MATH 316 Quiz 5 NAME:

1.(3pts) Solve

$$5x_1 - 2x_2 = 1$$
$$-x_1 - x_2 = -3$$

by Cramer's rule.

2.(3pts) Consider the linear transformation $T(\vec{x}) = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \vec{x}$ from R^2 to R^2. Let Ω denote the unit circle.

a. Sketch the image $T(\Omega)$.

b. Verify the identity that $area(T(\Omega)) = |det \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}| area(\Omega)$.

3.(2pts)

a. Let $A \in R^{n \times n}$ be such that both A and A^{-1} have integer entries. What are the possible values of $det(A)$?

b. Let $A \in R^{n \times n}$ be such that $det(A) = 4$. What is $det(A^T A)$?

4.(2pts) Find the determinant of the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$