MATH 316 Test 1 Summer '02 NAME:

Show your work to receive credit.

1.(10pts) Find all solutions x_1, x_2, x_3 and x_4 of the linear system

$$\left|\begin{array}{llll|l} x_1 & +4x_2 & +x_3 & & = 0 \\ & -3x_2 & & +6x_4 & = 0 \\ x_1 & & +4x_3 & & = 0 \end{array}\right|$$

2.(10pts) Find the reduced row echelon form for the following matrix A and specify its rank.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -1 \\ 1 & 2 & 0 & 0 & 1 & -1 \\ 1 & 2 & 2 & 0 & 1 & 1 \end{bmatrix}$$

3.(10pts) Find all vectors in R^3 that are perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \\ 4 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}$.

3.(10pts) Is it possible to write $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -4 \\ 0 \end{bmatrix}$?

Show your work.

4.(10pts) Let L be a line in R^3 that consists of all scalar multiples of $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

a. Find the orthogonal projection of $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ onto L.

b. Find the reflection of $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ about L.

c. Find the matrix of the reflection onto L.

5.(10pts) Given $T(\vec{v}) = A(\vec{v})$, give a geometric interpretation of this linear transformation for each of the following A below. Also find A^{-1} if it exists.

a. $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

b. $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

Bonus(5pts) True or False?

a. A system of three linear equations in two unknowns is always inconsistent.

b. There is a system of three linear equations in three unknowns that has exactly three solutions.

c. Let A be a 5×4 matrix. Then the rank of A is always less than or equal to 4.

d. The function $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ 3x_1 + x_2 \end{bmatrix}$ is a linear transformation of R^2 into R^2.

e. Matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ has its inverse.