MATH 162 Test 1 Spring 2001 NAME:

Problems 1-10 (multiple choice) worth 6 points each, problems 11-14 are 10pts each. For problems 1-10, circle the correct answers.

1. Express the statement "The quotient of p and q is at most 7".
   a $\frac{p}{q} \geq 7$
   b $\frac{p}{q} < 7$
   • c $\frac{p}{q} \leq 7$
   d $\frac{p}{q} > 7$
2. Rewrite the expression, $|2 - x|$ if $x < 2$ without using the absolute value.
   a $x - 2$
   • b $2 - x$
   c $-x - 2$
   d none of the above
3. Rewrite $\sqrt{x^2 + y^2}$ using the rational exponent.
   a $x + y$.
   • b $(x^2 + y^2)^{1/2}$
   c $\sqrt{x} + \sqrt{y}$
   d $\frac{x^2+y^2}{2}$
4. Simplify $(-2xy^2)^5 \left(\frac{x^7}{8y^3}\right)$.
   a $4x^{12}y^7$
   b $-4x^{35}y^7$
   c $\frac{-4x^{12}}{y^7}$
   • d $-4x^{12}y^7$
5. Simplify the expression and rationalize the denominator of $\sqrt[4]{\frac{5x^8y^3}{27x^2}}$.
   a $\frac{5x}{3}\sqrt[4]{15x^2y^3}$
   • b $\frac{x}{3}\sqrt[4]{15x^2y^3}$
   c $\frac{1}{3}\sqrt[4]{15x^6y^3}$
   d $\frac{5}{3}\sqrt[4]{15x^6y^3}$
6. Express $(x^2 + 2y)(x^2 - 2y)$ as a polynomial.
   a $x^4 - 4x^2y - 4y^2$
   • b $x^4 - 4y^2$
   c $x^4 + 4y^2$
   d $2x^2 - 4y^2$

7. Factor $64x^3 + 27$.
   a $(4x + 3)(4x^2 - 12x + 9)$
   b $(4x - 3)(16x^2 - 12x + 9)$
   c $(4x - 3)(4x^2 + 12x + 9)$
   • d $(4x + 3)(16x^2 - 12x + 9)$
8. Rationalize the numerator of $\frac{\sqrt{x+h}-\sqrt{x}}{h}$, $(h \neq 0)$.
   a $\frac{h}{\sqrt{x+h}-\sqrt{x}}$
   • b $\frac{1}{\sqrt{x+h}+\sqrt{x}}$
   c $\frac{\sqrt{h}}{h}$
   d $\frac{h}{\sqrt{x+h}-\sqrt{x}}$
9. Simplify the expression $\frac{x+2}{2x-3} \div \frac{x^2-4}{2x^2-3x}$.
   a $\frac{x+2}{x-2}$
   b $\frac{1}{2x-3}$
   • c $\frac{x}{x-2}$
   d $\frac{x}{2x-3}$
10. Given the formula for the area of a trapezoid, $A = \frac{1}{2}(b_1 + b_2)h$, solve the equation for b_1.
   • a $b_1 = \frac{2A-b_2h}{h}$
   b $b_1 = \frac{A-b_2h}{h}$
   c $b_1 = \frac{2A-b_2}{h}$
   d $b_1 = \frac{2A-b_2h}{2h}$
11. With water from one hose, a swimming pool can be filled in 8 hours. A second, larger hose used alone can fill the pool in 5 hours. How long would it take to fill the pool if both hoses were used simultaneously.

    Solution Let t be the number of hours for both hoses to fill the pool. Then

$$\frac{1}{t} = \frac{1}{8} + \frac{1}{5}.$$

Hence $\frac{1}{t} = 13/40$ and $t = 3\frac{1}{13} hrs$.

12. Solve the following equation for x, $\frac{2x}{x+3} + \frac{5}{x} - 4 = \frac{18}{x^2+3x}$

Solution

$$\frac{2x}{x+3} + \frac{5}{x} - 4 = \frac{18}{x^2+3x}$$
$$x(x+3)(\frac{2x}{x+3} + \frac{5}{x} - 4) = \frac{18}{x^2+3x}x(x+3)$$
$$2x^2 + 5(x+3) - 4x(x+3) = 18$$
$$2x^2 + 7x + 3 = 0$$
$$(2x+1)(x+3) = 0.$$

Now $x = -3$ does not satisfy the original equation, hence the solution is $x = -\frac{1}{2}$.

13. Simplify $\frac{\frac{5}{x+1}+\frac{2x}{x+3}}{\frac{x}{x+1}+\frac{7}{x+3}}$

Solution

$$\frac{\frac{5}{x+1} + \frac{2x}{x+3}}{\frac{x}{x+1} + \frac{7}{x+3}} = \frac{(\frac{5}{x+1} + \frac{2x}{x+3})(x+1)(x+3)}{(\frac{x}{x+1} + \frac{7}{x+3})(x+1)(x+3)}$$
$$= \frac{5(x+3) + 2x(x+1)}{x(x+3) + 7(x+1)}$$
$$= \frac{2x^2 + 7x + 15}{x^2 + 10x + 7}$$

14.

a. Find the solutions of $x^3 - 27 = 0$.
b. Write $\frac{2+9i}{1-i}$ in the form $a + bi$.

Solutions (a) $x^3 - 27 = (x-3)(x^2+3x+9)$, so $x = 3$ and using the quadratic formula, the solutions for $x^2 + 3x + 9 = 0$ are $\frac{-3 \pm 3\sqrt{3}i}{2}$.

(b) $\frac{2+9i}{1-i} = \frac{(2+9i)(1+i)}{1-i)(1+i)} = \frac{-7+11i}{2}$.

Bonus A couple does not wish to spend more than \$70 for dinner at a restaurant. If a sales tax of 6% is added to the bill and they plan to tip 15% after the tax has been added, what is the most they can spend for the meal?

Solution Let $\$x$ be the price of the meal. Then $meal + tax = x + 0.06x = 1.06x$ and the $tip = 0.15(1.06x)$. In total

$$1.06x + 0.15(1.06x) = \$70,$$

or $1.15(1.06)x = \$70$, solving for x, $x \simeq 57.37$.