MATH 316 Test 2- Fall, 2001

Show your work to receive credit.

1.(15pts) Consider the linear transformation $T\colon R^5 \to R^4$ given by the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 & 3 & 2 \\ 1 & 6 & 6 & 6 & 6 \\ 1 & 7 & 8 & 10 & 12 \\ 1 & 6 & 6 & 7 & 8 \end{bmatrix}.$$

It is given that the corresponding reduced row echelon form of A is

$$rref(A) = \begin{bmatrix} 1 & 0 & -6 & 0 & 6 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Find bases for the kernel of A, $ker(A)$, and for the image of A, $im(A)$.

(b) Does $\begin{bmatrix} 3 \\ 1 \\ -2 \\ 0 \end{bmatrix}$ belong to $im(A)$? Explain your answer.

2.(15pts) Decide which of the sets W are subspaces of V. Explain your answers in full. In the case of a subspace, find a basis for W.

(a) $W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x = y, z = 0 \right\}$, $V = R^3$.

(b) $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a \geq 0, d \geq 0 \right\}$, $V = R^{2\times 2}$.

(c) $W = \{p(t)\colon p(1) = 0\}$, $V = P_2$.

3.(10pts) Verify Rank-Nullity Theorem for the linear transformation $T\colon R^3 \to R^3$ defined by $T\vec{x} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \vec{x}$.

4.(10pts) Find the matrix of linear transformation

$$T(\vec{x}) = \begin{bmatrix} 7 & -1 \\ -6 & 8 \end{bmatrix} \vec{x}$$

with respect to the basis $B_T = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$.

5.(15pts) TRUE or FALSE? (**Extra Credits (10 points maximum:** 2 points each for proving a true statement and 2 points each for providing a counterexample for a false statement.)

(a) If $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent vectors, then $\vec{v}_1, \vec{v}_2$ are also linearly independent vectors.

(b) For invertible matrices A and B, $(A+B)^{-1} = A^{-1} + B^{-1}$.

(c) Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ are linearly independent vectors in V and $T\colon V \to W$ is a linear transformation. Then $T(\vec{v}_1), T(\vec{v}_2), \ldots, T(\vec{v}_n)$ are linearly independent vectors in W.

(d) If A is an invertible matrix, then A^2 is also invertible and $(A^2)^{-1} = (A^{-1})^2$.

(e) If $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n$ span R^4, then n must be equal to 4.