Exercises - Set 1

1.1 Construct the Lagrange interpolation polynomial of degree 2, $L_2(x)$, for $f(x) = \sin x$ with points $a_1 = 0, a_2 = 0.4, a_3 = 0.6$ using
(1) the fundamental Lagrange polynomials $\ell_j(x)$, $j = 1, 2, 3$.
(2) the Newton's divided difference formula.
(3) Evaluate $L_2(0.5)$ using (1) and (2).

1.2 Construct the Lagrange interpolation polynomial of degree 3, $L_3(x)$, for $f(x) = \sin x$ using the points $a_1 = 0, a_2 = 0.4, a_3 = 0.6$ and $a_4 = 1$ using
(1) the Newton's divided difference formula.
(2) Evaluate $L_3(0.5)$.
(3) Compute the value of $L_3(0.5)$ again, this time, using the Neville's algorithm.

1.3 Given the data, $f(1.00) = 0.2356, f(1.05) = 0.4602, f(1.10) = 0.3105, f(1.15) = 0.1602$, suppose that $L_3(x)$ is the Lagrange polynomial of degree 3 interpolating this data. Compute $L_3(1.07)$ using
(1) the Newton's forward difference formula
(2) the Newton's backward difference formula.

1.4 Suppose $a_j = j$, $j = 1, 2, 3, 4$ and

$$L_{1,2}(x) = x + 1, \quad L_{2,3}(x) = 3x - 1, \quad L_{2,3,4}(2.5) = 3.$$

Find $L_{1,2,3,4}(2.5)$.

1.5 A fourth-degree polynomial $L(x)$ satisfies $\triangle^4 L(0) = 24, \triangle^3 L(0) = 6$, and $\triangle^2 L(0) = 0$, where $\triangle L(x) = L(x + 1) - L(x)$. Compute $\triangle^2 L(10)$.

1.6 Exercise # 3 P.81 of Textbook

1.7 Exercise # 7 P.82 of Textbook

1.8 Exercise # 8 P.82 of Textbook

1.9 Exercise # 22 P.85 of Textbook

1.10 Given the following data;

a	0.1	0.2	0.3	0.4
$f(a)$	2	-4	1	6

(i) find the natural cubic spline.

(ii) Find the clamped cubic spline with $f'(0.1) = 0$ and $f'(0.4) = 1$.

1.11 Find the Hermite polynomial of degree 3 that interpolates

a	2	3
$f(a)$	-1	4
$f'(a)$	2	-1

(i) using the Newton divided difference formula

(ii) using the basis functions h_j and $\bar{h}_j$.

1.12 Given

a	2	3
$f(a)$	-1	4
$f'(a)$	2	-1
$f''(a)$	3	-1

generalize 1.11 (i) above to write a polynomial of degree 5 that interpolate this data.