MATH 212 Quiz 3 NAME:

1. Find the area of the surface obtained by rotating the curve $y = \sin x$ about the x-axis over $[0, \pi]$.

$$\int_0^{\pi} 2\pi \sin x \sqrt{1+\cos^2 x}\, dx = \int_{-1}^{1} 2\pi \sqrt{1+u^2} du$$

$$u = \cos x, \quad du = -\sin x dx$$

$$= 2\pi \int_{-\pi/4}^{\pi/4} \sqrt{1+\tan^2\theta}\sec^2\theta d\theta, \qquad u = \tan\theta$$

$$= 2\pi\{\frac{1}{2}[\sec\theta\tan\theta + \ln|\sec\theta+\tan\theta]\}|_{-\pi/4}^{\pi/4}$$

$$= \pi\{2\sqrt{2} + \ln|\frac{\sqrt{2}+1}{\sqrt{2}-1}|\} = 2\pi\{\sqrt{2}+\ln|\sqrt{2}+1|\}.$$

2. Eliminate the parameter to find the Cartesian equation of the curve $x = \cos t$ and $y = \cos 2t$. Sketch the curve and indicate with an arrow the direction in which the curve is traced.

Since $y = \cos 2t = 2\cos^2 t - 1$, $y = 2x^2 - 1$. Note that as $-1 \le \cos t \le 1$, $-1 \le x \le 1$. Hence the curve is the parabola $y = 2x^2 - 1$ where $-1 \le x \le 1$.

3. Given $x = t^4 - 1$ and $y = t - t^2$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-2t}{4t^3}.$$

Also

$$\frac{dy}{dx} = \frac{\frac{dy/dx}{dt}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}(\frac{1-2t}{4t^3})}{4t^3} = -\frac{3}{16t^7} + \frac{1}{4t^6}.$$