Chapter 3- Answers to even problems.

Section 3.1:

8. $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$.

16. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

18. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$. The line in $R^2$ spanned by $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

20. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. The line in $R^3$ spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

24. Kernel=span $\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \}$ and Image=span$\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \}$.

30. $A = \begin{bmatrix} 1 & 2 \\ 5 & 10 \end{bmatrix}$.

32. $A = \begin{bmatrix} 7 & 14 & 21 \\ 6 & 12 & 18 \\ 5 & 10 & 15 \end{bmatrix}$.

34. $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

36. $ker(T) = span\{\vec{v}\}$ and $im(T) = \{$all vectors perpendicular to $\vec{v}\}$.

38. As $A\vec{x} = \vec{0}$ implies $A^2\vec{x} = \vec{0}$, $ker(A) \subseteq ker(A^2)$. In general, $ker(A) \subseteq ker(A^2) \subseteq ker(A^3) \subseteq \cdots$.

40. $AB = 0$ since for any $\vec{x}$, $ker(A) = im(B)$ implies $A(B(\vec{x})) = \vec{0}$, hence $AB$ is a zero matrix.

44. (a) yes (b) no

Section 3.2:

2. $W$ is not a subspace. For example, let $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \in W$. Then $(-1) \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} \notin W$.

6. a. Yes b. No

8. $(-1) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + (-1) \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

20. Linearly Dependent.

24. If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in L^{\perp}$, then $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$. Hence $x + 2y + 3z = 0$. Let $y = s$ and $z = t$ so that $x = -2s - 3t$. Hence $L^{\perp} = span\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \}$.

36. Yes. Since $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\}$ are linearly dependent, there exist $c_1, c_2, \ldots, c_m$ not all zeroes such that

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m = \vec{0}.$$

Then

$$\begin{aligned} \vec{0} = T(\vec{0}) &= T(c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m) \\ &= c_1T(\vec{v}_1) + c_2T(\vec{v}_2) + \cdots + c_mT(\vec{v}_m). \end{aligned}$$

Since $c_1, c_2, \ldots, c_m$ are not all zeros, we see that $T(\vec{v}_1), T(\vec{v}_2), \ldots, T(\vec{v}_m)$ are linearly dependent.

42. Let $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\}$ be perpendicular vectors. Consider

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m = \vec{0}.$$

Taking the dot product on both sides with $\vec{v}_i$,

(1) $$\vec{v}_i \cdot (c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_m\vec{v}_m) = \vec{v}_i \cdot \vec{0}.$$

As $\vec{v}_j \cdot \vec{v}_i = 0$ if $i \neq j$, and $\vec{v}_i \cdot \vec{v}_i = 1$ as $\vec{v}_i$'s are unit vectors, we obtain from (1) that $c_i = 0$. Hence all the coefficients $c_i$'s are zero, making the vectors linearly independent.

Section 3.3

24. Let

$$A = \begin{bmatrix} 1 & 3 & 3 & 8 & 0 \\ 2 & 6 & 2 & 4 & 4 \\ 3 & 9 & 4 & 9 & 5 \\ 2 & 6 & 1 & 1 & 5 \\ 1 & 3 & 2 & 5 & 1 \end{bmatrix}.$$

$$rref(A) = \begin{bmatrix} 1 & 3 & 0 & 2 & 3 \\ 0 & 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

This shows that the first and the third columns of $A$ are linearly independent. Hence $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \\ 6 \\ 3 \end{bmatrix} \right\}$ span the subspace.

36. No, as $3 = dim(im(A)) + dim(ker(A))$ by the rank theorem, $dim(im(A)) \neq dim(ker(A))$, so $im(A)$ can not equal $ker(A)$.

38. a. Since $T\colon R^5 \to R^3$,

$$\begin{array}{rlcc} 5 & = dim(ker(A)) & + & dim(im(A)) \\ & = 5 & + & 0 \\ & = 4 & + & 1 \\ & = 3 & + & 2 \\ & = 2 & + & 3 \end{array}$$

b. Since $T\colon R^4 \to R^7$,

$$\begin{array}{rlcc} 4 & = dim(ker(A)) & + & dim(im(A)) \\ & = 4 & + & 0 \\ & = 3 & + & 1 \\ & = 2 & + & 2 \\ & = 1 & + & 3 \\ & = 0 & + & 4 \end{array}$$