Chapter 2- Answers to even problems.

Section 2.1:

2. Linear
4. $\begin{bmatrix} 9 & 3 & -3 \\ 2 & -9 & 1 \\ 4 & -9 & -2 \\ 5 & 1 & 5 \end{bmatrix}$.
6. Linear $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.
8. $A^{-1} = \begin{bmatrix} -20 & 7 \\ 3 & -1 \end{bmatrix}$
16. Dilation by the factor of 3 (scaling).
18. Contraction by the factor of 0.5 (scaling).
20. Reflection about the line $y = x$.
22. Reflection about the x-axis.
24. Rotation counter-clock-wise by $\pi/2$.
28. Dilation only in the direction of y axis by the factor 2.
32. $3 \cdot I_n$

Section 2.2:

2. $\begin{bmatrix} \cos(60°) & -\sin(60°) \\ \sin(60°) & \cos(60°) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
4. rotation by $\pi/4$ followed by dilation of factor $\sqrt{2}$.
6. $Proj_L \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{5}{9} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$
10. $\frac{1}{25} \begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix}$
24. (a) With $A = [\vec{v}\ \vec{w}]$, $A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \vec{v}$ and $A \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \vec{w}$. Since T preserves the length and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are unit vectors, $\vec{v}$ and $\vec{w}$ must be unit vectors. Also, since T preserves an angle and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are orthogonal, $\vec{v}$ and $\vec{w}$ must be orthogonal.

(b) Since $\vec{w}$ is orthogonal to $\vec{v} = \begin{bmatrix} a \\ b \end{bmatrix}$, it can be obtained by rotating $\vec{v}$ clockwise and counter clockwise by 90°. Hence

$$\vec{w} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -b \\ a \end{bmatrix} \quad \text{or} \quad \vec{w} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ -a \end{bmatrix}$$

(c) With $A = [\vec{v}\ \vec{w}]$ and part (b), we see that

$$A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \quad \text{or} \quad A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$$

where the first case represents a rotation and the second a reflection.

28. a-D, b-E, c-C, d-A, e-F

32. (a) Vector $\vec{v}$ of angle β is rotated by angle α, resulting in vector of angle $\alpha + \beta$.
(b)

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix} = \begin{bmatrix} \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \\ \cos(\beta)\sin(\alpha) + \cos(\alpha)\sin(\beta) \end{bmatrix}$$
$$= \begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \end{bmatrix}$$

Section 2.3

18. Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Since we must have $AB = BA$,

$$AB = \begin{bmatrix} a+2c & b+2d \\ c & d \end{bmatrix} \quad \text{and} \quad BA = \begin{bmatrix} a & 2a+b \\ c & 2c+d \end{bmatrix}$$

Comparing the $1-1$ entry $a + 2c = a$ which gives $c = 0$. When $1-2$ entries are compared, $b + 2d = 2a + b$, which implies $a = d$. Hence

$$B = \begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$$

30. (a) Let the angle between $vecx$ and P be denoted by A and the angle between $ref_P\vec{x}$ and Q by B. Note that $A + B = 30°$. So the angle between $\vec{x}$ and $T(\vec{x})$ is $A + B + 30° = 60°$.
(b) By (a), it is a rotation by 60°.
(c) $\begin{bmatrix} \cos(60°) & -\sin(60°) \\ \sin(60°) & \cos(60°) \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$
(d) 60° clockwise rotation from $\vec{x}$.

42. $A^n = A$ for all n.

Section 2.4

38. $A^{-1} = \frac{1}{(1)(-1)-(0)(k)} \begin{bmatrix} -1 & -k \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & k \\ 0 & -1 \end{bmatrix} = A.$

68. True

70. False

72. False

74. True

76. Since $A \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$,

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 8 & -3 \\ -1 & 1 \end{bmatrix}$$