

## SAMPLE MEAN

- $X_1, \dots, X_n$  iid r.v with PDF  $f_X(x)$ .  
The **SAMPLE MEAN** of  $X$  is the *random variable*

$$M_n(X) = (X_1 + \dots + X_n) / n$$

$$E[M_n(X)] = E[X] \quad , \quad Var[M_n(X)] = \frac{Var[X]}{n}$$

- $E[X]$  = 'expected value' of  $X$  is a number -- it is NOT an r.v

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## Markov & Chebyshev Inequalities

- Markov** : *non-negative* r.v  $X$ , and constant  $c > 0$ ,

$$P[X \geq c] \leq \frac{E[X]}{c}$$

- Chebyshev** : *arb.* r.v  $Y$ , constant  $c > 0$ ,

$$P[|Y - \mu_Y| \geq c] \leq \frac{Var[Y]}{c^2}$$

- Chernoff** : *arb.* r.v  $X$ , constant  $c$

$$P[X \geq c] \leq \min_{s \geq 0} \exp[-s c] \cdot \phi_X(s)$$

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## SAMPLE MEAN of LARGE NUMBERS

- For any constant  $c > 0$ , the sample mean r.v  $M_n(X)$  satisfies:

$$P[|M_n(X) - \mu_X| \geq c] \leq \frac{Var[X]}{nc^2} \equiv \alpha$$

$$P[|M_n(X) - \mu_X| < c] \leq 1 - \alpha$$

i.e., for small  $\alpha$ , there is a high probability that the r.v  $M_n(X)$  is  $(\mu_X - c, \mu_X + c)$

- $2c =$  confidence interval

$$\frac{Var[X]}{nc^2} \equiv \alpha \quad = \quad \underline{\text{confidence}}$$

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### weak LAW of LARGE NUMBERS

- If  $\sigma^2 = \text{Var}[X] < \infty$ , arb. Constant  $c > 0$ , the sample mean  $M_n(X)$  satisfies

$$P[|M_n(X) - \mu_X| \geq c] \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$P[|M_n(X) - \mu_X| < c] \rightarrow 1 \text{ as } n \rightarrow \infty$$

*i.e.*, the sample mean converges to  $E[X]$  as the number of samples  $n$  increases (by making  $c$  as small as we like)

- N.B This is convergence in probabilities -- hence 'weak' law

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### STRONG LAW OF LARGE NUMBERS

- If  $X_1, X_2, \dots$  iid r.v with sample mean  $M_n(X)$  then

$$M_n(X) \rightarrow E[X], \text{ as } n \rightarrow \infty, \text{ w.p.1}$$

"w.p. 1" = "with probability 1"

However this does NOT mean for every sample outcome -- but *almost always*

[basically, we can say nothing about sets of measure zero in integration theory]

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- $X_1, \dots, X_n$  iid r.v with sample

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