MULTIPLE CONTINUOUS r.v

- JCFD (joint CDF) for r.v X & Y:
  $F_{X,Y}(x,y) = P[X \leq x, Y \leq y]$
- JPDF : $$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \ \partial y}$$

Marginal pdf

- X, Y r.v's with JPDF $f_{X,Y}(x,y)$.
- the marginal pdf of X is just $f_X(x)$:
  $$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$$
- Similarly for the marginal pdf of Y:
  $$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$$

Functions of 2 r.v's

- 2 r.v's X, Y with JPDF $f_{X,Y}(x,y)$.

  We may be interested in some function
  W=g(X,Y)
- To determine the corresponding pdf and cdf of W : $f_W(w)$, and $F_W(w)$.

Expectation Values for 2 r.v's X & Y with respect to JPDF $f_{X,Y}(x,y)$

- E[W]=E[g(X,Y)] - w.r.t JPDF $f_{X,Y}(x,y)$
- E[X+Y] = E[X] +E[Y]
- **COVARIANCE** $Cov[X,Y] = E[X.Y] - \mu_X \mu_Y$
- **CORRELATION COEFFICIENT**

$$-1 \le \rho_{X,Y} = \frac{Cov[X,Y]}{\sigma_X . \sigma_Y} \le 1$$

CONDITIONAL PDF

- $f_{Y|X}(y\,|\,x) = \dfrac{f_{X,Y}(x,y)}{f_X(x)}$
- conditional expectation value of g(X,Y) given y : will use conditional pdf $f_{X|Y}(x|y)$

INDEPENDENT R.V's

- X, Y are independent $\Leftrightarrow$ $f_{X,Y}(x,y) = f_X(x).f_Y(y)$
- X, Y indep. then Cov[X,Y] = 0
  - E[g(X). h(Y)] = E[g(X)] . E[h(Y)]
  - Var[X+Y] = Var[X] + Var[Y]

JOINTLY GAUSSIAN R.V's

- *Jointly Gaussian* : if all the marginal PDF's of the r.v's are Gaussian
- bivariate Gaussian PDF (2 r.v's) :
  - marginal PDF's $f_X(x)$, $f_Y(y)$
  - conditional PDF of Y given X
  - correlation coefficient $\rho_{X,Y}$