

Chapter 2- Answers to even problems.

Section 2.1:

2. Linear
4.  $\begin{bmatrix} 1 & 3 & -3 \\ 2 & -9 & 1 \\ 4 & -9 & -2 \end{bmatrix}$ .
6. Linear  $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ .
8.  $A^{-1} = \begin{bmatrix} -20 & 7 \\ 3 & -1 \end{bmatrix}$  }
16. Dilation by factor of 2.
20. Reflection about the line  $y = x$ .
24. Rotation counter-clock-wise by  $\pi/2$ .
28. Dilation only in the direction of  $y$  axis by factor 2.
32.  $3 \cdot I_n$

Section 2.2:

2.  $\begin{bmatrix} \cos(60^\circ) & -\sin(60^\circ) \\ \sin(60^\circ) & \cos(60^\circ) \end{bmatrix}$
4. rotation by  $\pi/4$  followed by dilation of factor  $\sqrt{2}$ .
6.  $Proj_L \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{5}{9} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$
10.  $\frac{1}{25} \begin{bmatrix} 16 & 12 \\ 12 & 9 \end{bmatrix}$
24. Done in class.
28.  $\begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$
32. (a) Vector  $\vec{v}$  of angle  $\beta$  is rotated by angle  $\alpha$ , resulting in vector of angle  $\alpha + \beta$ . (b)  $\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix} = \begin{bmatrix} \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \\ \cos(\beta)\sin(\alpha) + \cos(\alpha)\sin(\beta) \end{bmatrix} = \begin{bmatrix} \cos(\alpha + \beta) \\ \sin(\alpha + \beta) \end{bmatrix}$  ■

Section 2.3

18.  $A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ .
30. Matrix is noninvertible for any  $b$  and  $c$ .
42. Yes. Yes.

Section 2.4

16. true

18. true
20. false
22. false
24. true
26. Done in class.
28. For example,  $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ .
38.  $\begin{bmatrix} -s+t & s \\ 0 & t \end{bmatrix}$  for any scalars  $s$  and  $t$ .
44.  $A = \begin{bmatrix} 8 & -3 \\ -1 & 1 \end{bmatrix}$ .
50. (a)  $EA = \begin{bmatrix} a & b & c \\ -3a+d & -3b+e & -3c+f \\ g & h & k \end{bmatrix}$ . Row operation of  $-3R_1 + R_2 \rightarrow R_2$ .
- (b)  $\frac{1}{4}R_2 \rightarrow R_2$
- (c) For example, if  $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ , then  $EA$  swaps the second and third row of  $A$ .
- (d) See the previous parts.
52. (a) See problem #50. (b)  $E_1E_2E_3A = I_2$  where  $E_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ,  $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$   
and  $E_1 = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$
66. In order for  $A$  to be invertible  $A_{11}$  and  $A_{22}$  must be invertible.

$$A^{-1} = \begin{bmatrix} A_{11}^{-1} & 0 \\ A_{22}^{-1}(-A_{21}A_{11}^{-1}) & A_{22}^{-1} \end{bmatrix}$$