Interference Avoidance Versus Iterative Water Filling in Multiaccess Vector Channels

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Abstract— **In this paper we present an analysis of the relationship between application of greedy interference avoidance and that of iterative water filling for vector multiaccess channels. The two methods converge to fixed points with similar properties corresponding to maximum sum capacity. However, the former is used for optimizing CDMA codewords, whereas the latter is used for optimizing transmit covariance matrices. We present numerical results from simulations and make a parallel between the two procedures discussing differences and similarities.**

I. Introduction

Interference avoidance has emerged in the literature as a method for distributed codeword adaptation in CDMA systems. Introduced originally in the context of DS-CDMA systems and MMSE receivers [1], [11] MMSE interference avoidance was followed by greedy interference avoidance which uses matched filter receivers and a minimum eigenvector approach [8], [9].

Concurrently and independently iterative water filling was established as a method for optimizing transmit covariances of users in a general multiaccess channel framework [16].

Motivated by the fact that these two methods converge to fixed points which correspond to maximum sum capacity and have similar properties, in this paper we discuss the relationship between them presenting similarities and differences. Numerical results obtained from simulations of the two methods will be used to support the presentation.

II. System Model: Vector Channels and CDMA

We consider a general multiuser communication system in which different users reside in different signal spaces, with different dimensions and potential overlap between them, and all being subspaces of the receiver signal space. We note that each user's signal space as well as the receiver signal space are of finite dimension as implied by a finite signaling interval and finite bandwidths W_ℓ for each user ℓ, respectively and W (which includes all W_ℓ's corresponding to all users) for the receiver [5]. Mathematically, this is described by the multiaccess vector channel equation [16]

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n} \tag{1}$$

where $\mathbf{x}_\ell$, of dimension N_ℓ, is the input vector corresponding to user ℓ, $\ell = 1, \dots, L$, $\mathbf{r}$, of dimension $N \geq N_\ell$, is the received vector at the common receiver corrupted by additive Gaussian noise vector $\mathbf{n}$ of the same dimension, with covariance matrix $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top]$. The $N \times N_\ell$ channel matrix $\mathbf{H}_\ell$ corresponding to user ℓ defines a linear transformation between user ℓ signal space and the receiver signal space, and in general incorporates channel attenuation and multipath [16]. Sum capacity for the multiaccess vector channel in equation (1) is expressed as

$$C_s = \frac{1}{2} \log \left| \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{X}_\ell \mathbf{H}_\ell^\top + \mathbf{W} \right| - \frac{1}{2} \log |\mathbf{W}| \tag{2}$$

In this signal space setting user ℓ transmits a frame of information symbols $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^\top$ using a multicode CDMA approach in which each symbol in the frame is assigned a distinct codeword for transmission, and the transmitted signal vector by user ℓ is expressed as $\mathbf{x}_\ell = \mathbf{S}_\ell \mathbf{b}_\ell$, $\ell = 1, \dots, L$, where $\mathbf{S}_\ell$ is the $N_\ell \times M_\ell$ codeword matrix corresponding to user ℓ whose columns $\mathbf{s}_m^{(\ell)}$, $m = 1, \dots, M_\ell$, are the codewords assigned to each of the M_ℓ symbols in user ℓ frame. We assume that codewords have unit norm, which implies that user ℓ transmitted power is $P_\ell = \text{Trace}\left[\mathbf{S}_\ell \mathbf{S}_\ell^\top\right] = M_\ell$. Therefore, the received signal at the common receiver can be rewritten as

$$\mathbf{r} = \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{b}_\ell + \mathbf{n} \tag{3}$$

User transmit covariance matrices are written in terms of user codeword matrices as $\mathbf{X}_\ell = E[\mathbf{x}_\ell \mathbf{x}_\ell^\top] = \mathbf{S}_\ell \mathbf{S}_\ell^\top$, and sum capacity in equation (2) becomes

$$C_s = \frac{1}{2} \log \left| \sum_{\ell=1}^{L} \mathbf{H}_\ell \mathbf{S}_\ell \mathbf{S}_\ell^\top \mathbf{H}_\ell^\top + \mathbf{W} \right| - \frac{1}{2} \log |\mathbf{W}| \tag{4}$$

III. Greedy Interference Avoidance

Interference avoidance offers a distributed procedure for optimizing CDMA codewords. Its application to the general multiaccess vector channel model in equation (3) is presented in [6], and is based on application of greedy interference avoidance [7] for a given user after whitening of the interference-plus-noise seen by that user and projection of the received

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signal onto the given user's signal space using the singular value decomposition (SVD).

For a particular user k the received signal in equation (3) is rewritten as

$$\mathbf{r} = \mathbf{H}_k\mathbf{S}_k\mathbf{b}_k + \underbrace{\sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell\mathbf{S}_\ell\mathbf{b}_\ell + \mathbf{n}}_{\mathbf{z}_k} \tag{5}$$

and the interference-plus-noise seen by user k, $\mathbf{z}_k$, with covariance matrix

$$\mathbf{Z}_k = E[\mathbf{z}_k\mathbf{z}_k^\top] = \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell\mathbf{S}_\ell\mathbf{S}_\ell^\top\mathbf{H}_\ell^\top + \mathbf{W} \tag{6}$$

is whitened by application of the whitening transformation

$$\mathbf{T}_k = \mathbf{\Delta}_k^{-1/2}\mathbf{E}_k^\top \tag{7}$$

where $\mathbf{E}_k$ and $\mathbf{\Delta}_k$ are the eigenvector, respectively eigenvalue matrix of $\mathbf{Z}_k$

$$\mathbf{Z}_k = \mathbf{E}_k\mathbf{\Delta}_k\mathbf{E}_k^\top \tag{8}$$

Application of the whitening transformation in equation (7) to equation (5) yields

$$\tilde{\mathbf{r}} = \mathbf{T}_k\mathbf{r} = \mathbf{T}_k\mathbf{H}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{T}_k\mathbf{z}_k = \tilde{\mathbf{H}}_k\mathbf{S}_k\mathbf{b}_k + \mathbf{w}_k \tag{9}$$

where $\tilde{\mathbf{H}}_k = \mathbf{T}_k\mathbf{H}_k$ is the channel matrix seen by user k in the new coordinates and $\mathbf{w}_k = \mathbf{T}_k\mathbf{z}_k$ is the equivalent "white noise" with covariance matrix $E[\mathbf{w}_k\mathbf{w}_k^\top] = \mathbf{T}_k\mathbf{Z}_k\mathbf{T}_k^\top = \mathbf{I}$ equal to the identity matrix.

We now apply the singular value decomposition (SVD) [10, p. 442] to the transformed channel matrix corresponding to user k

$$\tilde{\mathbf{H}}_k = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\top \tag{10}$$

Assuming, without loss of generality, that $\mathbf{H}_k$ has full rank[1] N_k the singular value matrix $\mathbf{D}_k$ is partitioned as

$$\mathbf{D}_k = \begin{bmatrix} \tilde{\mathbf{D}}_k \\ \mathbf{0} \end{bmatrix} \tag{11}$$

with $\tilde{\mathbf{D}}_k$ an $N_k \times N_k$ diagonal matrix containing the non-zero singular values along the diagonal and zeros in rest, and left inverse

$$\mathbf{D}_k^\dagger = \begin{bmatrix} \tilde{\mathbf{D}}_k^{-1} & \mathbf{0} \end{bmatrix} \Leftrightarrow \mathbf{D}_k^\dagger\mathbf{D}_k = \mathbf{I}_{N_k} \tag{12}$$

Application of the SVD for the transformed channel matrix $\tilde{\mathbf{H}}_k$ to equation (9) yields

$$\tilde{\mathbf{r}} = \mathbf{U}_k\mathbf{D}_k\mathbf{V}_k^\top\mathbf{S}_k\mathbf{b}_k + \mathbf{w}_k \tag{13}$$

We pre-multiply by $\mathbf{U}_k^\top$

$$\mathbf{r}_k = \mathbf{U}_k^\top\mathbf{r} = \mathbf{D}_k\mathbf{V}_k^\top\mathbf{S}_k\mathbf{b}_k + \mathbf{U}_k^\top\mathbf{w}_k \tag{14}$$

and define $\tilde{\mathbf{S}}_k = \mathbf{V}_k^\top\mathbf{S}_k$ and $\tilde{\mathbf{w}}_k = \mathbf{U}_k^\top\mathbf{w}_k$. Thus

$$\mathbf{r}_k = \mathbf{D}_k\tilde{\mathbf{S}}_k\mathbf{b}_k + \tilde{\mathbf{w}}_k \tag{15}$$

[1]This is not a restriction since if $\mathbf{H}_k$ is not full rank then some dimensions the user k signal space will have zero projection on the output space. Therefore we can redefine a reduced codeword matrix $\mathbf{S}_k$ which uses only dimensions with nonzero projections on the output space.

and at this point we define an equivalent problem for user k by pre-multiplying with the left inverse of $\mathbf{D}_k$ and obtain

$$\tilde{\mathbf{r}}_k = \mathbf{D}_k^\dagger\mathbf{r}_k = \tilde{\mathbf{S}}_k\mathbf{b}_k + \tilde{\mathbf{z}}_k \tag{16}$$

which is identical in form with the equation of the received signal in [8], [9] and allows straightforward application of greedy interference avoidance to optimizing the codeword matrix $\tilde{\mathbf{S}}_k$. For the equivalent problem in equation (16) direct application of greedy interference avoidance [9] is possible and consists of replacing codeword m of user k, that is $\tilde{\mathbf{s}}_m^{(k)}$, by the minimum eigenvector of the corresponding interference-plus-noise covariance matrix

$$\mathbf{R}_m^{(k)} = \tilde{\mathbf{S}}_k\tilde{\mathbf{S}}_k^\top - \tilde{\mathbf{s}}_m^{(k)}\tilde{\mathbf{s}}_m^{(k)\top} + \tilde{\mathbf{D}}_k^{-2} \tag{17}$$

We formally state application of greedy interference avoidance for multiaccess vector channels below:

Algorithm 1: Greedy Interference Avoidance

1) Start with a randomly chosen codeword ensemble specified by user codeword matrices $\{\mathbf{S}_k\}_{k=1}^L$
2) For each user $k = 1 \ldots L$
   a) Define the equivalent problem in equation (16)
   b) adjust user k codewords sequentially by applying greedy interference avoidance: the codeword corresponding to symbol m of user k is replaced by the minimum eigenvector of the corresponding $\mathbf{R}_m^{(k)}$ in equation (17)
   c) Iterate previous step until convergence (making use of escape methods [8] if necessary)
3) Repeat step 2 until a fixed point is reached for which further modification of codewords will bring no additional improvement in sum capacity.

It has been shown [7] that the resulting codeword ensemble satisfies a simultaneous water filling solution and maximizes sum capacity in equation (4).

IV. ITERATIVE WATER FILLING

Iterative water filling provides an algorithm for computing optimal transmit covariance matrices for users in the multiuser system in equation (1) which maximize sum capacity in equation (2) subject to power constraints on users. The iterative water filling algorithm [16] is formally stated here:

Algorithm 2: Iterative Water Filling

1) Initialize user transmit covariance matrices $\{\mathbf{X}_k\}_{k=1}^L$
2) For each user $k = 1 \ldots L$
   a) Determine corresponding interference-plus-noise covariance matrix $\mathbf{Z}_k = \sum_{\ell=1,\ell\neq k}^{L} \mathbf{H}_\ell\mathbf{X}_\ell\mathbf{H}_\ell^\top + \mathbf{W}$
   b) Update user k transmit covariance matrix
   $$\mathbf{X}_k = \arg\max_{\mathbf{X}} \log|\mathbf{H}_k\mathbf{X}\mathbf{H}_k^\top + \mathbf{Z}_k|$$
3) Repeat step 2 until sum capacity converges.

As noted in [16] in every step of the iterative water filling algorithm each user selects for itself the best signaling direction as well as the optimal power allocation while regarding all the other users in the system as Gaussian noise. Thus, the maximization problem in step 2(b) of the algorithm involves also whitening of the interference-plus-noise seen by user k followed by SVD of the corresponding channel matrix to determine the optimal signaling directions as well as the corresponding amount of power for each of these directions obtained using the traditional single-user water filling procedure [3, p. 253].

V. A Parallel Between Interference Avoidance and Iterative Water Filling

First and foremost we note that interference avoidance is concerned in essence with obtaining optimal codewords which maximize the SINR for power-constrained users in a CDMA system, while iterative water filling provides an algorithm for computing optimal transmit covariance matrices which maximize sum capacity for Gaussian multiple access channels with vector inputs and a vector output under similar power constraints. Thus, the main difference between the two methods is that interference avoidance yields optimal codeword ensembles for CDMA systems, while iterative water filling provides optimal transmit covariance matrices. Therefore, interference avoidance should not be regarded as a substitute for iterative water filling, and when optimization of transmit covariance matrices is the ultimate goal then iterative water filling is the right choice. However, when an optimal ensemble of codewords to be used for transmission is desired, as it is the case in CDMA systems, then interference avoidance offers a viable alternative to iterative water filling. This is due to the fact that in this case iterative water filling must be followed by a procedure that yields the ensemble of codewords based on the optimal transmit covariance matrices provided by iterative water filling. Such procedures can be found in [4], [12]–[14].

Next we note that it has been shown that codewords yielded by interference avoidance are also optimal with respect to global measures of performance for multiuser systems like total squared correlation (TSC) or generalized TSC and sum capacity [1], [8], [9], [11]. The fact that interference avoidance algorithms yields codeword ensembles which are optimal with respect to these global criteria shows that a social optimum is achieved through local, self-interested action of individual users. From this perspective interference avoidance and iterative water are similar since in the case of iterative water filling an individual user is supposed to perform single-user water filling while regarding all other users as noise which shows also self-interested action.

Maximization of sum capacity is another aspect where interference avoidance encounters iterative water filling. In the case of greedy interference avoidance sum capacity maximization was observed as an emergent property, and the aggregate water filling of the signal space performed by the basic eigen-algorithm is a useful byproduct [9]. In contrast, iterative water filling was obtained as solution of the convex optimization problem of maximizing sum capacity subject to power constraints imposed on users [16]. However, since both interference avoidance and iterative water filling converge to a fixed point which corresponds to maximum sum capacity, it is natural to expect that the two different procedures exhibit close similarities. This is observed in the fact that application of greedy interference avoidance with vector channels leads to an ensemble of codewords that satisfies a simultaneous water filling solution, as shown by optimal user codeword covariance matrices $\mathbf{S}_\ell \mathbf{S}_\ell^\top$. We have performed simulations of both greedy interference avoidance and iterative water filling, and have observed that usually the covariances corresponding to optimal codeword matrices yielded by the greedy interference avoidance algorithm and optimal transmit covariance matrices obtained using iterative water filling are identical. We present the following numerical example for illustration: $L = 2$ users with powers $P_1 = 3$ and $P_2 = 4$, residing in signal spaces of dimensions $N_1 = 3$ and $N_2 = 4$ with receiver signal space of dimension $N = 5$, channel matrices

$$\mathbf{H}_1 = \begin{bmatrix} -0.4326 & 1.1909 & -0.1867 \\ -1.6656 & 1.1892 & 0.7258 \\ 0.1253 & -0.0376 & -0.5883 \\ 0.2877 & 0.3273 & 2.1832 \\ -1.1465 & 0.1746 & -0.1364 \end{bmatrix} \tag{18}$$

and

$$\mathbf{H}_2 = \begin{bmatrix} 0.1139 & 0.2944 & 0.8580 & -0.3999 \\ 1.0668 & -1.3362 & 1.2540 & 0.6900 \\ 0.0593 & 0.7143 & -1.5937 & 0.8156 \\ -0.0956 & 1.6236 & -1.4410 & 0.7119 \\ -0.8323 & -0.6918 & 0.5711 & 1.2902 \end{bmatrix} \tag{19}$$

and background noise with covariance matrix

$$\mathbf{W} = \begin{bmatrix} 4.7353 & -0.3664 & 0.5396 & -5.1288 & 0.5888 \\ -0.3664 & 2.0546 & -1.9778 & 0.8970 & -0.0618 \\ 0.5396 & -1.9778 & 4.6881 & 1.4089 & 1.4424 \\ -5.1288 & 0.8970 & 1.4089 & 9.5795 & -0.0061 \\ 0.5888 & -0.0618 & 1.4424 & -0.0061 & 1.0282 \end{bmatrix} \tag{20}$$

Greedy interference avoidance was run with initial codeword matrices selected randomly

$$\mathbf{S}_1 = \begin{bmatrix} 0.1078 & 0.5397 & 0.0597 \\ -0.4567 & -0.0656 & -0.4310 \\ -0.3363 & 0.1944 & -0.3795 \end{bmatrix} \tag{21}$$

and

$$\mathbf{S}_2 = \begin{bmatrix} 0.1556 & -0.1552 & 0.4162 & 0.0075 \\ -0.3331 & -0.0502 & -0.2189 & -0.3510 \\ 0.2739 & -0.2244 & 0.1382 & -0.3311 \\ 0.1995 & -0.4160 & 0.0988 & -0.1309 \end{bmatrix} \tag{22}$$

for which the resulting optimal codeword matrices are

$$\mathbf{S}_1^{\text{opt}} = \begin{bmatrix} 0.4020 & -0.6896 & -0.0273 \\ -0.6755 & 0.7186 & 0.4348 \\ -0.6182 & 0.0895 & 0.9001 \end{bmatrix} \tag{23}$$

and

$$\mathbf{S}_2^{\text{opt}} = \begin{bmatrix} -0.3931 & -0.4070 & -0.7215 & 0.8201 \\ -0.5766 & -0.4246 & 0.6544 & -0.3218 \\ 0.6928 & -0.0179 & -0.2153 & -0.4303 \\ 0.1816 & 0.8086 & 0.0701 & 0.1967 \end{bmatrix} \quad (24)$$

In terms of user codeword matrices the sum capacity of the multiaccess vector channel in equation (3) is given by equation (4) and for the optimal codeword matrices obtained this is equal to

$$C_s = 7.1668 \quad \text{[bits/channel use]} \quad (25)$$

Iterative water filling was run with initial transmit covariance matrices selected randomly

$$\mathbf{X}_1 = \begin{bmatrix} 1.8206 & -0.1710 & -0.7419 \\ -0.1710 & 0.2782 & 0.0264 \\ -0.7419 & 0.0264 & 0.9012 \end{bmatrix} \quad (26)$$

and

$$\mathbf{X}_2 = \begin{bmatrix} 0.5991 & 0.9466 & -0.4701 & 0.0350 \\ 0.9466 & 2.5526 & -0.5789 & 0.3313 \\ -0.4701 & -0.5789 & 0.5730 & -0.1682 \\ 0.0350 & 0.3313 & -0.1682 & 0.2753 \end{bmatrix} \quad (27)$$

for which the resulting optimal transmit covariance matrices are

$$\mathbf{X}_1^{\text{opt}} = \begin{bmatrix} 0.6380 & -0.7790 & -0.3348 \\ -0.7790 & 1.1617 & 0.8732 \\ -0.3348 & 0.8732 & 1.2003 \end{bmatrix} \quad (28)$$

and

$$\mathbf{X}_2^{\text{opt}} = \begin{bmatrix} 1.5133 & -0.3365 & -0.4627 & -0.2896 \\ -0.3365 & 1.0445 & -0.3943 & -0.4655 \\ -0.4627 & -0.3943 & 0.7119 & 0.0116 \\ -0.2896 & -0.4655 & 0.0116 & 0.7303 \end{bmatrix} \quad (29)$$

In terms of transmit covariance matrices the sum capacity of the multiaccess vector channel in equation (1) is given by equation (2) and the optimal transmit covariance matrices obtained imply the same value for sum capacity as in equation (25). We note that, while initially $\mathbf{X}_\ell \neq \mathbf{S}_\ell \mathbf{S}_\ell$, the optimal matrices $\mathbf{X}_\ell^{\text{opt}} = \mathbf{S}_\ell^{\text{opt}} \mathbf{S}_\ell^{\text{opt}}$, $\ell = 1, 2$. We also note that at the optimal fixed point user 1 occupies only two of the $N_1 = 3$ available signal dimensions and user 2 occupies only three of the $N_2 = 4$ available signal dimensions, as can be observed by examining the rank of covariance matrices

$$\text{rank}(\mathbf{X}_1^{\text{opt}}) = \text{rank}(\mathbf{S}_1^{\text{opt}} \mathbf{S}_1^{\text{opt}\top}) = 2 \quad (30)$$

$$\text{rank}(\mathbf{X}_2^{\text{opt}}) = \text{rank}(\mathbf{S}_2^{\text{opt}} \mathbf{S}_2^{\text{opt}\top}) = 3 \quad (31)$$

In terms of the number of ensemble iterations required to obtain the optimal matrices, the two algorithms require the same number of ensemble iterations. In the case of the particular example presented above after two ensemble iterations the maximum sum capacity point was practically achieved. As far as the complexity of an individual iteration is concerned, we note that both interference avoidance and iterative water filling involve the whitening of the interference-plus-noise seen by a given user followed by the SVD of the channel matrix. In addition, greedy interference avoidance requires also computation of minimum eigenvectors for codeword adaptation and may therefore seem slower and computationally more costly. However, this increase in computational complexity has no correspondent in the iterative water filling procedure, and a fair comparison should take into account the eventual step following iterative water filling in which codeword ensembles are obtained based on the optimal transmit covariance matrices [4], [12]–[14]. Such a comparison is beyond the scope of the current paper and will be the object of future investigations.

Finally we make an interesting observation involving the dimensionality of the subspaces in which users place their transmitted signals. This is given by the rank of the user transmit covariance matrix, but when the user employs multicode CDMA, this can also be observed in the number of codewords assigned to the user for transmission. When a given user ℓ is assigned a number of codewords $M_\ell \geq N_\ell$ then its codeword matrix may span the entire user ℓ signal space, otherwise, when $M_\ell < N_\ell$, user ℓ is restricted to a subspace of dimension at most M_ℓ. We note that usually water filling schemes do not place restrictions on the subspaces in which users can place their transmitted signal, and assume that the transmit covariance matrix has full rank and the user can water fill over its entire signal space. We also note that, by restricting the rank of user transmit covariance matrices, water filling may not occur over all signal dimensions with minimum noise energy. It has been pointed out to us that in such cases the iterative water filling procedure may not reach the maximum sum capacity point [15]. This is mainly because rank constraints are non-convex, and maximization of sum capacity subject to the usual power constraints on users *and additional rank constraints on user transmit covariance matrices* is no longer a convex optimization problem and does not enjoy the usual global convergence properties that characterize convex optimization problems [2]. Convergence to a fixed-point for both the eigen-algorithm for multiaccess vector channels and iterative water filling is ensured however by the monotonic increase in sum capacity.

We have performed simulations of both greedy interference avoidance and iterative water filling for cases in which users were restricted to lower dimensional subspaces for transmission, and have observed that the two algorithms have also converged to the same fixed point. For illustration we consider the same system in the previous example in which users are constrained to two dimensional subspaces. This implies that in the multicode CDMA approach users will employ $M_1 = M_2 = 2$ codewords for transmission, while in the iterative water filling procedure the rank of transmit covariance matrices will be constrained to be equal to 2. User powers are not changed to allow comparison with the sum capacity value achieved in the previous case.

Greedy interference avoidance was run with initial codeword matrices selected randomly

$$\mathbf{S}_1 = \begin{bmatrix} -1.2058 & 0.2539 \\ -0.1913 & 1.1642 \\ 0.0968 & -0.2831 \end{bmatrix} \quad (32)$$

and

$$\mathbf{S}_2 = \begin{bmatrix} 0.5179 & 0.2322 \\ 0.6640 & 0.2605 \\ 0.7818 & -1.1036 \\ -0.8244 & -0.8126 \end{bmatrix} \quad (33)$$

for which the resulting final codeword matrices are

$$\mathbf{S}_1^{\text{fin}} = \begin{bmatrix} 0.1488 & 0.7464 \\ -0.6119 & -0.9188 \\ -1.0504 & -0.3140 \end{bmatrix} \quad (34)$$

and

$$\mathbf{S}_2^{\text{fin}} = \begin{bmatrix} 1.0889 & -0.7750 \\ -0.8757 & -0.7139 \\ -0.0649 & 0.6802 \\ 0.2081 & 0.6535 \end{bmatrix} \quad (35)$$

and imply sum capacity computed using equation (4)

$$C_s = 7.0757 \quad \text{[bits/channel use]} \quad (36)$$

which is smaller than in the case when users were not rank constrained.

With rank constrained transmit covariance matrices iterative water filling was run with random, rank constrained, initial transmit covariance matrices

$$\mathbf{X}_1 = \begin{bmatrix} 1.8173 & -0.4152 & -0.8685 \\ -0.4152 & 0.1472 & 0.0183 \\ -0.8685 & 0.0183 & 1.0356 \end{bmatrix} \quad (37)$$

and

$$\mathbf{X}_2 = \begin{bmatrix} 1.7623 & 0.5689 & -0.3806 & 0.6709 \\ 0.5689 & 0.8201 & 0.2461 & 0.9591 \\ -0.3806 & 0.2461 & 0.2960 & 0.2855 \\ 0.6709 & 0.9591 & 0.2855 & 1.1215 \end{bmatrix} \quad (38)$$

and yielded

$$\mathbf{X}_1^{\text{fin}} = \begin{bmatrix} 0.5793 & -0.7769 & -0.3912 \\ -0.7769 & 1.2189 & 0.9319 \\ -0.3912 & 0.9319 & 1.2019 \end{bmatrix} \quad (39)$$

and

$$\mathbf{X}_2^{\text{fin}} = \begin{bmatrix} 1.7878 & -0.4054 & -0.5961 & -0.2772 \\ -0.4054 & 1.2777 & -0.4270 & -0.6480 \\ -0.5961 & -0.4270 & 0.4652 & 0.4295 \\ -0.2772 & -0.6480 & 0.4295 & 0.4692 \end{bmatrix} \quad (40)$$

and imply sum capacity computed using equation (2) equal to that in equation (36). Codeword covariances are also equal to transmit covariances, as it was the case when users were not rank constrained.

We note that, whether the fixed point to which the two algorithms converge in the case when users are rank constrained, corresponds to maximum sum capacity under the given constraints is an open question whose answer is contingent upon solving the more general problem of maximizing sum capacity subject to power *and* rank constraints imposed on users.

VI. Conclusions

In this paper we presented an analysis of the relationship between interference avoidance and iterative water filling in multiaccess vector channels. The two procedures converge to fixed points where sum capacity is maximized, and which have similar properties. As main difference between the two methods we note that interference avoidance is a codeword adaptation method and yields optimal codeword ensembles for CDMA systems, while iterative water filling provides an algorithm for computing optimal transmit covariance matrices.

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