MATH 212 Test 1-(Re-take) Spring 2001

1.(84pts) Evaluate the following integrals.

i. $\int xe^{2x}\,dx$

Let $u = x$ and $dv = e^{2x}dx$ so that $du = dx$ and $v = \frac{1}{2}e^{2x}$. By integration by parts,

$$\begin{aligned}\int xe^{2x}\,dx &= \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}dx \\ &= \frac{1}{2}xE^{2x} - \frac{1}{4}e^{2x} + C.\end{aligned}$$

ii. $\int \sin^2 x\,dx$

$$\begin{aligned}\int \sin^2 x\,dx &= \int \frac{1-\cos 2x}{2}\,dx \\ &= \frac{1}{2}\{x - \frac{1}{2}\sin 2x\} + C.\end{aligned}$$

iii. $\int x\sqrt{x^2+16}\,dx$

Let $u = x^2 + 16$ so that $du = 2xdx$. Thus

$$\begin{aligned}\int x\sqrt{x^2+16}\,dx &= \frac{1}{2}\int \sqrt{u}du \\ &= \frac{1}{3}u^{3/2} + C = \frac{1}{3}(x^2+16)^{3/2} + c.\end{aligned}$$

iv. $\int \frac{3x+5}{x(x-1)}\,dx$

By partial fractions, $\frac{3x+5}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$, from which $A = -5$ and $B = 8$. Hence,

$$\begin{aligned}\int \frac{3x+5}{x(x-1)}\,dx &= \int \frac{-5}{x} + \frac{8}{x-1}dx \\ &= -5\ln|x| + 8\ln|x-1| + C.\end{aligned}$$

v. $\int \sin^5 x\cos^3 x\,dx$

$$\begin{aligned}\int \sin^5 x\cos^3 x\,dx &= \int \sin^5 x\cos^2 x\cos x\,dx \\ &= \int \sin^5 x(1-\sin^2 x)\cos x\,dx \\ &= \int u^5(1-u^2)du, \qquad u = \sin x \\ &= \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C.\end{aligned}$$

vi. $\int_0^\infty e^{-x}dx$

$$\begin{aligned}\int_0^\infty e^{-x}dx &= \lim_{b\to\infty}\int_0^b e^{-x}dx\\ &= \lim_{b\to\infty}(-e^{-x})|_0^b\\ &= \lim_{b\to\infty}(-e^{-b}+1) = 1.\end{aligned}$$

vii. $\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

$$\begin{aligned}\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx &= \int_0^{\sec^{-1}2} \frac{\sqrt{\sec^2\theta-1}}{\sec\theta}\sec\theta\tan\theta\,d\theta\\ &\quad x=\sec\theta \text{so that } dx=\sec\theta\tan\theta d\theta\\ &= \int_0^{\sec^{-1}2} \tan^2\theta d\theta\\ &= \int_0^{\sec^{-1}2} \sec^2\theta - 1 d\theta\\ &= \tan\theta - \theta|_0^{\sec^{-1}2}\\ &= \sqrt{3}-\frac{\pi}{3}.\end{aligned}$$

2. (16pts)

Consider the region in the first quadrant that is bounded by $y = x^2$ and $y = x^3$. Set up the integrals (do not evaluate) that compute the volume of the solid obtained when the region is rotated about

a. the x-axis.

$$V = \int_0^1 \pi((x^2)^2 - (x^3)^2)dx. \qquad \text{Solids of revolution}$$

b. the y-axis.

$$V = \int_0^1 2\pi x(x^2 - x^3)dx. \qquad \text{Cylindrical shell}$$

a. the line $y = 3$

$$V = \int_0^1 \pi[(3-x^3)^2 - (3-x^2)^2]dx. \qquad \text{Solids of revolution}$$

b. the line $x = -2$.

$$V = \int_0^1 2\pi(x+2)(x^2 - x^3)dx. \qquad \text{Cylindrical shell}$$