You have 90 minutes to answer the following 5 (five) questions on 2 (two) pages. Show all your work. There is a total of 175 points on the test (including 10 extra credit points).

1. (20 + 20 = 40 points)
   For each of the following matrices, find the determinant
   
   \[
   \begin{bmatrix}
   2 & 0 & 2 & 2 \\
   1 & 0 & 2 & 2 \\
   1 & 2 & 2 & 2 \\
   1 & 0 & 1 & 2 \\
   \end{bmatrix}
   \quad \begin{bmatrix}
   1 & 9 & 8 & 7 \\
   0 & 2 & 9 & 6 \\
   0 & 0 & 3 & 5 \\
   0 & 0 & 0 & 4 \\
   \end{bmatrix}
   \]

2. (25 points)
   Use the determinant to find out for which values of the constant \( k \) the following matrix is invertible
   
   \[
   \begin{bmatrix}
   0 & 1 & k \\
   2 & 2k & 5 \\
   4 & 9 & 15 \\
   \end{bmatrix}
   \]
3. (20 + 20 = 40 points)
   For each of the following matrices
   a) Find all the eigenvalues and eigenvectors
   b) Determine whether an eigenbasis exists

   $\begin{bmatrix} 3 & -2 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 2 \end{bmatrix}$

   $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

4. (20 + 20 = 40 points)
   For each of the following $A$ matrices
   a) Determine if $A$ is diagonalizable
   b) If so, find an invertible $S$ matrix and diagonal $D$ matrix such that $D = S^{-1}AS$

   $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

   $\begin{bmatrix} 3 & -4 & 0 \\ 2 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. (20 + 10 = 30 points)
   Given the matrix

   $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

   Find a formula for $A^t$, where $t$ is a positive integer

   (Extra Credit) Express $A^t$ in terms of $A$