MATH 316 Test 1 Spring '02 NAME:

Show your work to receive credit.

1.(10pts) Find all solutions x_1, x_2, x_3, x_4 and x_5 of the linear system

$$\left|\begin{array}{llll} x_2 & +4x_4 & +x_5 & =0 \\ & -3x_4 & +6x_5 & =0 \end{array}\right|$$

2.(10pts) Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 5 \end{bmatrix}$,

a. Find the rank of A.

b. Can you express $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ as a linear combination of the columns of A? Explain.

3.(10pts) Find all vectors in R^3 that are perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$.

4.(15pts) Let L be a line in R^2 that consists of all scalar multiples of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

a. Find the orthogonal projection of $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ onto L.

b. Find the reflection of $\begin{bmatrix} 4 \\ 4 \end{bmatrix}$ about L.

c. Find the matrix of the orthogonal projection onto L.

5.(10pts)

a. For which choices of the constant k is the matrix $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}$ invertible?

b. For which choices of the constant k are all entries of $\begin{bmatrix} 2 & 3 \\ 5 & k \end{bmatrix}^{-1}$ integers?

6.(5pts) True or False?

a. There is a 4×5 matrix with rank 5.

b. There is a 2×2 matrix A such that $A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $A \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

c. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are nonzero vectors in R^2, then $\vec{w}$ is a linear combination of $\vec{u}$ and $\vec{v}$.

d. If A and B are invertible, then $A + B$ is invertible as well.

e. The function $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 1 \end{bmatrix}$ is a linear transformation.