MATH 212 Quiz 5 NAME:

1. Determine the convergence or divergence of the following sequences. If it converges, find its limit.

   (a) $\{\sqrt{n+5}-\sqrt{n}\}$

   •

$$\begin{aligned}\lim_{n\to\infty}\sqrt{n+5}-\sqrt{n} &= \lim_{n\to\infty}\sqrt{n+5}-\sqrt{n}\left(\frac{\sqrt{n+5}+\sqrt{n}}{\sqrt{n+5}+\sqrt{n}}\right)\\ &= \lim_{n\to\infty}\frac{5}{\sqrt{n+5}+\sqrt{n}}\\ &= 0.\end{aligned}$$

   (b) $\{\frac{\sqrt{n}+1}{n+5}\}$

   • $\lim_{n\to\infty}\frac{\sqrt{n}+1}{n+5}=0$ by the L'Hôpital rule.

2. Test for convergence of the following series. You must provide a complete proof for each.

   (a) $\sum_{n=1}^{\infty}8^{-n}5^{n+2}$

   • The series converges by the geometric series with $r=\frac{5}{8}$.

   (b) $\sum_{n=1}^{\infty}\frac{1}{n^2+4n+8}$

   • The series converges by the direct comparison test, compared with $\sum_{n=1}^{\infty}\frac{1}{n^2}$ and

$$\frac{1}{n^2+4n+8}<\frac{1}{n^2}$$

.

   (c) $\sum_{n=1}^{\infty}\frac{1+2^n}{5+2^n}$

   • The series diverges by the test for divergence, since $\lim_{n\to\infty}\frac{1+2^n}{5+2^n}=1\neq 0$.

   (d) $\sum_{n=1}^{\infty}(-1)^n\frac{1}{\ln(n+1)}$

   • The series converges by the alternating series test.