Solutions to Selected HW from Chapter 2

2.1

8. The number of multi/divisions is

$$\sum_{j=1}^{n-1} \sum_{i=j+1}^{n} (n-j+1) = \frac{n^3}{3} - \frac{n}{3}.$$

The number of add/substractions is

$$\sum_{i=1}^{n} (n-i) = \frac{n^2}{2} - \frac{n}{2}.$$

2.2

8. Information gives that $\frac{n^3}{3} + \frac{n^2}{2}$ operations per second which is about 2.7×10^6 operations per second with $n = 200$. To solve four systems with $n = 500$ requires $\frac{n^3}{3} + 4\frac{n^2}{2}$ operations and thus $(\frac{n^3}{3} + 4\frac{n^2}{2})/(2.7 \times 10^6) \approx 16$ seconds.

2.3

8. Here $A^{-1} = \begin{bmatrix} -\frac{1}{\delta} & \frac{1}{\delta} \\ 1+\frac{1}{\delta} & -\frac{1}{\delta} \end{bmatrix}$ and $cond(A) = (2+\delta)(1+\frac{2}{\delta})$.

2.4

4b.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{1}{2} & \frac{1}{5} & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{5}{2} & \frac{1}{2} \\ 0 & 0 & \frac{7}{5} \end{bmatrix}$$

2.5

5. (a) With out loss of generality, we assume that an eigenvector is $\|v\| = 1$ and $|v_m| = 1$. Then the mth component of $Av = \lambda v$ is

$$\sum_{j=1}^{n} A_{mj} v_j = \lambda v_m \Longrightarrow |\lambda - A_{mm}| \leq \sum_{j \neq m}^{n} |A_{mj}|.$$

(b) If A is singular then $\lambda = 0$ is an eigenvalue and from part (a), we get

$$|A_{mm} \leq \sum_{j \neq m}^{n} |A_{mj}|$$

which is a contradiction to the property of strictly diagonally dominant.