Solutions to Selected Problems in Test 1

1.

(b) Since r lie between a_n and x_c or x_c and b_n,

$$|x_c - r| \le \frac{1}{2}|b_n - a_n| = \frac{1}{2^2}|b_{n-1} - a_{n-1}| = \cdots = \frac{1}{2^{n+1}}|b_0 - a_0|$$

where $a_0 = a$ and $b_0 = b$.

(c) Solving

$$\frac{1}{2^{n+1}} < 0.5 \times 10^{-6},$$

we get $n > 19.9$, so it takes 20 steps.

3.

(b)

$$|\cos x - P_2(x)| \le \frac{\frac{d^3}{dx^3}\cos x|_{x=c}}{3!}|(x-0)(x-0.4)(x-0.6)|$$

Now,

$$\max_{0\le x\le 1} |(x-0)(x-0.4)(x-0.6)| < 0.24$$

with maximum occurring at $x = 1$ (use simple calculus to show this) and hence,

$$\max_{0\le x\le 1} |\cos x - P_2(x)| \le 0.04.$$

(d) From the table,

x	$f[x]$	$f[x,x]$	$f[x,x,x]$	$f[x,x,x,x]$	$f[x,x,x,x,x]$
1	−1				
		2			
1	−1		0		
		−2		−4	
2	1		−4		9/2
		−2		5	
2	1		6		
		4			
3	5				

we see that

$$\begin{aligned} H(x) &= -1 + 2(x-1) + 0(x-1)^2 - 4(x-1)^2(x-2) + \frac{9}{2}(x-1)^2(x-2)^2 \\ &= -1 + 2(x-1) - 4(x-1)^2(x-2) + \frac{9}{2}(x-1)^2(x-2)^2 \end{aligned}$$

satisfies the conditions.

(e) (508) Note that $P(x) = \sum_{j=1}^{n} L_j(x) - 1$ is a polynomial of degree $n-1$ having n zeros. Hence, $P(x) \equiv 0$ for all x.

4. Please consult the textbook.