MATH 316 Test 3- Fall, 2001

Show your work to receive credit.

1.(15pts) Let $T: U^{2\times 2} \to U^{2\times 2}$, where $U^{2\times 2}$ is the space of all 2×2 upper triangular matrices, be defined by

$$T(M) = M \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} M.$$

(i) Show that T is a linear transformation.

(ii) Is it an isomorphism?

(iii) Find the matrix of T with respect to the basis $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$.

2.(10pts) Show that $f_1(x) = 1 + 2x + 3x^2$, $f_2(x) = 4 + 5x + 6x^2$, $f_3(x) = 7 + 8x + 10x^2$ are linearly independent in P_2, the linear space of all polynomials of degree ≤ 2.

3.(10pts) Find the determinant of the following matrix. Explain the method which you used in detail.

$$\begin{bmatrix} 1 & 0 & 0 & 9 & 8 \\ 1 & 1 & 0 & 5 & 4 \\ 1 & 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 7 & 9 \\ 0 & 0 & 0 & 3 & 1 \end{bmatrix}.$$

4.(10pts) Find the eigenvalue(s) and the corresponding eigenvector(s) for the following matrix A. Specify their algebraic and geometric multiplicities.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

5.(10pts) True or False?

(a) The equation $det(-A) = det(A)$ holds for all 10×10 matrices.

(b) $det \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = 1.$

(c) There is a 3×3 matrix A such that $A^2 = -I_3$.

(d) The eigenvalues of a 2×2 matrix A are the solutions of the equation $\lambda^2 - (tr\ A)\lambda + (det\ A) = 0$.

(e) If 0 is an eigenvalue of a matrix A, then $det(A) = 0$.

Bonus:(5pts) Prove or disprove the statements in Exercise # 5.