero singular values $d_i^{(k)} \neq 0$.

Let us denote by ρ_k the rank of user k's transformed MIMO channel matrix $\tilde{\mathbf{H}}_k$, equal to the number of non-zero singular values. It is obvious that

$$\rho_k = \mathrm{rank}(\tilde{\mathbf{H}}_k) \leqslant \min(N, N_k) \tag{14}$$

Then, the dimension of the column space of matrix $\tilde{\mathbf{H}}_k$ will be equal to ρ_k. Also, the dimension of the null space of $\mathbf{H}_k$ is $N_k - \rho_k$ and the dimension of the left null space is $N - \rho_k$. Because there are only ρ_k non-zero singular values and we are interested only in their corresponding eigenvectors, we can partition matrix $\mathbf{D}_k$ containing the singular values as

$$\mathbf{D}_k = \begin{bmatrix} \bar{\mathbf{D}}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{15}$$

with a $\rho_k \times \rho_k$ diagonal matrix $\bar{\mathbf{D}}_k$ which contains the non-zero singular values and zero matrices of appropriate dimensions.

Returning to Equation (11) in which we apply the SVD for matrix $\tilde{\mathbf{H}}_k$ we obtain

$$\tilde{\mathbf{r}} = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\top\mathbf{S}_k\mathbf{b}_k + \mathbf{w}_k \tag{16}$$

We can premultiply by $\mathbf{U}_k^\top$

$$\tilde{\mathbf{r}}_k = \mathbf{U}_k^\top\mathbf{r}_k = \mathbf{D}_k\mathbf{V}_k^\top\mathbf{S}_k\mathbf{b}_k + \mathbf{U}_k^\top\mathbf{w}_k \tag{17}$$

By defining $\tilde{\mathbf{S}}_k = \mathbf{V}_k^\top\mathbf{S}_k$ and $\tilde{\mathbf{w}}_k = \mathbf{U}_k^\top\mathbf{w}_k$ we have

$$\tilde{\mathbf{r}}_k = \mathbf{D}_k\tilde{\mathbf{S}}_k\mathbf{b}_k + \tilde{\mathbf{w}}_k \tag{18}$$

The partition in Equation (15) on the singular value matrix $\mathbf{D}_k$ induces the following partition of transformed precoder matrix $\tilde{\mathbf{S}}_k$:

$$\tilde{\mathbf{S}}_k = \begin{bmatrix} \tilde{\mathbf{S}}_{k1} \\ \tilde{\mathbf{S}}_{k2} \end{bmatrix} \tag{19}$$

with $\tilde{\mathbf{S}}_{k1}$ of dimension $\rho_k \times M_k$ and $\tilde{\mathbf{S}}_{k2}$ of dimension $(N_k - \rho_k) \times M_k$.

Note that because both $\mathbf{U}_k$ and $\mathbf{V}_k$ are orthogonal matrices they preserve norms of vectors. Thus, columns of $\tilde{\mathbf{S}}_k$ are also unit norm as were the columns of $\mathbf{S}_k$. Also, because the equivalent noise term $\mathbf{w}_k$ is white, then $\tilde{\mathbf{w}}_k$ will also be white.

In light of the partitions in Equations (15) and (19) we can safely ignore the last $N - \rho_k$ dimensions of the received vector $\tilde{r}_k$ in Equation (18) and reduce dimensionality of the problem to the rank of the channel matrix ρ_k. This can be done because, on one hand, no transmitted signal due to user k will be observed on these dimensions, and on the other hand noise components on these dimensions are statistically independent of the remaining noise components. Thus, the reduced dimension problem in which only the first ρ_k components of the received vector appear can be written as

$$\bar{\mathbf{r}}_k = [\mathbf{I}_{\rho_k}\mathbf{0}]\tilde{\mathbf{r}}_k = \bar{\mathbf{D}}_k\bar{\mathbf{S}}_k\mathbf{b}_k + \bar{\mathbf{w}}_k \tag{20}$$

with $\bar{\mathbf{S}}_k = \tilde{\mathbf{S}}_{k1}$ and $\bar{\mathbf{w}} = [\mathbf{I}_{\rho_k}\mathbf{0}]\tilde{\mathbf{w}}_k$. The covariance matrix of the 'new' noise vector is also an identity matrix of dimension ρ_k.

We are now ready for application of the greedy interference avoidance procedure, and recall that this consists of replacing one codeword m in user k transformed precoder matrix $\bar{\mathbf{S}}_k$ with the minimum eigenvector of the corresponding interference plus noise covariance matrix [11, 12]

$$\mathbf{R}_m^{(k)} = \bar{\mathbf{S}}_k\bar{\mathbf{S}}_k^\top - \bar{\mathbf{s}}_m^{(k)}\bar{\mathbf{s}}_m^{(k)\top} + \bar{\mathbf{D}}_k^{-2} \tag{21}$$

We note that, in addition to maximizing the SINR corresponding to codeword m of user k, this procedure also monotonically increases sum capacity [11, 12]. We also note that numerous interference avoidance algorithms can be formulated based on repeated application of the greedy interference avoidance procedure, depending on the particular order in which codewords/users are selected for replacement, and that, as it is also emphasized in References [11, 12], these are in general not water filling schemes although they yield a simultaneously water filling codeword ensemble.

After application of the greedy interference avoidance codeword update procedure the full dimension updated precoder matrix is given by

$$\mathbf{S}_k = \mathbf{V}_k \begin{bmatrix} \bar{\mathbf{S}}_k \\ \mathbf{0} \end{bmatrix} \tag{22}$$

which ensures that each input codeword vector is a linear combination of only those $\mathbf{v}_i^{(k)}$ which actually appear at the channel output.

A straightforward way to implement a precoder optimization algorithm for MIMO systems based on interference avoidance is to sequentially update all codewords of a given user k until convergence and then iterate this procedure for all users. This procedure defines the eigen-algorithm for multiuser MIMO systems and is formally stated below:

3.1. The eigen-algorithm for multiuser MIMO systems

1. Start with a randomly chosen set of precoder matrices $\{\mathbf{S}_\ell\}_{\ell=1}^{L}$
2. For each user $k = 1, \ldots, L$

   (a) Compute the transformation matrix $\mathbf{T}_k$ in Equation (10) that whitens the interference-plus-noise seen by user k.
   (b) Change co-ordinates and compute transformed user k's MIMO channel matrix $\tilde{\mathbf{H}}_k = \mathbf{T}_k\mathbf{H}_k$.
   (c) Apply SVD for $\tilde{\mathbf{H}}_k$ and project the problem onto user k's signal space to obtain $\tilde{\mathbf{r}}_k$ in Equation (18).
   (d) Reduce dimensionality to ρ_k the rank of the MIMO channel matrix and obtain the reduced dimension problem in Equation (20).
   (e) Define the equivalent problem for user k [11, 12] by premultiplying Equation (20) with $\bar{\mathbf{D}}_k^{-1}$

$$\bar{\mathbf{r}}_{k,\text{inv}} = \bar{\mathbf{D}}_k^{-1}\bar{\mathbf{r}}_k = \bar{\mathbf{S}}_k\mathbf{b}_k + \bar{\mathbf{D}}_k^{-1}\bar{\mathbf{w}}_k \tag{23}$$

   (f) Adjust user k's transformed precoder matrix by replacing its columns sequentially: column m of $\bar{\mathbf{S}}_k$ ($\bar{\mathbf{s}}_m^{(k)}$) is replaced by the minimum eigenvector of corresponding interference-plus-noise covariance matrix in Equation (21).
   (g) Iterate previous step until convergence (making use of escape methods [9] if the procedure stops in suboptimal points).

3. Repeat step 2 iteratively for each user until a fixed point is reached for which further modification of codewords will bring no additional improvement.

We note that steps 2(f) and (g) represent application of the basic eigen-algorithm [9, 10] and 'water fill' user k's signal space while regarding the remaining users in the system as noise. Therefore, applied iteratively by each user, the eigen-algorithm for MIMO systems is an instance of iterative water filling and is thus guaranteed to converge to codeword ensembles which maximize sum capacity of the multiple access vector channel in Equation (6).

However, we also note that the distributed and asynchronous nature of independent users and codeword updates might not admit such a simple tightly co-ordinated sequential approach. Fortunately, interference avoidance can still be applied under the assumption of asynchronous codeword updates since each update increases sum capacity. Various provably convergent flavors of the algorithm can be used, but perhaps the most satisfying feature of distributed interference avoidance is that empirically in numerical simulations convergence does not seem to depend on the specific update method employed. Thus, interference avoidance appears to be robust.

We also note, that from a practical point of view the dimensionality of the problem can be further reduced in step 2(d) of the algorithm by taking advantage of the water filling result implied by the eigen-algorithm. More precisely, the noise levels over which water filling occurs in steps 2(f) and (g) of the algorithm are given by the inverse of the non-zero singular values of the MIMO channel matrix $\tilde{\mathbf{H}}_k$, that is

$$\bar{\mathbf{D}}_k^{-2} = \begin{bmatrix} d_1^{(k)^{-2}} & & & & \\ & \ddots & & & \\ & & d_i^{(k)^{-2}} & & \\ & & & \ddots & \\ & & & & d_{\rho_k}^{(k)^{-2}} \end{bmatrix} \tag{24}$$

in decreasing order of their magnitudes as yielded by the SVD

$$d_1^{(k)} \geqslant \cdots \geqslant d_i^{(k)} \geqslant \cdots \geqslant d_{\rho_k}^{(k)} \tag{25}$$

This implies that their inverses will be in increasing order of their magnitudes, i.e.

$$d_1^{(k)^{-2}} \leqslant \cdots \leqslant d_i^{(k)^{-2}} \leqslant \cdots \leqslant d_{\rho_k}^{(k)^{-2}} \tag{26}$$

Because each user k is limited to a fixed amount of transmitted power equal to $\text{Trace}[\mathbf{S}_k\mathbf{S}_k^\top] = M_k$ (coming from the fact that each user k sends M_k symbols each with unit energy) we can determine how many of the ρ_k dimensions will actually carry information by looking at the 'watermark' in the corresponding water filling diagram (see Figure 2). If we denote by n the number of dimensions water-filled by the transmitted power then the 'watermark' is defined as

$$c_k^* = \frac{M_k + \sum_{i=1}^{n} d_i^{(k)^{-2}}}{n} \tag{27}$$

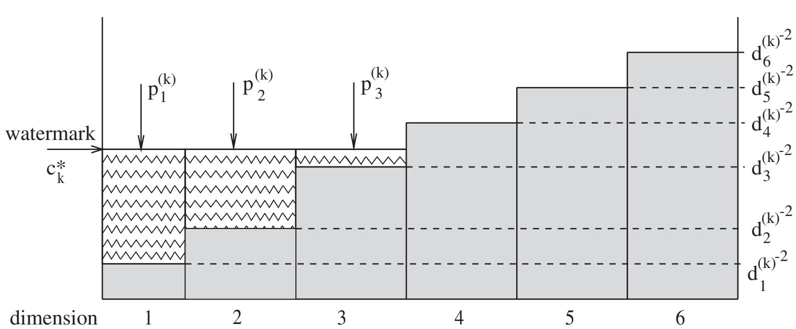

Figure 2. An example of water filling diagram in a signal space with 6 dimensions for which the total power M_k of user k is split only on the first three dimensions with minimum 'noise' energy. The identities implied by water filling $c_k^* = p_1^{(k)} + d_1^{(k)^{-2}} = p_2^{(k)} + d_2^{(k)^{-2}} = p_3^{(k)} + d_3^{(k)^{-2}}$ determine $c_k^* = [M_k + (d_1^{(k)^{-2}} + d_2^{(k)^{-2}} + d_3^{(k)^{-2}})]/3 \leqslant d_4^{(k)^{-2}}$.

and can be found by algorithmically by checking the following inequalities:

$$\left.\begin{aligned} c_m^{(k)^*} &= \frac{1}{m}\left(M_k + \sum_{i=1}^{m} d_i^{(k)^{-2}}\right) > d_{m+1}^{(k)^{-2}} \\ c_{m+1}^{(k)^*} &= \frac{1}{m+1}\left(M_k + \sum_{i=1}^{m+1} d_i^{(k)^{-2}}\right) \leqslant d_{m+2}^{(k)^{-2}} \end{aligned}\right\} \Rightarrow \quad n = m+1 \tag{28}$$

The first inequality in (28) tells us that if we were to use only the first m dimensions to do water filling, the resulting watermark $c_m^{(k)^*}$ will be larger than the $m+1$ 'noise level'. However, the next inequality assures that when the $(m+1)$st dimension is used, the resulting watermark $c_{m+1}^{(k)^*}$ will be less than or equal to the corresponding 'noise level' and therefore no additional dimensions will be water-filled.

Reducing the number of dimensions to only those which are actually water-filled by user k is advantageous from a numerical point of view since the eigen-algorithm will be performed on a lower dimensional problem. We note however, that sum capacity of the MIMO channel does not change since it is still obtained by water filling over the maximum number of dimensions with smallest 'background' noise energy in user k's inverted channel signal space.

4. SIMULATION RESULTS

In this section, we perform simulations and look at performance in terms of signal-to-noise ratios (SNRs) at the receiver and channel capacity. Capacity is analysed in the context of fading channels which are characteristic of wireless communications. Here, we use Rayleigh channel models which depending upon one's perspective, provide a best or a worst case view. That is, satellite channels are generally Ricean with a strong direct path and smaller scattered paths. Thus, a Rayleigh model will show a good deal of spread between optimal performance where the channel is known all the time and coding is done for each channel realization, and average performance where codes are chosen for the average channel. A good Ricean channel model will show much less spread. However, the large capacity increases possible with compact antennas over fading channels is owed directly to the multiplicity of independent paths introduced by scattering. So in this sense, Rayleigh channels constitute a best case result of sorts.

Regardless, due to the randomness of channel realizations, the resulting capacity is a random variable and we plot Complementary Cumulative Distribution Functions (CCDFs) similar to those in Reference [4] from which one can see what capacity can be achieved with a given probability. More precisely, we say that an outage occurs whenever capacity is below a given value, and identify the probability of outage P_{out} from the corresponding CCDF.

The MIMO channel model used for simulations is derived by using the same set of basis functions for the signal space as in References [11, 15] consisting of real sinusoids (sine and cosine functions). Similar to [11, 15] we assume that the frame duration $\mathscr{T} \geqslant T_{ij}^{(\ell)}\ \forall \ell, i, j$. Consequently, sinusoids are eigenfunctions for all the channels in the multiple antenna link, and let us denote by N_c the number of frequencies used. The number of transmit antennas for user ℓ is T_ℓ and the number of receiver antennas is R.

Decomposition of each channel into orthogonal sub-channels implied by the sinusoidal basis functions leads to multicarrier modulation for transmission of information on each pair of transmit/receive antennas. In this context the equivalent vector channel representation for Equation (1) is

$$\mathbf{y}_{ij}^{(\ell)} = \mathbf{\Lambda}_{ij}^{(\ell)1/2}\mathbf{x}_i^{(\ell)} \tag{29}$$

where $\mathbf{x}_i^{(\ell)}$ and $\mathbf{y}_{ij}^{(\ell)}$ are the $2N_c$-dimensional input and output vectors, and $\mathbf{\Lambda}_{ij}^{(\ell)}$ the $2N_c \times 2N_c$ matrix containing the eigenvalues corresponding to channel having impulse response $h_{ij}^{(\ell)}(t)$. We note that in the context of multicarrier modulation when real sinusoids are approximately channel eigenfunctions, channel eigenvalues correspond to the real channel gains for the frequencies that span the signal space. The received vector at receive antenna j corresponding to the received waveform in Equation (2) is then given by

$$\mathbf{r}_j = \sum_{\ell=1}^{L}\sum_{i=1}^{T_\ell}\mathbf{y}_{ij}^{(\ell)} + \mathbf{n}_j = \sum_{\ell=1}^{L}\sum_{i=1}^{T_\ell}\mathbf{\Lambda}_{ij}^{(\ell)1/2}\mathbf{x}_i^{(\ell)} + \mathbf{n}_j \tag{30}$$

where $\mathbf{n}_j$ is the additive noise vector at receive antenna j with covariance matrix $E[\mathbf{n}_j\mathbf{n}_j^\top] = \mathbf{W}_j$.

By stacking together all received signal vectors from all receive antennas we can write

$$\begin{bmatrix}\mathbf{r}_1\\ \vdots\\ \mathbf{r}_j\\ \vdots\\ \mathbf{r}_R\end{bmatrix} = \sum_{\ell=1}^{L}\begin{bmatrix}\mathbf{\Lambda}_{11}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{i1}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{T_\ell 1}^{(\ell)1/2}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{\Lambda}_{1j}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{ij}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{T_\ell j}^{(\ell)1/2}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{\Lambda}_{1R}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{iR}^{(\ell)1/2} & \dots & \mathbf{\Lambda}_{T_\ell R}^{(\ell)1/2}\end{bmatrix}\begin{bmatrix}\mathbf{x}_1^{(\ell)}\\ \vdots\\ \mathbf{x}_i^{(\ell)}\\ \vdots\\ \mathbf{x}_{T_\ell}^{(\ell)}\end{bmatrix} + \begin{bmatrix}\mathbf{n}_1\\ \vdots\\ \mathbf{n}_j\\ \vdots\\ \mathbf{n}_R\end{bmatrix} \tag{31}$$

or simply

$$\mathbf{r} = \sum_{\ell=1}^{L}\mathbf{H}_\ell\mathbf{x}_\ell + \mathbf{n} \tag{32}$$

where we have denoted by $\mathbf{H}_\ell$ the $2N_cT_\ell \times 2N_cR$ matrix containing channel eigenvalue matrices of all channels in the multiple antenna link between user ℓ and the base station, $\mathbf{x}_\ell$ the $2N_cT_\ell$-dimensional transmitted vector of user ℓ, and $\mathbf{n}$ the $2N_cR$-dimensional noise vector at the base station. Under the assumption that noise vectors at different antennas are independent then the noise covariance matrix will be block diagonal, each block containing the covariance of the noise that corrupts the received signal at the corresponding receive antenna

$$E[\mathbf{n}\mathbf{n}^\top] = \mathbf{W} = \begin{bmatrix}\mathbf{W}_1 & & \\ & \ddots & \\ & & \mathbf{W}_{N_r}\end{bmatrix}$$

with $\mathbf{W}_j = E[\mathbf{n}\mathbf{n}^\top]$.

The above MIMO channel model has been derived under the implicit assumption that the N_c periods of spanning sinusoids are large compared to the propagation delays between antenna elements so that the sine and cosine components are still approximately synchronized at the receiver even in the presence of multiple transmit and receive antennas. In addition, for simplicity, carrier synchronization for received signals has also been assumed.

We start with the single user multiple antenna case which was the case considered in previous work [4, 6]. For this case, we analyse SNR distribution when random precoding matrices are used as well as when the optimal precoding matrices yielded by the eigen-algorithm are used, and we also look at improvement in SNR generated by an increase in number of antennas. CCDFs for sum capacity for single user as well as multiple user cases are also plotted.

4.1. Receiver SNR distribution

In order to compute the SNRs at the receiver antennas, we return to Equation (30) which for only one user becomes

$$\mathbf{r}_j = \sum_{i=1}^{T} \mathbf{y}_{ij} + \mathbf{n}_j = \sum_{i=1}^{T} \mathbf{\Lambda}_{ij}^{1/2} \mathbf{x}_i + \mathbf{n}_j \tag{33}$$

Note that we have dropped the user index ℓ and replaced the number of transmit antennas by T since there is only one user in the system. The average energy of the signal at receive antenna j is given by

$$\begin{aligned} E[\mathbf{r}_j^\top \mathbf{r}_j] &= \text{Trace}\,[E[\mathbf{r}_j \mathbf{r}_j^\top]] \\ &= \text{Trace}\left[\sum_{p=1}^{T} \sum_{r=1}^{T} \mathbf{\Lambda}_{pj}^{1/2} E[\mathbf{x}_p \mathbf{x}_r^\top] \mathbf{\Lambda}_{rj}^{1/2} + \mathbf{W}_j\right] \\ &= \text{Trace}\left[\sum_{p=1}^{T} \sum_{r=1}^{T} \mathbf{\Lambda}_{pj}^{1/2} E[\mathbf{x}_p \mathbf{x}_r^\top] \mathbf{\Lambda}_{rj}^{1/2}\right] + \text{Trace}\,[\mathbf{W}_j] \end{aligned} \tag{34}$$

where $\mathbf{x}_p$, $\mathbf{x}_r$ represent signals coming from transmit antennas p and r, respectively, which can be obtained explicitly by partitioning the $2N_cT \times M$ precoding matrix $\mathbf{S}$ in T blocks of dimensions $2N_c \times M$ stacked together

$$\mathbf{x} = \mathbf{S}\mathbf{b} = \begin{bmatrix} \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_i \\ \vdots \\ \mathbf{S}_T \end{bmatrix} \mathbf{b} = \begin{bmatrix} \mathbf{S}_1\mathbf{b} \\ \vdots \\ \mathbf{S}_i\mathbf{b} \\ \vdots \\ \mathbf{S}_T\mathbf{b} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_i \\ \vdots \\ \mathbf{x}_T \end{bmatrix} \tag{35}$$

With these partitions we get $E[\mathbf{x}_p \mathbf{x}_r^\top] = \mathbf{S}_p \mathbf{S}_r^\top$ and we compute the SNR at receive antenna j as the ratio of the first term on the right-hand side of Equation (34) to the second term

$$\text{SNR}_j = \frac{\text{Trace}\,[\sum_{p=1}^{T} \sum_{r=1}^{T} \mathbf{\Lambda}_{pj}^{1/2} \mathbf{S}_p \mathbf{S}_r^\top \mathbf{\Lambda}_{rj}^{1/2}]}{\text{Trace}\,[\mathbf{W}_j]} \tag{36}$$

Note that knowledge of channels composing the multiple antenna link implies different SNRs at different receive antennas even in the context of white noise with the same average power. This is different from [4] where the same average SNR at all receive antennas has been assumed.

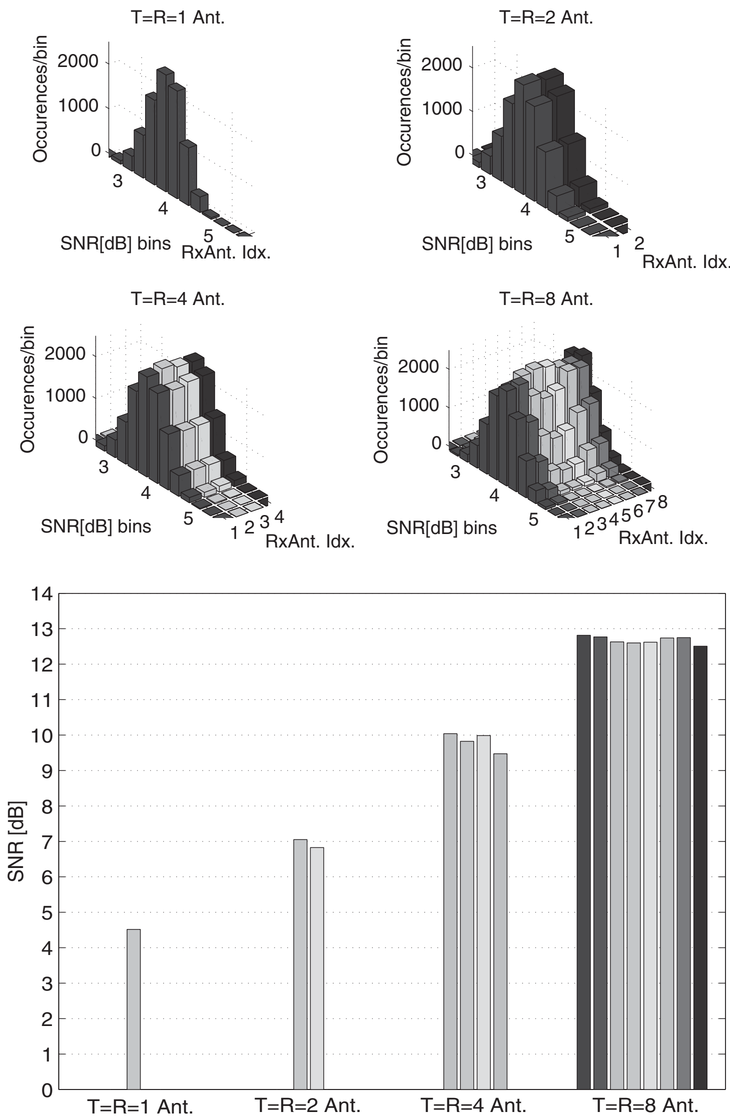

Figure 3. SNR distributions for a single user multiple antenna system with random precoding matrices (upper plot). SNRs with optimal precoding matrices yielded by the interference avoidance algorithm for a single user multiple antenna system (lower plot). Signal space dimension is $N = 10$ and average power of white noise is $N_0 = 0.5$ at each receive antenna.

However, we have a similar constraint on the total transmitted power which is constant regardless of the number of transmit antennas used.

With random precoding matrices and for a particular set of channels comprising the multiple antenna link, the SNRs at different receive antennas have the bell-shaped distribution shown in Figure 3 (upper plots). The SNR at all receiver antennas has been recorded for a number of $N_c = 5$ spanning frequencies (corresponding to 10 real sinusoids), with equal number of antennas at transmitter and receiver $T = R = 1, 2, 4, 8$, and average power of the white noise $N_0 = 0.5$ at each receive antenna for the same set of channels but for different (random) precoding matrices. However, application of interference avoidance water fills the channels appropriately and results in a fixed set of SNRs at each receive antenna for given instances of the channel(s). As it can be seen from Figure 3 doubling the number of antenna elements in both transmitter and receiver results in about a 3 dB improvement in the SNR. Also note that after interference avoidance the SNR is approximately the same for all receive antennas, even though no *a priori* assumption about equal SNRs at each antenna [4] has been made.

4.2. Fading channels and outage capacity

In the case of fading environments, which is characteristic of wireless communications, the impulse responses of channels in the multiple antenna link change over time and it becomes difficult, if not impossible, to apply interference avoidance to determine optimal precoding matrices corresponding to all channel realizations that occur during the duration of the transmission. However, in such cases interference avoidance is applied using average characteristics of the channels [16] to determine precoder matrices which are optimal for the average channel.

We analyse the effect of fading on the sum capacity of our MIMO channel model by using precoding matrices which are optimal for the average channel (obtained by application of the eigen-algorithm) and computing the corresponding sum capacity for various realizations of the fading channel models assumed. The results are used to plot the corresponding CCDFs.

We assume the same frequency-selective fading channel model as in Reference [16] which is suited for the multicarrier modulation scheme used for transmission [17]. The model assumes flat fading of the carriers—the amplitudes of distinct carriers being scaled at the receiver by different constants. In a Rayleigh fading environment the amplitude scaling $\kappa_n^{(ij)}$ of the nth carrier due to the channel between transmit antenna i and receive antenna j is a Rayleigh random variable with the probability density function

$$f_{\kappa_n^{(ij)}}(\kappa_n^{(ij)}) = \frac{\kappa_n^{(ij)}}{\sigma_n^{(ij)^2}} \mathrm{e}^{-\kappa_n^{(ij)^2}/2\sigma_n^{(ij)^2}} \tag{37}$$

where the parameter $\sigma_n^{(\ell k)^2}$ is related to the second moment of the Rayleigh random variable $E[\kappa_n^{(ij)^2}] = 2\sigma_n^{(ij)^2}$. The second moment of this random variable characterizes the average channel being the gain (eigenvalue) corresponding to carrier n for channel linking transmit antenna i with receive antenna j in the average channel model, i.e. $E[\kappa_n^{(ij)^2}] = 2\sigma_n^{(ij)^2} = \lambda_n^{(ij)}$.

For a single user system, we perform a set of experiments in which we first determine a precoding matrix which is optimal for the average channel defined in terms of a set of average values of the Rayleigh random variables, and then compute capacity values for distinct realizations of these Rayleigh random variables using equation

$$C = \tfrac{1}{2}\log[\det(\mathbf{HSS}^\top\mathbf{H}^\top + \mathbf{W})] - \tfrac{1}{2}\log(\det \mathbf{W}) \tag{38}$$

where $\mathbf{H}$ is the MIMO channel matrix of the considered user. Again there is no need for user index ℓ as there is only one user in the system. The resulting set of capacity values are used to derive the CCDFs presented in Figure 4. In these plots we compare the case of only one transmit and one receive antenna with the case of two transmit and two receive antennas, and four transmit and four receive antennas respectively. Simulations are done with a set of $N = 5$ carrier

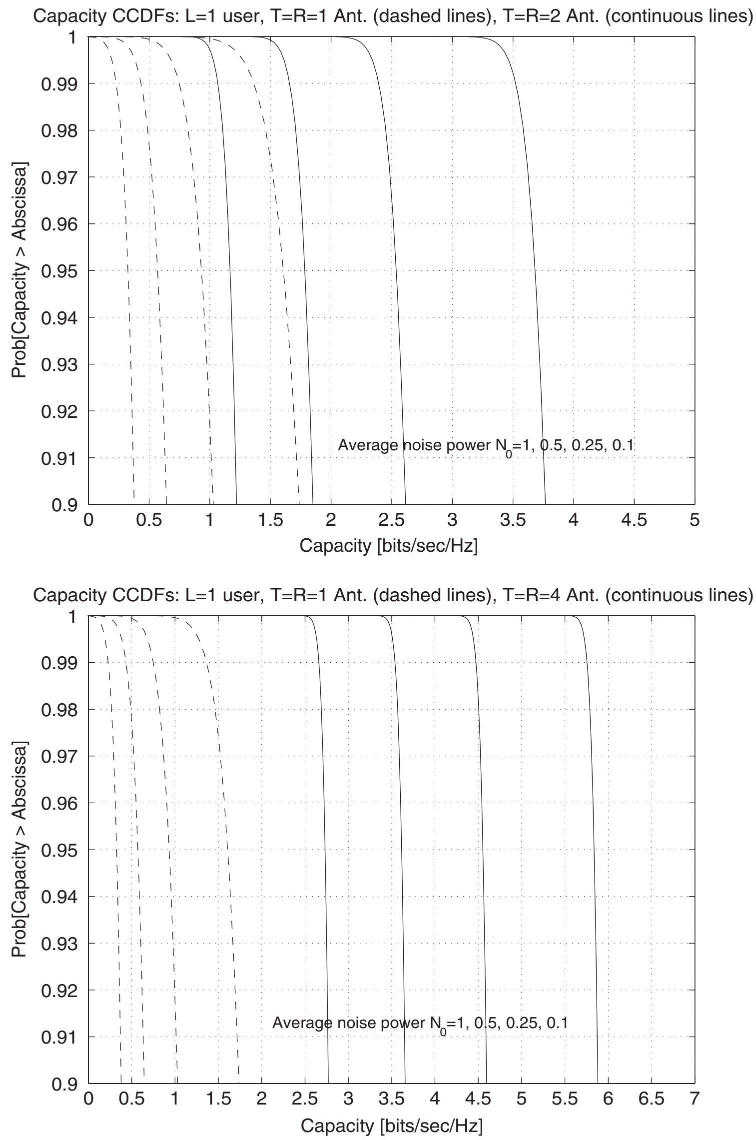

Figure 4. Capacity CCDFs for single user MIMO system. The $T = R = 1$ antenna case is compared with the $T = R = 2$ antenna case (upper plot) and with the $T = R = 4$ antenna case (lower plot).

frequencies for different average white noise power at the receiver $N_0 = 1, 0.5, 0.25, 0.1$. These values correspond to a 3 dB increase in the SNR at receiver antennas and for the average channel imply SNRs of approximately[¶] 1, 4, 7, and 10 dB for the $T = R = 1$ antenna case, 4, 7,

[¶] As it has been seen, the SNRs at receive antennas are not necessarily the same. However, when optimal codewords are used they are approximately equal.

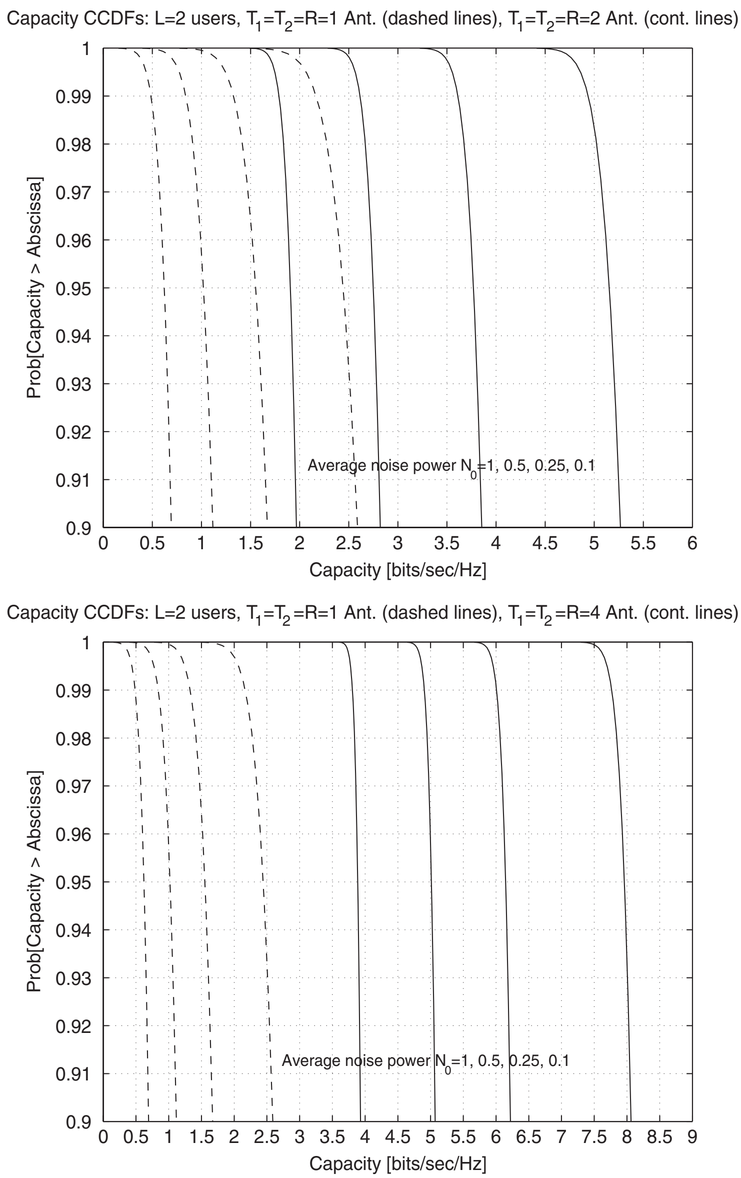

Figure 5. Sum capacity CCDFs for a two-user MIMO system. The $T_1 = T_2 = R = 1$ antenna case is compared with the $T_1 = T_2 = R = 2$ antenna case (upper plot) and with the $T_1 = T_2 = R = 4$ antenna case (lower plot).

10 and 13 dB for the $T = R = 2$ antenna case, and 7, 10, 13, and 16 dB for the $T = R = 4$ antenna case.

From the CCDFs in Figure 4 see that for the analysed scenario, the use of multiple antennas at both transmitter and receiver improves outage capacity. For example, for an outage probability $P_{\text{out}} = 1\%$, capacity is increased for low SNR from less than 0.5 bits/s/Hz to approximately 1.1 bits/s/Hz for two transmit and receive antennas, and almost 2.7 bits/s/Hz for four transmit and receive antennas. For high SNR, the rates increase from about 1.5 bits/s/Hz to about 3.6 bits/s/Hz for two transmit and receive antennas and 5.7 bits/s/Hz for four transmit and receive antennas.

Next we consider a multiuser MIMO system with $L = 2$ users and perform similar simulations as in the single user case with $N = 5$ carrier frequencies for different average white noise power at the receiver $N_0 = 1, 0.5, 0.25, 0.1$. Optimal precoding matrices for the average channel for two users are determined, and then sum capacity values for distinct realizations of these Rayleigh random variables is computed using equation

$$C = \frac{1}{2}\log\left[\det\left(\sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}^\top + \mathbf{W}\right)\right] - \frac{1}{2}\log(\det \mathbf{W}) \tag{39}$$

and the resulting set of capacity values are used to derive the CCDFs presented in Figure 5. In these plots, we compare sum capacity for the two user system in the case of only one transmit antenna per user and one receive antenna with the case of two transmit antennas per user and two receive antennas, and four transmit antennas per user and four receive antennas, respectively. From the CCDFs in Figure 5 we see similar improvements in sum capacity for the analysed scenario and conclude that the use of multiple antennas in the transmitters as well as in the receiver improves outage capacity.

5. CONCLUSIONS

Application of interference avoidance methods to multiuser MIMO systems has been presented and analysed in the paper. Such systems are associated with the uplink of a wireless system in which users and the base station have multiple antennas.

Our approach is based on application of interference avoidance to general multiaccess vector channels in Reference [12] for the particular multiaccess vector channel corresponding to the multiuser MIMO system. We note that the approach is completely general and applicable to any MIMO system models regardless of the choice of signal space basis functions. Information is sent in frames using a multicode CDMA approach with spreading over the available dimensions implied by a precoding matrix. Optimal precoding matrices for which sum capacity is maximized can be obtained for all users in the system through sequential application of the greedy interference avoidance procedure. The update method used illustratively, which extends the eigen-algorithm [11, 9, 10], is an instance of iterative water filling [13] and provides optimal ensembles for both single and multiple users. However in practice, we suspect that more asynchronous operation would be typical with individual codewords with poor performance being updated as necessary. Interference avoidance is empirically robust with respect to the codeword update sequence so such application should not pose any problems.

Numerical results based on simulations were also presented in the paper. We note that these results are consistent with the well-known results in the multiple antenna literature, namely that the use of multiple antennas in both the transmitter and the receiver is beneficial for system performance.

For a single user system, the SNR distribution at receiver antennas was computed with both random precoding matrices as well as with optimal precoding matrices yielded by the eigen-algorithm, and it has been noted that with the latter the SNR is approximately the same for all receive antennas in the absence of any *a priori* assumptions (like those made for example in Reference [4]). It has also been noted that doubling the number of antennas (both in the transmitter and in the receiver) translates to a gain of about 3 dB in the SNR when optimal precoding matrices are used.

Sum capacity of the multiuser MIMO channel was also investigated in the context of fading environments. A frequency-selective fading channel model was assumed and precoding matrices optimal for the average channel were used and sum capacity was treated as a random variable in this case for which CCDF curves were plotted. We note that single user CCDFs are comparable to those in Reference [4] showing similar capacity improvements when multiple antennas are used in both the transmitter and the receiver. The CCDFs for multiple users show also improvement in sum capacity when multiple antennas are used in conjunction with designing precoding matrices that are optimal for the average channel for all users in the system.

We close by emphasizing that interference avoidance is a *distributed method* of transceiver optimization and could be of particular use in satellite systems owing to processor power limitations on the satellite. Specifically, the original development of interference avoidance [10] posited codeword updates calculated at the receiver and then disbursed to the transmitters through a feedback channel. However, application of interference avoidance requires only that each user know its own channel (or its average channel) and the received covariance at the receiver. So, if the receiver (satellite) can broadcast its covariance on a side channel beacon and users can learn their own channels, then individual users can asynchronously calculate codeword updates. Furthermore, since the algorithm is robust with respect to the codeword update procedure, various 'lagged' versions of the algorithm [11] wherein codewords, rather than being abruptly changed, are gradually moved toward target codewords in smaller steps could be used which would allow the receiver (satellite) to track codeword changes in exactly the same way an adaptive equalizer tracks channel variations. For satellites where power is at a premium, off-loading the computation and co-ordination associated with codeword update could be especially beneficial.

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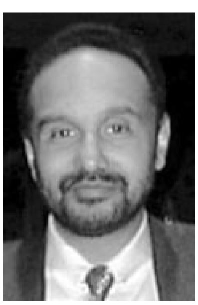

Christopher Rose received the BS (1979), MS (1981) and PhD (1985) degrees all from the Massachusetts Institute of Technology in Cambridge, Massachusetts. Following graduate school, Dr Rose joined AT&T Bell Laboratories in Holmdel, N.J. as a member of the Network Systems Research Department. Dr Rose is currently an associate professor of Electrical and Computer Engineering at Rutgers University in New Jersey an Associate Director of the Wireless Networks Laboratory (WINLAB), as well as a Henry Rutgers Research Fellow. He is an editor for the ACM Wireless Networks (WINET) journal, the Elsevier Computer Networks Journal, and has served on many conference technical program committees and was technical program co-chair for MobiCom'97, Co-chair of the WINLAB Focus'98 on the U-NII, the WINLAB Berkeley Focus'99 on Radio Networks for Everything and the Berkeley WINLAB Focus 2000 on Picoradio Networks. Dr Rose, a past member of the ACM SIGMobile Executive Committee is currently a member of the ACM MobiCom Steering Committee and has also served as General Chair of ACM SIGMobile MobiCom 2001 (Rome, July 2001). In December 1999 he served on an international panel to evaluate engineering teaching and research in Portugal. Closer to home Dr Rose has served on the Scientific Fields Advisory Committee of the New Jersey Commission on Science and Technology. His current technical interests include mobility management, novel mobile communications networks, applications of genetic algorithms to control problems in communications networks and most recently, interference avoidance methods using universal radios to foster peaceful coexistence in what will be the wireless ecology of the recently allocated 5 G Hz U-NII bands.

