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A Three-dimensional Time-domain Boundary Element Method for the Computation of Exact Green's Functions in Acoustic Analogy

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A three-dimensional time-domain Boundary Element Method is formulated for solving exact Green's functions numerically. The use of triangular boundary elements and interior collocation points allows application of the method to complex boundaries. A March-On-Time scheme is used to determine exact Green's functions at several frequencies in a single calculation. Long-time stability is accomplished via a modified Burton-Miller boundary integral equation. A Fast Fourier Transform of the time-dependent numerical solution is used to compare numerical results with the exact solution for a sphere in the frequency domain. The effects of changing two boundary integral equation parameters and the time step size are discussed. By decreasing the time step size, the numerical solution becomes more accurate until a minimum value that depends on the frequency is reached. With proper parameter choices and time step size, the modified Burton-Miller time-domain Boundary Element Method produces accurate and stable numerical results.

I. Introduction

A Green's function that satisfies all the boundary conditions is often referred to as the exact or the tailored Green's function. It is well known in the theory of acoustic analogy that when an exact Green's function is used, the far field sound can be expressed as a volume integral involving the Lighthill tensors and the double divergence of the Green's function. The use of the exact Green's function allows the development of noise prediction strategy based on steady flow simulations with direct noise source modelings.[1] The computation of the exact Green's function, as well as its double divergence, becomes an integral part of this noise prediction strategy.

In earlier studies, a spectral collocation boundary element method for solving the convective wave equation in the frequency domain with a constant mean flow was developed for arbitrary two-dimensional bodies.[2,3] In this paper, we present a time-domain Boundary Element Method for the computation of exact Green's functions for general three-dimensional surfaces. The extension of exact Green's function computation to three-dimensional geometries is by no means trivial. The main challenge is achieving the computational efficiency necessary for routine use of the exact Green's function as a practical tool. This involves overcoming the computational difficulty that results from the extremely large degrees of freedom at mid to high frequencies of typical airframe acoustic radiation when the body of the entire aircraft, or a large component of which, is included. The time-domain approach has several advantages. First, the inversion of a large, dense matrix is not required. Next, exact Green's functions at all frequencies are contained in a single computation. Finally, the computational complexity can be reduced to $O(N_t N \log^2 N)$ where N_t is the number of time steps and N is the total number of spatial basis functions, making the method feasible for routine application at mid to high frequencies.[4,5]

The time-domain boundary integral equation will be solved using a spectral collocation method on triangular boundary elements. The use of triangular elements makes our method flexible and easily applicable

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