Probability & Stochastic Processes

Yates & Goodman

- Probability & Set Theory
- Discrete r.v
- Continuous r.v
- Statistical Inference
- Stochastic Processes

I. Probability & Set Theory

- many situations occur in which it is *impossible* to *exactly* replicate what is important --> probability
- 3 major aims of this course :
  - logic behind probability theory
  - develop intuition into applying the theory
  - applications to engineering problems
  - i.e., critical and mathematical skills

Probability & Set Theory

- *<u>OUTCOME</u>* of an experiment : any possible observation
- *<u>SAMPLE SPACE</u>* : set of all possible outcomes
- An *<u>EVENT</u>* : some set of outcomes
  - *EVENT SPACE* : set of events

Probability Axioms

- probability measure P[.] maps
  Sample Space --> Real interval [0, 1]

  – P[A] $\geq$ 0 , P[S] = 1,
  – for mutually exclusive events

$$P[A \cup B \cup C \cup ....] = P[A] + P[B] + P[C] + ...$$

Conditional Probability

$$P[A \mid B] = \frac{P[AB]}{P[B]}$$

– law of total probability

– Bayes' theorem

Statistical Independence

- 2 events A, B are *independent* iff
  P[AB] = P[A] . P[B]

  – extend to 3 or more events

Sequential Experiments, Tree Diagrams

- Bayes' theorem in tree structure
  - at each branch level, the probability sum = 1

Counting

- if expt. A has *n* outcomes, and expt B has *k* outcomes then performing expts A *and* B yields *nk* outcomes
- *n* distinguishable objects:
  - total number of *k* -permutations = $\frac{n!}{(n-k)!}$
  - total number of ways to choose *k* objects = $\frac{n!}{k!(n-k)!}$

Independent Trials

- In *n* indep. Trials : for *k* successes
  - each outcome has probability $p^k\,(1-p)^{n-k}$
  - there are $\frac{n!}{k!(n-k)!}$ such outcomes
- reliability
- multiple outcomes