Math 316 - Test 3 - Name:

1.(10pts) Let P_2 denote the space of all polynomials of degree ≤ 2. Show that

$$W = \{p(t) \colon p(1) = 0\}$$

is a subspace of P_2. Also find a basis for W.

2.(10pts) Let $T \colon R^{2\times 2} \to R^{2\times 2}$ be defined by

$$T(A) = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} A.$$

Show that T is linear. Is T an isomorphism? Explain your answer.

3.(10pts) Show that

$$\begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

are linearly independent in $R^{2\times 2}$.

4.(10pts) Find the determinant of

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 4 & 3 & 0 \\ -1 & 8 & -3 & 0 \\ 4 & 0 & 0 & 3 \end{bmatrix}$$

using the Laplace expansion or Gauss-Jordan elimination.

5.(9pts) Answer the following with justifications.

a. Consider an $n \times n$ matrix A such that both A and A^{-1} have integer entries. What are possible values of $det(A)$.

b. If $det(A) = 8$ for some $n \times n$ matrix A, what is $det(A^T A)$?

c. Two $n \times n$ matrices A and B are said to be orthogonal if $A^T A = I_n$. What can you conclude about $det(A)$?

6.(15pts) Find all the eigenvalues and the corresponding eigenvectors of

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Specify algebraic as well as geometric multiplicities of each eigenvalue.

Bonus:(6pts) Using the definition of the determinant, find

$$\begin{bmatrix} 3 & 0 & 1 & 0 & 0 \\ 4 & 3 & 6 & 1 & 9 \\ 9 & 0 & 4 & 1 & 4 \\ 0 & 0 & 4 & 0 & 0 \\ 6 & 0 & 3 & 0 & 2 \end{bmatrix}$$