Chapter 1- Answers to even problems.

Section 1.1:

12. $x = 3 + t$, $y = t$ for $t \in R$. It is the line $x = 3 + y$.

20. $\left| \begin{array}{ll} a & = 1000 + 0.1b \\ b & = 780 + 0.2a \end{array} \right|$ Solving, we obtain $a = 1100$ and $b = 1000$.

22. Done in class.

Section 1.2:

2. $\begin{bmatrix} 3 & 4 & -1 & \vdots & 8 \\ 6 & 8 & -2 & \vdots & 3 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & \frac{4}{3} & -\frac{1}{3} & \vdots & \frac{8}{3} \\ 0 & 0 & 0 & \vdots & -13 \end{bmatrix}$, so no solution.

18. (a) No (b) Yes (c) No (d) Yes

20. 4 types. $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

34. $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x + 3y - z = 0$. Solving the last equation, $x = -3s + t$, $y = s$ and $z = t$ for $s, t \in R$. Plane spanned by the vectors $\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

36. $\begin{bmatrix} 1 & 2 & 4 & \vdots & -8 \\ 4 & 5 & 6 & \vdots & -1 \\ 7 & 8 & 9 & \vdots & 2 \\ 5 & 3 & 1 & \vdots & 15 \end{bmatrix} \Longrightarrow \begin{bmatrix} 1 & 0 & 0 & \vdots & 2 \\ 0 & 1 & 0 & \vdots & 3 \\ 0 & 0 & 1 & \vdots & -4 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$ so that

$$2 \begin{bmatrix} 1 \\ 4 \\ 7 \\ 5 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 5 \\ 8 \\ 3 \end{bmatrix} - 4 \begin{bmatrix} 4 \\ 6 \\ 9 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ -1 \\ 2 \\ 15 \end{bmatrix}$$

Section 1.3

2. Rank is 3.
4. Rank is 2.
6. Since $\vec{v}_1$ and $\vec{v}_2$ are parallel, every linear combination $x\vec{v}_1 + y\vec{v}_2$ also becomes a vector parallel to them. Hence a vector $\vec{v}_3$ which is not parallel $\neq x\vec{v}_1 + y\vec{v}_2$.
8. Note that $\vec{v}_4$ can be written as a linear combination of $\vec{v}_1$ and $\vec{v}_2$. So $x\vec{v}_1 + y\vec{v}_2 + z\vec{v}_3 = \vec{v}_4$ has at least one solution. Hence, by Fact 1.3.3, we can find infinitely many solutions.

30. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

32. True.