

Sample Final B

7. The circle $r = 3 \cos \theta$ and the cardioid $r = 1 + \cos \theta$ intersect at $\theta = -\frac{\pi}{3} = \frac{\pi}{3}$. Hence

$$\begin{aligned} \text{Area} &= \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} [3 \cos \theta]^2 - [1 + \cos \theta]^2 d\theta \\ &= \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 8 \cos^2 \theta - 2 \cos \theta - 1 d\theta \\ &= \frac{1}{2} \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 8 \left(\frac{1 + \cos 2\theta}{2} \right) - 2 \cos \theta - 1 d\theta \\ &= \pi. \end{aligned}$$

8. As $y = t^2 + 3t + 1$ and $x = \ln(2t + 1)$, $\frac{dy}{dt} = 2t + 3$ and $\frac{dx}{dt} = \frac{2}{2t+1}$. Hence $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{2}(2t + 3)(2t + 1)$. When $t = 1$, $\frac{dy}{dx} = \frac{15}{2}$.
 $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \frac{dy}{dx}}{\frac{dx}{dt}} = 2(t + 1)(2t + 1)$. When $t = 1$, $\frac{d^2y}{dx^2} = 12$.