

## Gauss-Jordan Elimination - Supplement to Section 1.3

**Elementary Row Operations:** The following three operations on the rows of a matrix is called Elementary Row Operations. Each operation does not affect the solution of the system of linear equations.

1. Multiply the  $i$ th row,  $R_i$ , by a nonzero scalar  $\lambda$ . ( $R_i \implies \lambda R_i$ )
2. Interchange two rows,  $R_i$  and  $R_j$ . ( $R_i \leftrightarrow R_j$ )
3. Add a scalar  $\lambda$  multiple of the  $i$ th row,  $R_i$ , to the  $j$ th row,  $R_j$ . ( $\lambda R_i + R_j \rightarrow R_j$ )

**Example:**

$$\left[ \begin{array}{ccc|c} 2 & 8 & -4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array} \right] \xRightarrow{\frac{1}{2}R_1 \rightarrow R_1} \left[ \begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array} \right] \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -4R_1 + R_3 \rightarrow R_3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & -3 & 5 & 3 \\ 0 & -6 & 7 & -3 \end{array} \right] \begin{array}{l} -\frac{1}{3}R_2 \rightarrow R_2 \\ -\frac{1}{6}R_3 \rightarrow R_3 \end{array} \xRightarrow{} \left[ \begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 0 & 1 & -\frac{5}{3} & -1 \\ 0 & 1 & -\frac{7}{6} & \frac{1}{2} \end{array} \right] \begin{array}{l} -4R_2 + R_1 \rightarrow R_1 \\ -R_2 + R_3 \rightarrow R_3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & \frac{14}{3} & 5 \\ 0 & 1 & -\frac{5}{3} & -1 \\ 0 & 0 & \frac{1}{2} & \frac{3}{2} \end{array} \right] \xRightarrow{2R_3 \rightarrow R_3} \left[ \begin{array}{ccc|c} 1 & 0 & \frac{14}{3} & 5 \\ 0 & 1 & -\frac{5}{3} & -1 \\ 0 & 0 & 1 & 3 \end{array} \right] \begin{array}{l} \frac{5}{3}R_3 + R_2 \rightarrow R_2 \\ -\frac{14}{3}R_3 + R_1 \rightarrow R_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

**Exercises:**

$$1. \left[ \begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 0 & 3 & 1 & -1 \\ -1 & 2 & 1 & 1 \end{array} \right]$$

$$2. \left[ \begin{array}{ccc|c} 1 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ -6 & 2 & 3 & 0 \end{array} \right]$$