Solutions to Selected HW from Chapter 3

3.2

3 (a) By Theorem 3.3,

$$|f(x) - P_9(x)| = \left| \frac{(x-0)(x-\frac{1}{9})\cdots(x-1)}{10!} \right| 2^{10} e^{-2c}$$

where $c \in (0,1)$. At $x = \frac{1}{2}$ the error is approximately 7.06×10^{-11}.

(b) Since $7.06 \times 10^{-11} \leq 0.5 \times 10^{-9}$, the approximation is of at least 9 correct decimal places.

3.3

3 By Theorem 3.3,

$$e^x - Q_5(x) = \frac{(x-x_1)\cdots(x-x_6)}{6!} f^{(4)}(c)$$

where $c \in (-1,1)$. Hence

$$|e^x - Q_5(x)| \leq \frac{|f^{(4)}(c)|}{2^5 6!} = \frac{e^c}{2^5 6!} \leq \frac{e}{2^5 6!} \approx 1.1798 \times 10^{-4}.$$

As $1.1798 \times 10^{-4} < 0.5 \times 10^{-3}$, they agree to 3 decimal places over $[-1,1]$.

8

$$T_n(0) = \begin{cases} (-1)^{\frac{n}{2}} & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd.} \end{cases}$$

3.4

3 (a) The second derivatives must agree at $x = 1$. This gives $c = \frac{9}{4}$. Since $S_1''(0) = 0$ and $S_2''(2) = 0$, the spline is natural. The spline is not a not-a-knot since $S_1'''(1) = \frac{9}{2} \neq -\frac{9}{2} = S_2'''(1)$. It is not parabolically terminated because of the degree 3 terms.

(b) Equating $S_1'''(1) = S_2'''(1)$, we get $c = 4$. the spline is parabolically terminated and is also not-a-knot because $S_1'''(1) = S_2'''(1) = 0$. It is not natural since $S_1''(0)8 \neq 0$.

(c) Setting the first derivatives equal at $x = 1$, $c = \frac{5}{2}$. Since $S_1''(0) \neq 0$, it is not natural. It is not parabolically terminated due to the degree 3 terms. The spline is not-a-knot because $S_1'''(1) = S_2'''(1) = -6$ and $S_2'''(2) = S_3'''2) = -6$.

6 $S(x) = 1$. Spline is also not-a-knot and natural.

12 Since any not-a-knot cubic spline, $S_1(x) = S_2(x)$ over $[1,3] \cup [3,4]$ and $S_2(x) = S_3(x)$ over $[3,4] \cup [4,5]$, so the spline can be expressed by a single expression. Namely, the Lagrange interpolation polynomial,

$$P(x) = 3 - \frac{1}{3}(x-1) - \frac{1}{24}(x-1)(x-3)(x-4).$$