Solutions to Test 2

1.(14pts)

(a) $\frac{f(0.99)-2f(1)-f(1.01)}{0.01^2} \simeq 2.7183045$

(b) See P. 245

2.(14pts)

(a) $\frac{\pi/4}{2}[\sin 0 + 2\sin\frac{\pi}{4} + 2\sin\frac{\pi}{2} + 2\sin\frac{3\pi}{4} + \sin\pi] = 1.896118898$

(b) See P. 259

3.(12pts)

(a) Note that $p_3(x) = \frac{1}{2^3 3!}\frac{d^3}{dx^3}(x^2-1)^3 = \frac{24}{2^3 3!}(5x^3 - 3x)$. Hence $-\sqrt{\frac{3}{5}}, 0, \sqrt{\frac{3}{5}}$ are three zeroes.

(b) $\frac{5}{9}f(-\sqrt{\frac{3}{5}}) + \frac{8}{9}f(0) + \frac{5}{9}f(\sqrt{\frac{3}{5}}) \simeq 1.7120204520191$

4.(12pts)

(a) $w_1 = 1$ and $w_2 = 1.00250625$

(b) Since $f(t,y) = ty + t^3$, $f_y = t$, $|f_y| \le 1$ for all $t \in [0,1]$. Hence we may take $L = 1$. Also, $y'' = (3t^2+1)e^{t^2/2} - 2$ and $|y''| \le 4e^{1/2} + 2 \simeq 8.6$. Hence we let $M = 8.6$. Hence $\frac{Mh}{2L}|(e^{L(t_i - a)} - 1)| \le 0.17225$.

5.(14pts) Let $W_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. As $F(t,Y) = \begin{bmatrix} y_1 + y_2 \\ -y_1 + y_2 \end{bmatrix}$,

$$\begin{aligned} W_1 &= W_0 + hF(t_0 + \frac{h}{2}, W_0 + \frac{h}{2}F(W_0 + \frac{h}{2}F(t_0, W_0)) \\ &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{5}F(\frac{1}{10}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{10}\begin{bmatrix} 1 \\ -1 \end{bmatrix}) \\ &= \begin{bmatrix} 1.2 \\ -0.24 \end{bmatrix} \end{aligned}$$

6.(14pts)

(a) Since $w_{i+1} = w_i + hf(t_i, w_i)$, we have $w_{i+1} = w_i(1-10h) + 10h$.

(b) $\lambda - (1-10h) = 0$ is the characteristic equation and $w_i^P = 1$, so $w_i = c(1-10h)^i + 1$. Using $w_0 = \frac{1}{2}$, we see that $c = -\frac{1}{2}$. Hence $w_i = -\frac{1}{2}(1-10h)^i + 1$.

(c) $(1+10h)w_{i+1} - w_i = 10h$ and its chara. eq. is $(1+10h)\lambda - 1 = 0$. hence $w_i = c(1+10h)^{-i} + 1$ and using $w_0 = \frac{1}{2}$, we see that $c = -\frac{1}{2}$.

(d) (b) converges only when $h < \frac{1}{10}$ whereas (c) converges for all h.

7.(10pts) Note that $w_{i-1} - 2w_i + w_{i+1} - h^2(2+4t_i^2)w_i = 0$ for $t_i = \frac{i}{5}$ and $i = 1, \ldots, 4$. Since $w_0 = 1$ and $w_5 = e$, the first and the last equations are respectively

$$-[2 + h^2(2 + 4t_1^2)]w_1 + w_2 = -1$$

and

$$w_3 - [2 + h^2(2 + 4t_4^2)]w_4 = -e$$

The system $AW = B$ of linear equations for w_i's is

$$A = \begin{bmatrix} d_1 & 1 & 0 & 0 \\ 1 & d_2 & 1 & 0 \\ 0 & 1 & d_3 & 1 \\ 0 & 0 & 1 & d_4 \end{bmatrix}$$

where $d_i = -[2 + h^2(2 + 4t_i^2)]$, $i = 1, 2, 3, 4$ and

$$W = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -e \end{bmatrix}$$

8.(10pts) The characteristic equation is $\lambda^2 - 2\lambda - 3 = 0$ whose roots are 3 and -1. Hence $w_i^H = c_1 3^i + c_2(-1)^i$. A particular solution is $w_i^P = -\frac{1}{4}$. Using $w_0 = 0$ and $w_1 = 1$, we get $w_i = \frac{3}{8}3^i - \frac{1}{8}(-1)^i - \frac{1}{4}$.

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Solutions will be posted by noon tomorrow (6/19) at
http://www.lions.odu.edu/ hkaneko/teaching/Summer07/408-su07.htm