MATH 316 - Quiz 1 - Fall 2009 NAME:

You must show your work to receive credit.

1. Find the reduced row-echelon form of

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -1 \\ 1 & 2 & 0 & 0 & 1 & -1 \\ 1 & 2 & 2 & 0 & -1 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 2 & -1 \\ 1 & 2 & 2 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{-R_1+R_3 \to R_3} \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 2 & 0 & -2 & 2 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 0 & -2 & 2 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 2 & -1 \end{bmatrix}$$

2. Find all solutions of

$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + 2y + z &= 1 \\ 7x + 2y - 3z &= 1 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 3 & 2 & 1 & 1 \\ 7 & 2 & -3 & 1 \end{array}\right] \xrightarrow[-7R_1+R_3 \to R_3]{-3R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -4 & -8 & -2 \\ 0 & -12 & -24 & -6 \end{array}\right] \xrightarrow[-\frac{1}{12}R_3 \to R_3]{-\frac{1}{4}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & \frac{1}{2} \\ 0 & 1 & 2 & \frac{1}{2} \end{array}\right]$$

$$\xrightarrow[-R_2+R_3 \to R_3]{-2R_2+R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

z is free variable

Let $z = t$, $y = \frac{1}{2} - 2t$, $x = t$

$$\begin{cases} x = t \\ y = \frac{1}{2} - 2t \\ z = t \end{cases}$$

3. Compute $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$.

$$= \begin{bmatrix} -2+6+15 \\ -4+9+20 \end{bmatrix}$$

$$= \begin{bmatrix} 19 \\ 25 \end{bmatrix}$$

4. Consider a linear system of five equations in four unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix look like? Explain your answer.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The right side should contain 0 in the 5th position.