MATH 316 Test 2 Spring '02 NAME:

Show your work to receive credit.

1.(10pts) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$. Find a basis for the image of A, $im(A)$, and a basis for the kernel of A, $ker(A)$.

2.(10pts) Verify that

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + y - 2z = 0 \right\}$$

is a subspace of R^3. Also, specify a basis for W.

3.(10pts) Find all 2×2 matrices that commute with $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$.

4.(15pts)

a. Give an example of 2×3 matrix A such that $im(A)$ is spanned by $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

b. Give an example of 3×3 matrix A such that $ker(A)$ is spanned by $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$.

c. Describe the image and the kernel of the orthogonal projection onto the plane $x + y + z = 0$ in R^3.

4.(10pts)

a. Show that $\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix} \}$ are linearly dependent.

b. Show that $\{ \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 9 \end{bmatrix} \}$ do not span R^3.

5.(10pts) Find the coordinate vector $[\vec{x}]_{\mathcal{B}}$ for $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ with respect to $\mathcal{B} = \{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \}$.

Bonus (10pts) Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m$ be linearly dependent vector in R^n and let $T\colon R^n \to R^p$ be a linear transformation. Show that $T(\vec{v}_1), T(\vec{v}_2), \ldots, T(\vec{v}_m)$ are linearly dependent.