

MATH 212 Quiz 6 NAME:

1. Test for convergence or divergence of the following series.

(a) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

• The series diverges by the limit comparison test, compared with $\sum_{n=1}^{\infty} \frac{1}{n}$, since $\lim_{n \rightarrow \infty} \frac{\frac{n^2+1}{n^3+1}}{\frac{1}{n}} = 1$ and this limit is finite and positive.

(b) $\sum_{n=1}^{\infty} \frac{3^n}{5^n+n}$

• The series converges by the Direct Comparison Test, compared with $\sum_{n=1}^{\infty} \frac{3^n}{5^n}$ as $\frac{3^n}{5^n+n} < \frac{3^n}{5^n}$.

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$

• The series converges by the Direct Comparison Test, compared with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (a convergent p series) as $\frac{1}{n^2+n} < \frac{1}{n^2}$.

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$

• The series converges by the Alternating Series Test.

(e) $\sum_{n=1}^{\infty} e^{-n} n!$

• The series diverges by the Ratio Test, as $\lim_{n \rightarrow \infty} \left| \frac{e^{-(n+1)}(n+1)!}{e^{-n}n!} \right| = \infty$.

2. Find the radius of convergence and interval of convergence of the series, $\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$.

• $\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{\ln(n+1)} \frac{\ln n}{x^n} \right| = |x|$. By Ratio test, the power series converges for $|x| < 1$. Radius= 1. Check the endpoints convergence. For $x = -1$, $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ is convergent by Alt. Ser. Test, whereas for $x = 1$, $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ diverges as $\frac{1}{\ln x} > \frac{1}{n}$ and apply D.C.T. Hence the interval of convergence is $[-1, 1)$.