

MATH 211 Test 2 Fall '01 NAME:

Show your work to receive credit.

1.(40pts) Differentiate the following functions.

- (i) $f(x) = x^4 + \frac{1}{\sqrt{x}}$
- (ii) $g(x) = t^2 \sin t - \tan t$
- (iii) $f(x) = \frac{x^2-1}{x^2+1}$
- (iv) $h(t) = (1+t^2)^6$
- (v) $h(x) = \ln(\cos x) + 2^x$
- (vi) $f(t) = \frac{\sqrt{1-t^2}}{t}$
- (vii) $g(x) = \tan^{-1}(x^3)$
- (viii) $h(t) = xe^{\frac{1}{x}}$

2.(10pts) Using the implicit differentiation, find $\frac{dy}{dx}$, then use it to find an equation of the tangent line to $\frac{x^2}{9} + \frac{y^2}{36} = 1$ at $(-1, 4\sqrt{2})$.

3.(15pts) Find the following limits.

- (i) $\lim_{x \rightarrow 0^+} x \ln x$
- (ii) $\lim_{h \rightarrow 0} \frac{\sin 3h}{\sin 5h}$
- (iii) $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$

4.(25pts) Graph $f(x) = \frac{x^2}{x^2+1}$ by answering the following parts.

- (i) Verify that $f'(x) = \frac{2x}{(x^2+1)^2}$.
- (ii) Classify the regions according to increasing or decreasing of f .
- (iii) Find all the local extrema of f .
- (iv) Verify that $f''(x) = \frac{2(1-3x^2)}{(x^2+1)^3}$.
- (v) Classify the regions according to the concavities of f .
- (vi) Find the inflection point(s) of f .
- (vii) Find all the asymptotes of f .
- (vi) Graph f .

5.(10pts) Air is being pumped into a spherical balloon at the rate of $5 \text{ in}^3/\text{sec}$. At what rate is the radius of the balloon increasing when the radius r is 9 inches.