Exercises - Set 2

2.1 Derive the approximation formula

$$f'(x) \simeq \frac{1}{12h}[f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)]$$

(1) using (4.1-5), P.90.
(2) using the method of undetermined coefficients.

2.2 Derive an $O(h^4)$ five-point formula to approximate $f'(x)$ that uses $f(x-h), f(x), f(x+h), f(x+2h)$ and $f(x+3h)$.

2.3 Compute the values of A, W_0, W_1, W_2 and W_3 in the Table 4.5, P.120 for $n = 3$. (In class, we derived these coefficients for $n = 1$ and $n = 2$.)

2.4 Compute the values of A, W_1, W_2 and W_3 in the Table 4.6, P.121 for $n = 4$. (In class, we derived these coefficients for $n = 2$ and $n = 3$.)

2.5 Using the Richardson extrapolation, approximate the value of $f'(1)$ for $f(x) = \ln|x+1|$ to the accuracy of $O(h^6)$ where use $h = 0.1$.

2.6 Suppose $T(h)$ approximate M as

$$M = T(h) + c_1 h^2 + c_2 h^4 + c_3 h^6 + \cdots$$

where $h > 0$. Use $T(h), T(\frac{h}{3})$ and $T(\frac{h}{9})$ to produce an $O(h^6)$ approximation to M.

2.7 Do exercise # 16 parts (a)-(d), P.151. (Here γ_r's are defined in exercise # 14.)

2.8 Do exercise # 17, P.151 concerning the Christoffel-Darboux identity.

2.9 **Bonus Problem:** Do exercise # 19, p. 152. (Use Exercise # 14.)

2.10 Verify that the following formula is exact for polynomials of degree ≤ 4 and discuss how such a formula is derived:

$$\int_0^1 f(x)dx \approx \frac{1}{90}[7f(0) + 32f(\frac{1}{4}) + 12f(\frac{1}{2}) + 32f(\frac{3}{4}) + 7f(1)]$$

2.11 Find the formula

$$\int_0^1 f(x)dx \approx A_0 f(0) + A_1 f(1)$$

that is exact for all functions of the form $f(x) = ae^x + b\cos(\frac{\pi x}{2})$.

2.12 For what value of α is this formula exact on P_3, the space of all polynomials of degree ≤ 3,

$$\int_0^2 f(x)dx \approx f(\alpha) + f(2-\alpha)$$