Solutions to Selected Even Numbered Exercises in Chapter 4

Section 4.2

2. Since A is 3×3 and $\det A = -1$, $\det(\frac{1}{2}A) = (\frac{1}{2})^3 \det A = -\frac{1}{8}$, $\det(-A) = (-1)^3 \det A = 1$, $\det(A^2) = \det A \det A = 1$ and $\det(A^{-1}) = \frac{1}{\det A} = -1$.

22.

$$\det\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{matrix} \cdots\cdots \\ \text{after some} \\ \text{row operations} \end{matrix} = \det\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = 1$$

$$\det\begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} = \begin{matrix} \cdots\cdots \\ \text{after some} \\ \text{row operations} \end{matrix} = \det\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -3 \\ 0 & 0 & -\frac{2}{3} \end{bmatrix} = (1)(-2)(-\frac{2}{3}) = 3$$

Section 4.3

24.

$$C_A^T = \begin{bmatrix} 6 & -1 \\ -3 & 2 \end{bmatrix}, \quad \text{and} \quad C_A^T A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} = (\det A) I_2.$$

$$C_B^T = \begin{bmatrix} 0 & 0 & -3 \\ 42 & -21 & 6 \\ -35 & 14 & -3 \end{bmatrix}, \quad \text{and} \quad C_B^T B = \begin{bmatrix} -21 & 0 & 0 \\ 0 & -21 & 0 \\ 0 & 0 & -21 \end{bmatrix} = (\det B) I_3.$$

Section 4.4

2. See Section 4.3, #25 for A. With $B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, $C^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ and $BC^T = I_3$ so that $C^T = B^{-1}$.

18. (a) For $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$, we have $\det A = 3$. Also the transpose of the matrix of cofactors, -i.e., the adjoint of A is,

$$adj(A) = C^T = \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 3 \end{bmatrix}$$

and $A^{-1} = \frac{1}{det(A)} adj(A) = \frac{1}{3} C^T$.

(b) For $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, we have $\det A = 4$. Also the transpose of the matrix of cofactors, -i.e., the adjoint of A is,

$$adj(A) = C^T = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

and $A^{-1} = \frac{1}{det(A)} adj(A) = \frac{1}{4} C^T$.