1. (10pts) Find all solutions $x_1, x_2, x_3$ and $x_4$ of the linear system

\[
\begin{align*}
    x_1 + 4x_2 + x_3 &= 0 \\
    -3x_2 + 6x_4 &= 0 \\
    x_1 + 4x_3 &= 0
\end{align*}
\]

2. (10pts) Find the reduced row echelon form for the following matrix $A$ and specify its rank.

\[
A = \begin{bmatrix}
    0 & 0 & 1 & 2 & -1 \\
    1 & 2 & 0 & 1 & -1 \\
    1 & 2 & 0 & 1 & 1
\end{bmatrix}
\]

3. (10pts) Find all vectors in $\mathbb{R}^3$ that are perpendicular to

\[
\begin{bmatrix}
    1 \\
    1 \\
    1
\end{bmatrix},
\begin{bmatrix}
    2 \\
    3 \\
    4
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
    0 \\
    -1 \\
    2
\end{bmatrix}.
\]

4. (10pts) Is it possible to write

\[
\begin{bmatrix}
    1 \\
    3
\end{bmatrix}
\]

as a linear combination of

\[
\begin{bmatrix}
    -1 \\
    0 \\
    2
\end{bmatrix},
\begin{bmatrix}
    0 \\
    3 \\
    0
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
    2 \\
    -4 \\
    0
\end{bmatrix}.
\]

Show your work.

5. (10pts) Let $L$ be a line in $\mathbb{R}^3$ that consists of all scalar multiples of

\[
\begin{bmatrix}
    2 \\
    1 \\
    1
\end{bmatrix}.
\]

a. Find the orthogonal projection of

\[
\begin{bmatrix}
    2 \\
    2 \\
    2
\end{bmatrix}
\]

onto $L$.

b. Find the reflection of

\[
\begin{bmatrix}
    2 \\
    2 \\
    2
\end{bmatrix}
\]

about $L$.

c. Find the matrix of the reflection onto $L$.

5. (10pts) Given $T(\vec{v}) = A(\vec{v})$, give a geometric interpretation of this linear transformation for each of the following $A$ below. Also find $A^{-1}$ if it exists.

a. $A = \begin{bmatrix}
    1 & 0 \\
    0 & 3
\end{bmatrix}$

b. $A = \begin{bmatrix}
    2 & -1 \\
    1 & 2
\end{bmatrix}$

Bonus (5pts) True or False?

a. A system of three linear equations in two unknowns is always inconsistent.
b. There is a system of three linear equations in three unknowns that has exactly three solutions.

c. Let $A$ be a $5 \times 4$ matrix. Then the rank of $A$ is always less than or equal to 4.

d. The function $T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ 3x_1 + x_2 \end{bmatrix}$ is a linear transformation of $\mathbb{R}^2$ into $\mathbb{R}^2$.

e. Matrix $\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ has its inverse.