Chapter 1- Answers to selected even problems.

Section 1.1:

12. $x = 3 + t$, $y = t$ for $t \in R$. It is the line $x = 3 + y$.

20. $\begin{vmatrix} a & = 1000 + 0.1b \\ b & = 780 + 0.2a \end{vmatrix}$ Solving, we obtain $a = 1100$ and $b = 1000$.

Section 1.2:

18. (a) No (b) Yes (c) No (d) Yes

20. 4 types. $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

34. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x + 3y - z = 0$. Solving the last equation, $x = -3s + t$, $y = s$ and $z = t$ for $s, t \in R$. Plane spanned by the vectors $\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

36. $\begin{bmatrix} 1 & 2 & 4 & \vdots & -8 \\ 4 & 5 & 6 & \vdots & -1 \\ 7 & 8 & 9 & \vdots & 2 \\ 5 & 3 & 1 & \vdots & 15 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -4 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$ so that

$$2 \begin{bmatrix} 1 \\ 4 \\ 7 \\ 5 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix} - 4 \begin{bmatrix} 4 \\ 6 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ -1 \\ 2 \\ 15 \end{bmatrix}$$

Section 1.3

2. Rank is 3.
4. Rank is 2.
6. Since $\vec{v}_1$ and $\vec{v}_2$ are parallel, every linear combination $x\vec{v}_1 + y\vec{v}_2$ also becomes a vector parallel to them. Hence a vector $\vec{v}_3$ which is not parallel $\neq x\vec{v}_1 + y\vec{v}_2$.
8. Note that $\vec{v}_4$ can be written as a linear combination of $\vec{v}_1$ and $\vec{v}_2$. So $x\vec{v}_1 + y\vec{v}_2 + z\vec{v}_3 = \vec{v}_4$ has at least one solution. Hence, by Fact 1.3.3, we can find infinitely many solutions.
30. Since A is of rank 1, we may assume that it is in the form

$$\begin{bmatrix} 1 & a & b \\ c & ca & cb \\ d & da & db \end{bmatrix}$$

Then $A\vec{x} = \vec{y}$ yields

$$\begin{aligned} 5 + 3a - 9b &= 2 \\ c(5 + 3a - 9b) &= 0 \\ d(5 + 3a - 9b) &= 1. \end{aligned}$$

From the second and the first equations, $c = 0$. From the third and the first equation $d = \frac{1}{2}$. The first equation gives $3a - 9b = -3$ which has infinitely many solutions. For example, with $b = 0$, $a = -1$. Thus one example of rank 1 A may be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}.$$

32. There are many choices for A. Let

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

. From $A\vec{x} = \vec{y}$, the first equation is

$$5a + 3b - 9c = 2$$

Two free variables, b and c can be randomly assigned. For example, let $b = c = 1$, then $a = \frac{8}{5}$. Similarly, the second and the third equations can be handled to get $e = f = 1$ and $d = \frac{6}{5}$, $h = i = 1$ and $g = \frac{7}{5}$. Hence,

$$A = \begin{bmatrix} \frac{8}{5} & 1 & 1 \\ \frac{6}{5} & 1 & 1 \\ \frac{7}{5} & 1 & 1 \end{bmatrix}$$