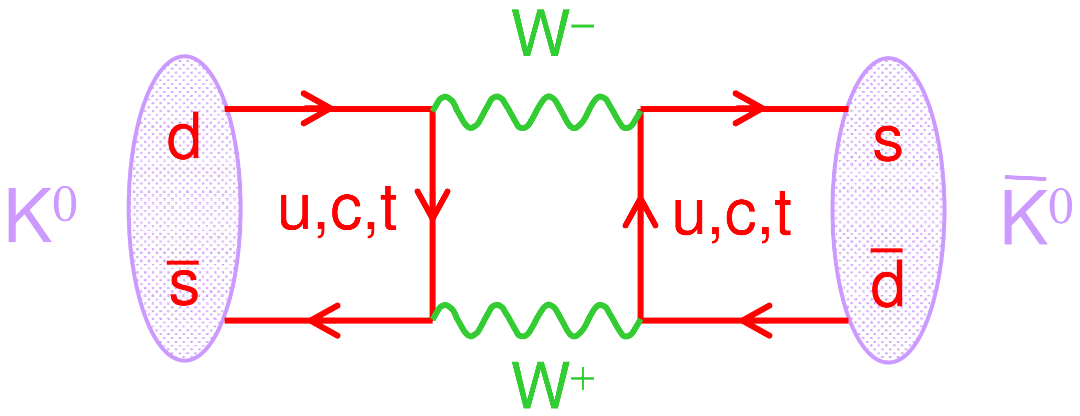
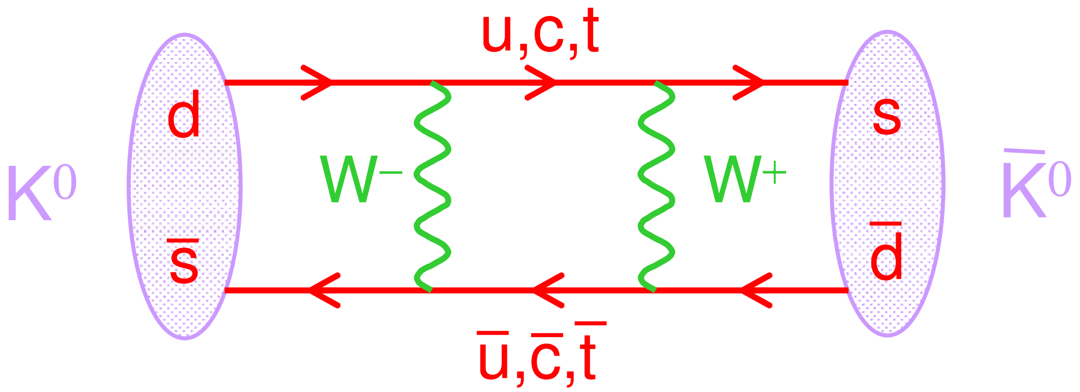
CP Violation in the SM

- In the Standard Model, $K^0 \Leftrightarrow \bar{K}^0$ oscillations occur via the following diagrams:





From matrix element M_{fi} for these diagrams:

$$\Delta m \propto |M_{\mathrm{fi}}|$$

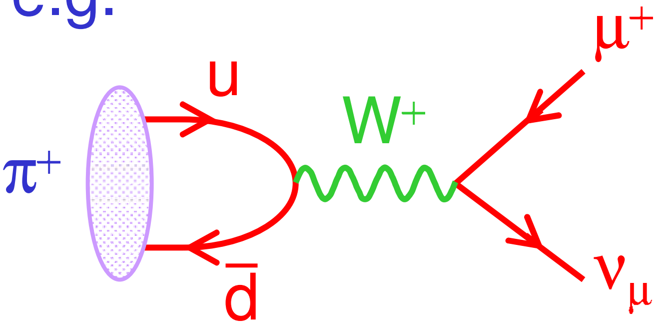
$$|\varepsilon| \propto \mathrm{Im}(M_{\mathrm{fi}})$$

but first need to look more closely at coupling of quarks to W bosons ….

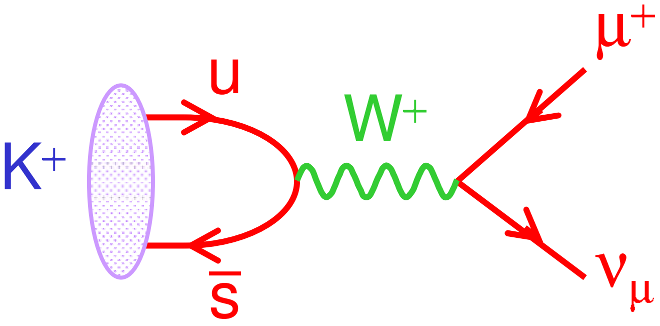
The Cabibbo Angle

1) Decay rates for strange hadrons were much lower than expected:

e.g.



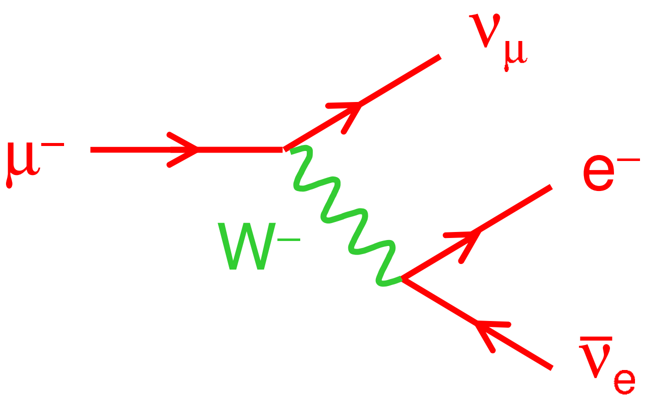
$\pi^+ \rightarrow \mu^+ \nu_\mu$



$K^+ \rightarrow \mu^+ \nu_\mu$

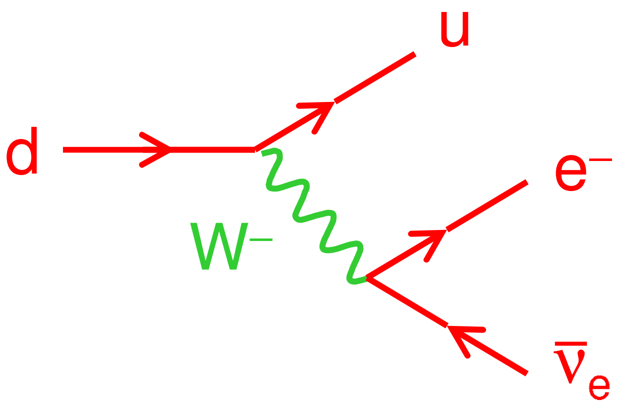
Only ~ 5% of expected value relative to $\pi^+ \rightarrow \mu^+ \nu_\mu$

2) Values of G_F measured in μ decay and nuclear β decay were slightly different:



$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$



$n \rightarrow p\, e^- \bar{\nu}_e$

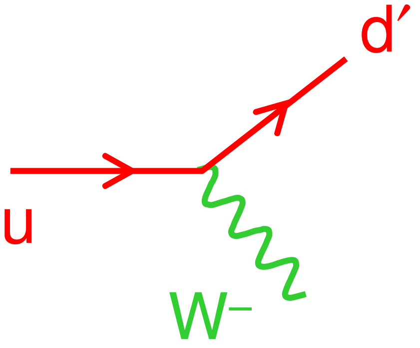
$G_F = 1.136 \times 10^{-5}\ \mathrm{GeV}^{-2}$

- Both observations were elegantly accounted for by the Cabibbo hypothesis (1963) :

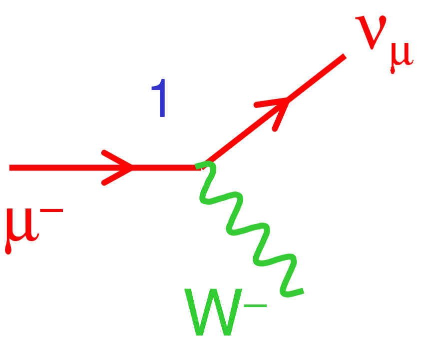
W± couples to u quark with same strength as to leptons, but turns it into

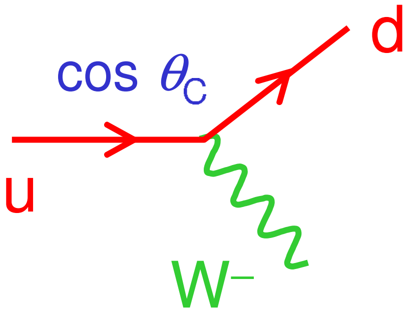
$$d' = d\cos\theta_C + s\sin\theta_C$$

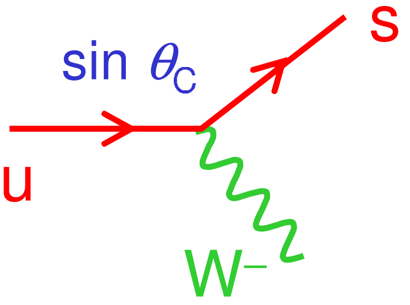
where θ_C = Cabibbo angle



i.e. weak W± vertices include the factors:







- Then:

1) $\pi^+ \to \mu^+ \nu_\mu \quad \propto \quad \cos^2\theta_C$

$K^+ \to \mu^+ \nu_\mu \quad \propto \quad \sin^2\theta_C$

2) $\mu^- \to e^- \bar{\nu}_e \nu_\mu \quad \propto \quad 1$

$n \to p\, e^- \bar{\nu}_e \quad \propto \quad \cos^2\theta_C$

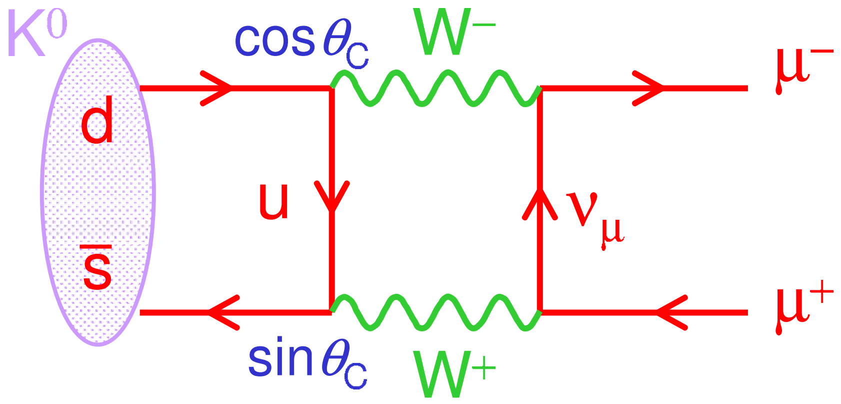
and a value $\theta_C \approx 13.1^\circ$ nicely explains all the observations

$$\left\{ \begin{array}{l} \cos^2\theta_C \approx 0.95 \\ \sin^2\theta_C \approx 0.05 \end{array} \right.$$

(and the νp, νn cross sections considered earlier should all be multiplied by $\cos^2\theta_C \approx 0.95$)

The GIM Mechanism

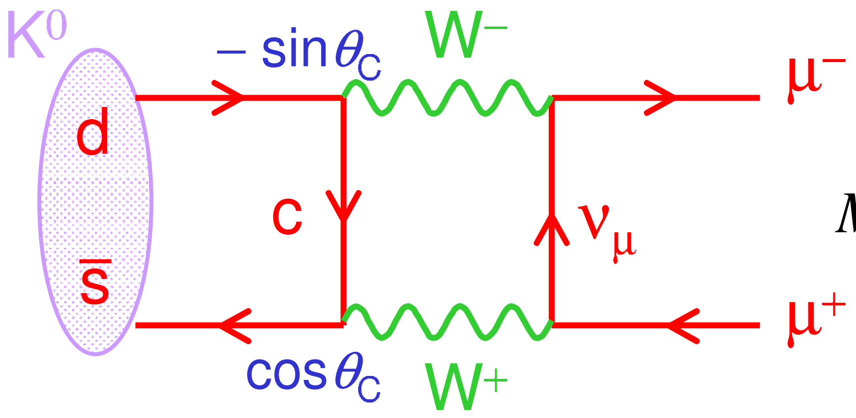
- Another puzzle was $K^0 \rightarrow \mu^+\mu^-$, which had a much lower BR than expected (BR ~ 7 x10^{-9})



$$M_{fi} \propto g_W^4 \cos\theta_C \sin\theta_C$$

- Led Glashow, Illiopoulos and Maiani to postulate existence of an extra quark

before discovery of charm quark



$$M_{fi} \propto -g_W^4 \cos\theta_C \sin\theta_C$$

Vertex factors enter with opposite overall sign

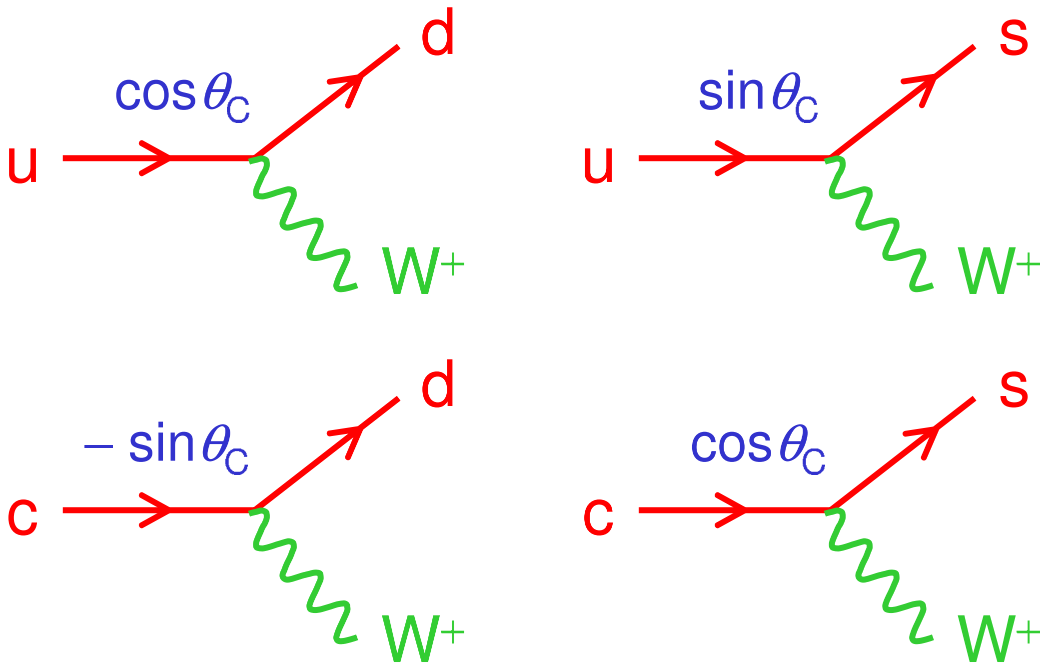
$\Longrightarrow$ two diagrams approximately cancel

(not quite, because $m_u \neq m_c$)

Known as GIM mechanism

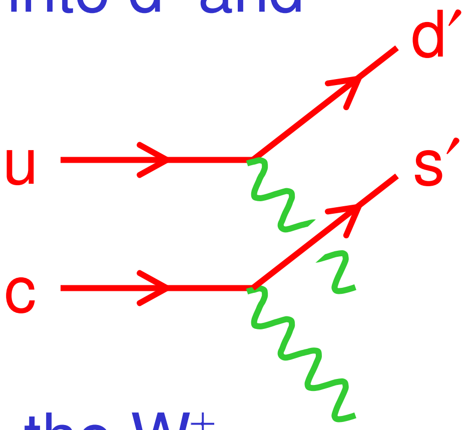
Vindicated by charm discovery in 1974

- Thus the weak $W^{\pm}$ vertex factors are:



or, equivalently, the $W^{\pm}$ turns u into d′ and c into s′ where

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$



i.e. the quark states coupling to the $W^{\pm}$

(the weak eigenstates d′, s′)

are **not** the same as the quark states produced in the strong interactions

(the mass eigenstates d, s)

Note: ascribing this difference to the charge –1/3 quarks d,s rather than to the charge +2/3 quarks u,c is purely conventional

- The small BR for $K^0 \to \mu^+ \mu^-$ was even harder to understand following the discovery of <u>neutral</u> <u>currents</u>: (1972)

$$\boxed{\bar{\nu}_\mu + e^- \;\to\; \bar{\nu}_\mu + e^-}$$

$\bar{\nu}_\mu$ (E_ν ~ 1-2 GeV)

F.J. Hasert et al., Phys. Lett. **46B** (1973) 121

Saw <u>one</u> candidate event in Gargamelle bubble chamber at CERN

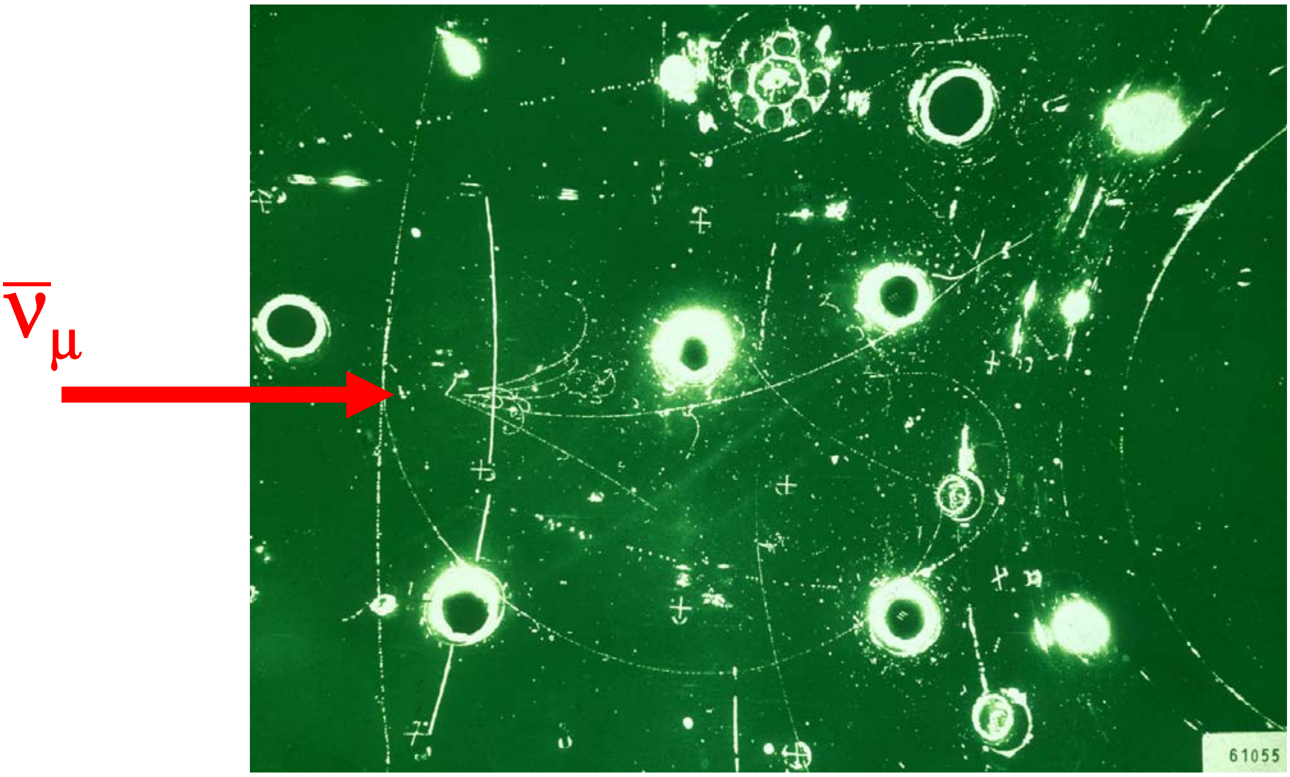
filled with liquid freon (CF_3Br)

Probability that this event was due to some other (background) reaction was < 3%

- Also saw neutral current hadronic events :

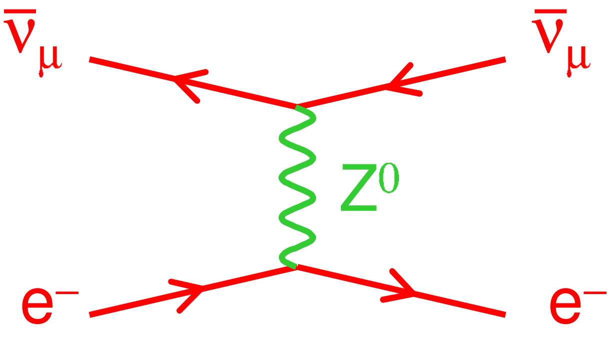
$$\boxed{\nu_\mu + N \rightarrow \nu_\mu + X} \qquad \boxed{\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + X}$$

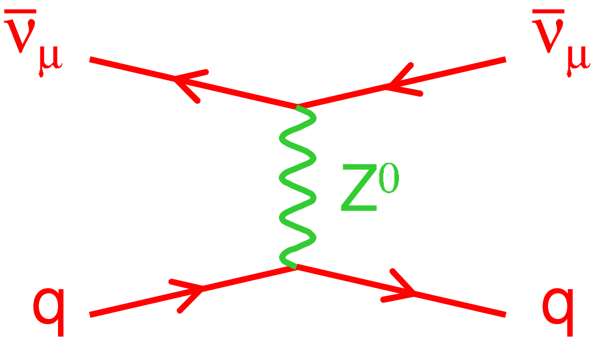
No μ^- or μ^+ in the final state



F.J. Hasert et al., Phys. Lett. **46B** (1973) 138

- Can only proceed via Z^0 exchange:



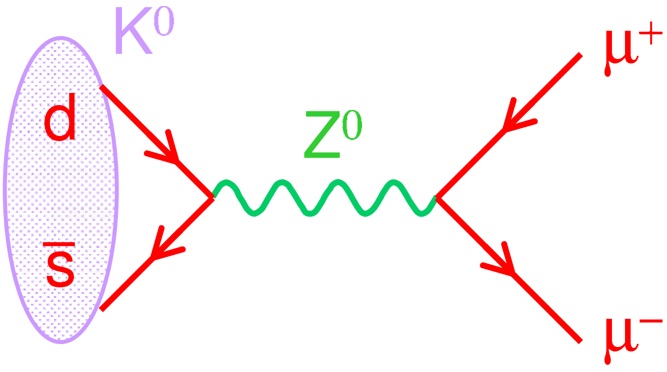


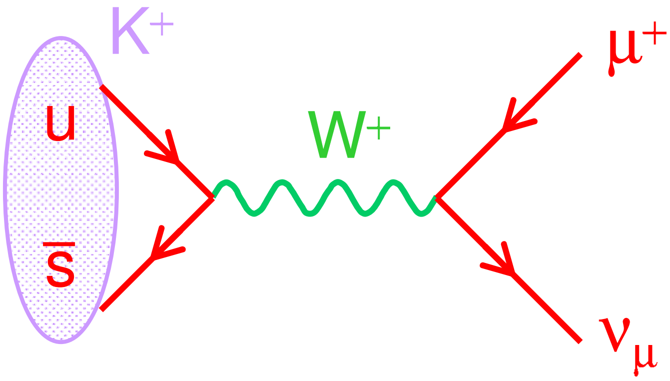
No W diagrams possible

⇒ First (indirect) evidence for Z^0 boson

- With <u>neutral</u> as well as <u>charged</u> currents :

would expect $K^0 \rightarrow \mu^+\mu^-$ at similar rate to $K^+ \rightarrow \mu^+\nu_\mu$





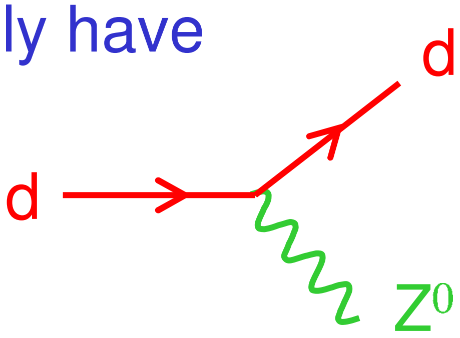
whereas experiment gives

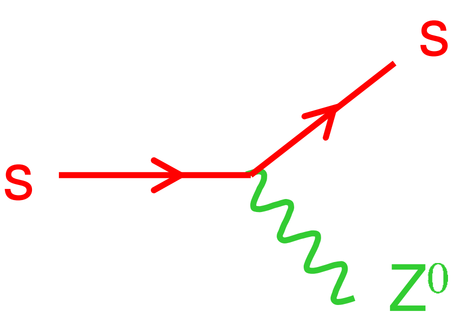
$$\frac{\Gamma(K_L \rightarrow \mu^+\mu^-)}{\Gamma(K^+ \rightarrow \mu^+\nu_\mu)} = 2.6\times10^{-9}$$

- Conclude that

$\boxed{\text{the } Z^0 \text{ cannot change quark flavour}}$

i.e. only have





etc.

i.e. no <u>flavour changing neutral currents</u> (FCNC)

$\boxed{\text{only the W can change flavour}}$

The CKM Matrix

- The GIM mechanism can be generalised straightforwardly to 6 quark flavours:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

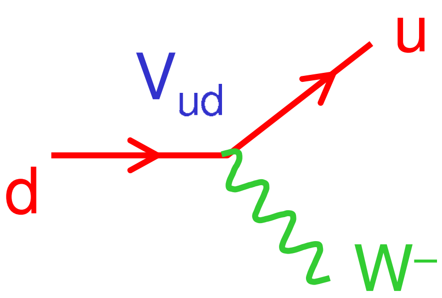
weak eigenstates

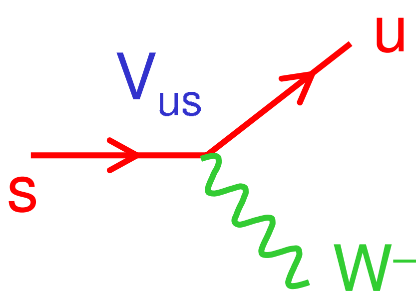
CKM matrix

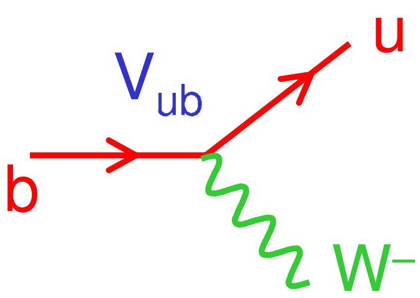
strong eigenstates

(Cabibbo, Kobayashi, Maskawa)

i.e. the weak $W^{\pm}$ vertices contain factors





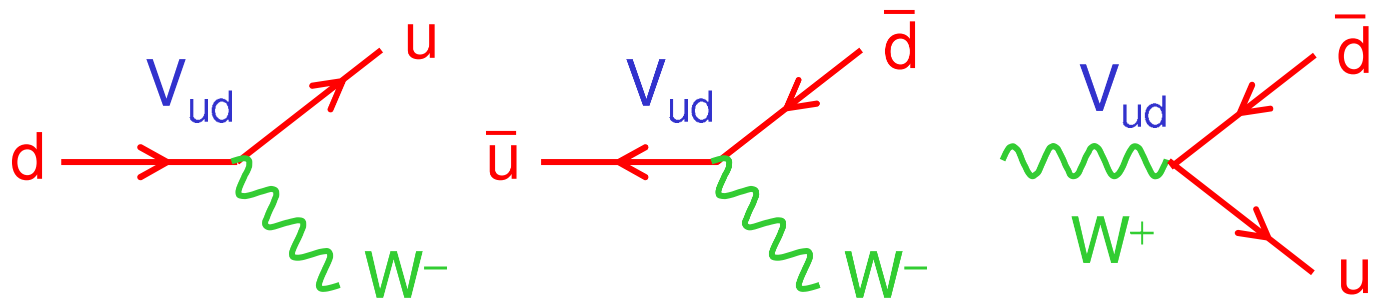


etc.

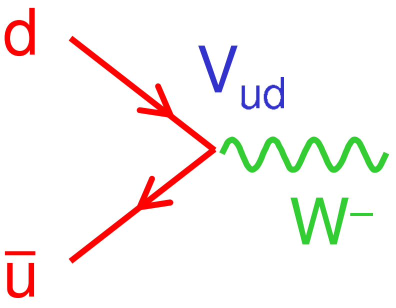
- The V_{ij} are <u>complex</u> constants
- The CKM matrix is <u>unitary</u>: $V_{CKM}^{+} V_{CKM} = I$
- The V_{ij} are <u>not predicted</u> by the SM
- A complex CKM matrix allows <u>CP violation</u>

Extension of Feynman Rules

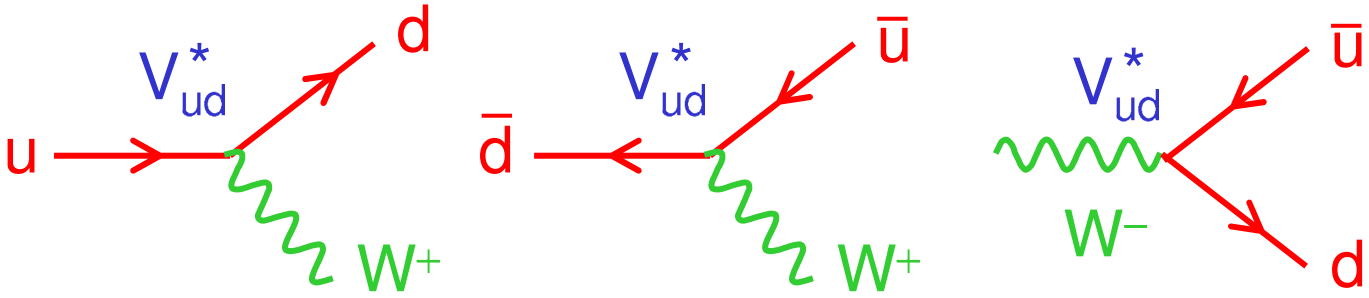
- If arrows flow from d → u, extra factor V_{ud}



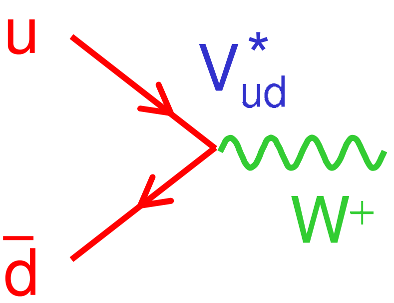
$$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\,\gamma^\mu\,\tfrac{1}{2}(1-\gamma^5)}$$



- If arrows flow from u → d, extra factor V^*_{ud}



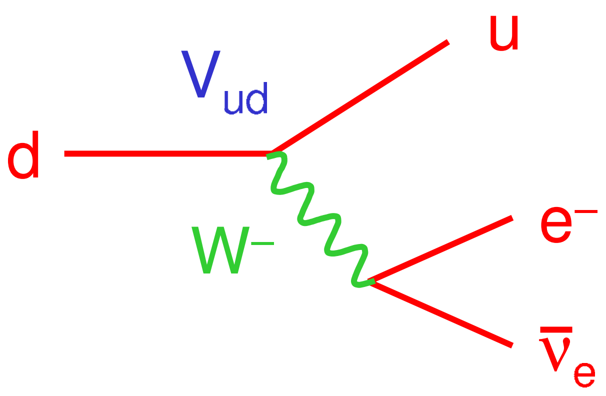
$$\boxed{-i\frac{g_W}{\sqrt{2}}V^*_{ud}\,\gamma^\mu\,\tfrac{1}{2}(1-\gamma^5)}$$



Measurements of $|V_{ij}|$

General theme : use semileptonic final states

1) $|V_{ud}|$: use nuclear β-decay

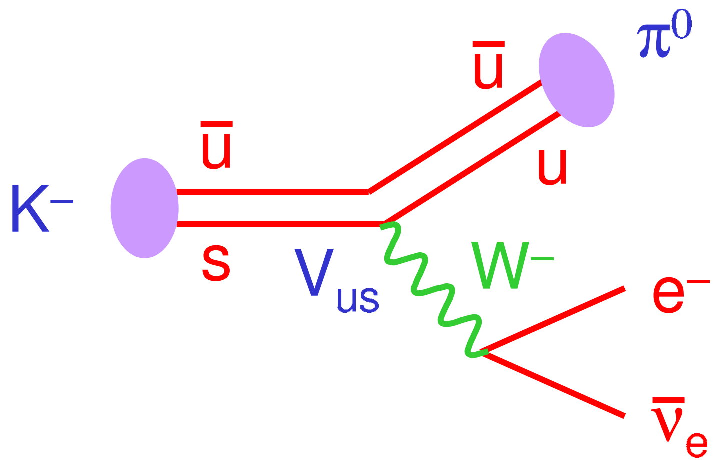


$$\Gamma \propto |V_{ud}|^2$$

superallowed $0^+ \rightarrow 0^+$ decays are especially free of nuclear model uncertainties

$$\Rightarrow \quad \boxed{|V_{ud}| = 0.9738 \pm 0.0005} \quad (\approx \cos\theta_C)$$

2) $|V_{us}|$: use kaon decays



e.g. $K^- \rightarrow \pi^0 e^- \bar{\nu}_e$

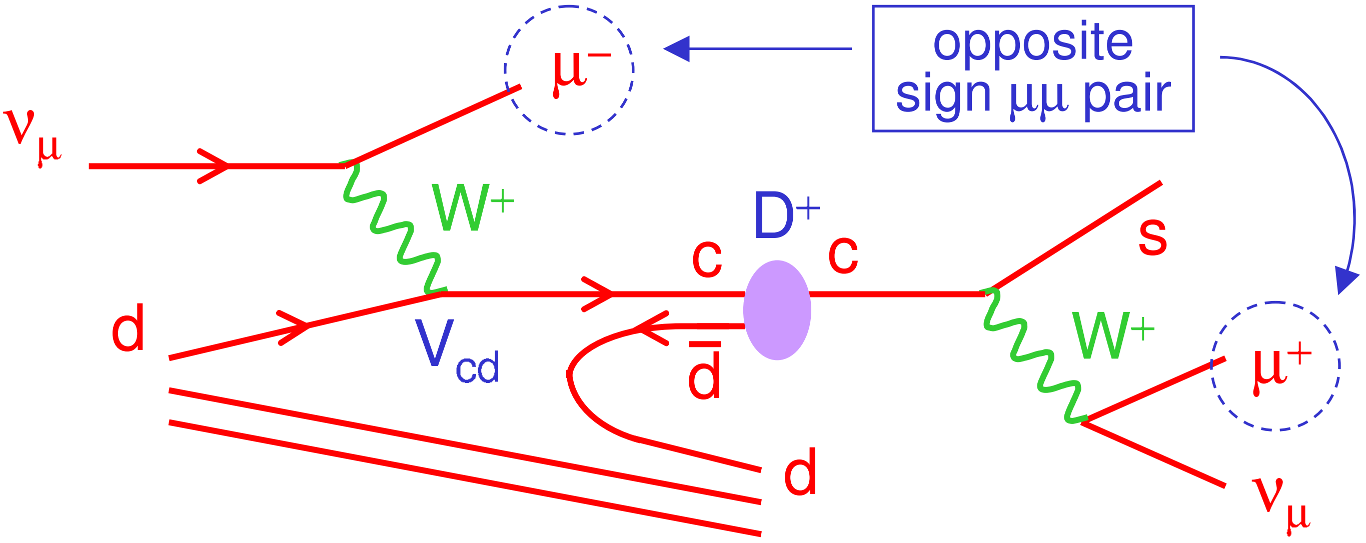
$$\Gamma \propto |V_{us}|^2$$

$$\Rightarrow \quad \boxed{|V_{us}| = 0.2259 \pm 0.0023} \quad (\approx \sin\theta_C)$$

3) $|V_{cd}|$: use neutrino scattering

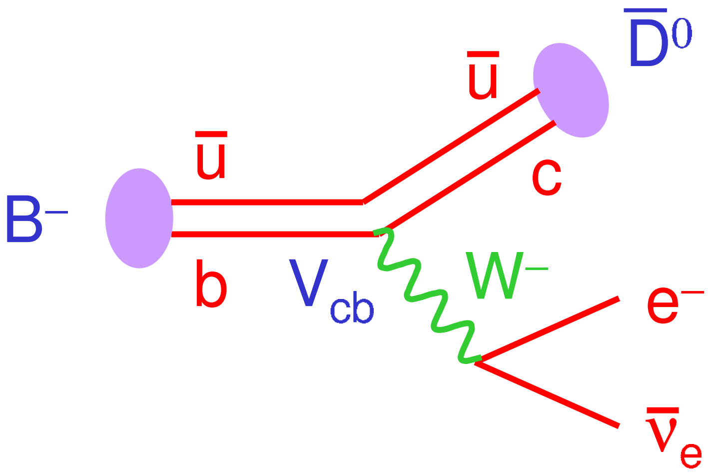
turns out to be better than c → d decays
as these are swamped by c → s decays

Look for dimuon events: $\nu_\mu + N \rightarrow \mu^+ \mu^- X$


opposite sign μμ pair

$$\Rightarrow \quad \boxed{|V_{cd}| = 0.224 \pm 0.012}$$

4) $|V_{cb}|$: use B hadron decays



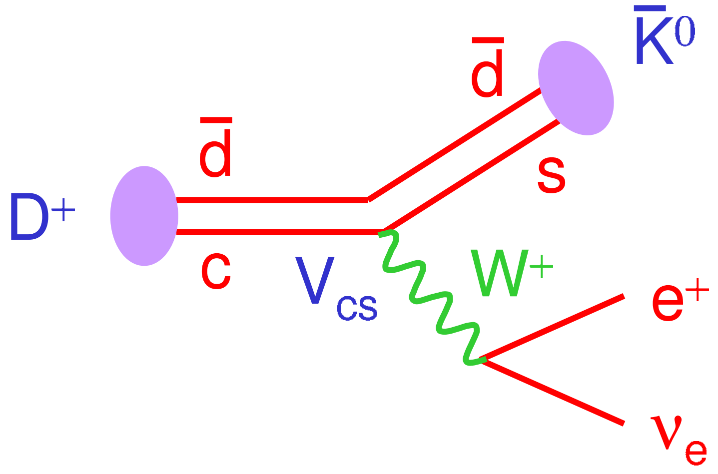
e.g. $B^- \rightarrow \bar{D}^0 e^- \bar{\nu}_e$

$$\Gamma \propto |V_{cb}|^2$$

⇒ τ_B quite "long" (~1.5 ps)

$$\Rightarrow \quad \boxed{|V_{cb}| = 0.0414 \pm 0.0007}$$

5) $|V_{cs}|$: use charm decays



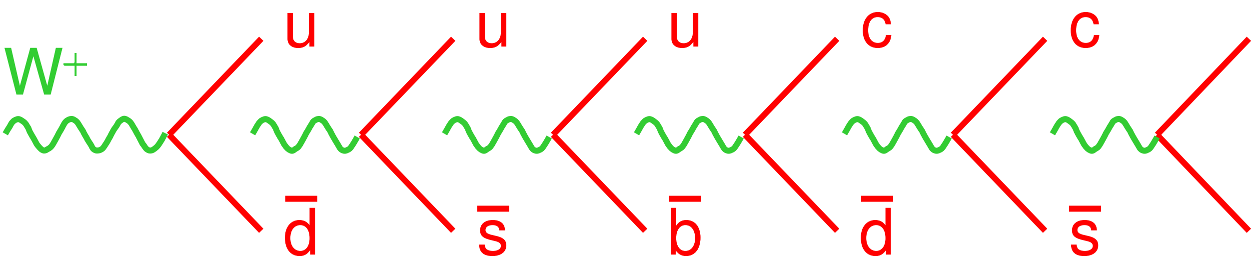
e.g. $D^+ \to \bar{K}^0 e^+ \nu_e$

$$\Gamma \propto |V_{cs}|^2$$

$$\Rightarrow \quad |V_{cs}| = 1.04 \pm 0.16$$

relatively poorly measured: limited statistics and large hadronic uncertainties

Can improve on this using indirect measurement of V_{cs} from W decays:

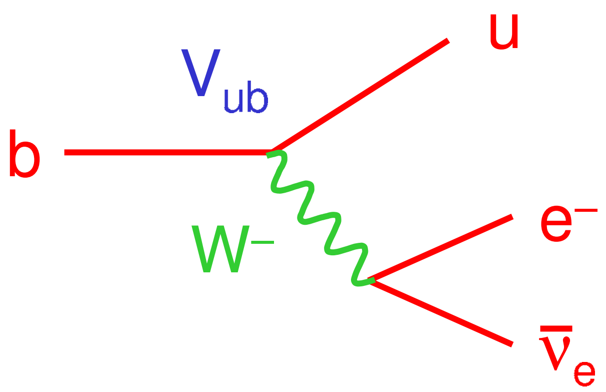


$$\Gamma(W \to q\bar{q}') \propto \left(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 + |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2\right)$$

$$\Rightarrow \quad \boxed{|V_{cs}| = 0.996 \pm 0.013}$$

compared to V_{cs} , the other elements are either small or well measured

6) $|V_{ub}|$: use b → u decays



b → c is dominant decay
mode so hard to extract
b → u

Use end-point of lepton spectrum:

$$\left.\begin{array}{l} b \rightarrow c l^- \bar{\nu}_l \\ b \rightarrow u l^- \bar{\nu}_l \end{array}\right\}$$

u quark mass smaller than c

⇓

maximum lepton energy
larger for b → u than b → c

$$\Rightarrow \qquad \boxed{|V_{ub}| = 0.0041 \pm 0.0007}$$

The story so far …

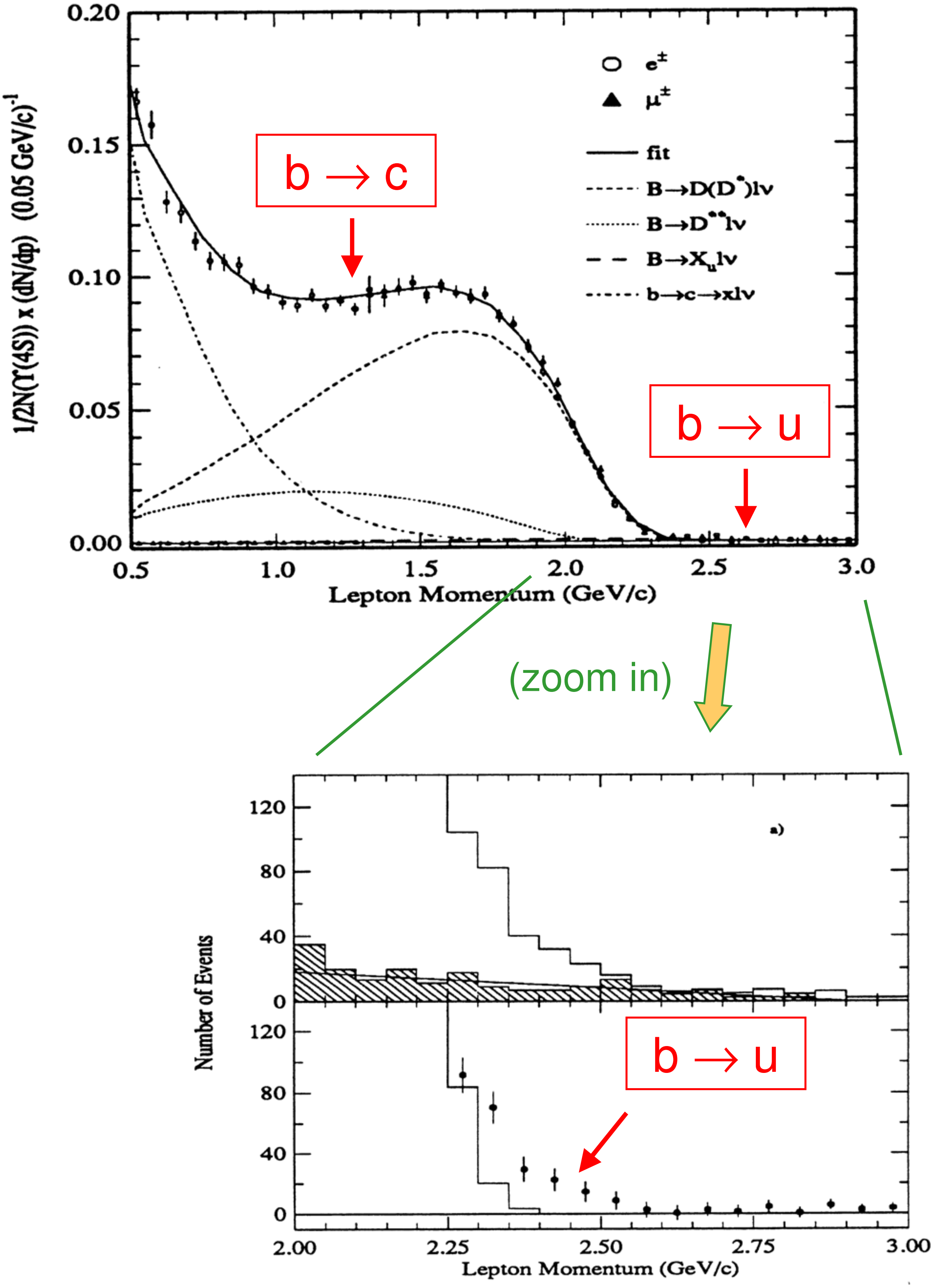
$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ 0.22 & 0.99 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

Could use unitarity to fill in the rest

$$\Rightarrow \qquad V_{td} \sim 0, \quad V_{ts} \sim 0, \quad V_{tb} \sim 1$$

(sum of squares within each column = 1)

Measurement of V_{ub}



b → c

b → u

(zoom in)

b → u

Direct information on CKM elements involving the t quark :

7) $|V_{tb}|$: use $t \rightarrow b l^{+} \nu$ decays

$$\Rightarrow \quad \boxed{\left|V_{\mathrm{tb}}\right| = 0.94 \pm 0.29}$$

(only a handful of events)

8) $|V_{ts}|$: use $\mathrm{B}_s^0 - \overline{\mathrm{B}}_s^0$ mixing

9) $|V_{td}|$: use $\mathrm{B}_d^0 - \overline{\mathrm{B}}_d^0$ mixing

} analagous to $\mathrm{K}^0 - \overline{\mathrm{K}}^0$ mixing

◆ B_d mixing has already been observed:

$$\Rightarrow \quad \boxed{\left|V_{\mathrm{tb}} V_{\mathrm{td}}\right| = 0.0084 \pm 0.0018}$$

◆ B_s mixing not yet seen

$$\Rightarrow \quad \boxed{\left|V_{\mathrm{td}} / V_{\mathrm{ts}}\right| < 0.27}$$

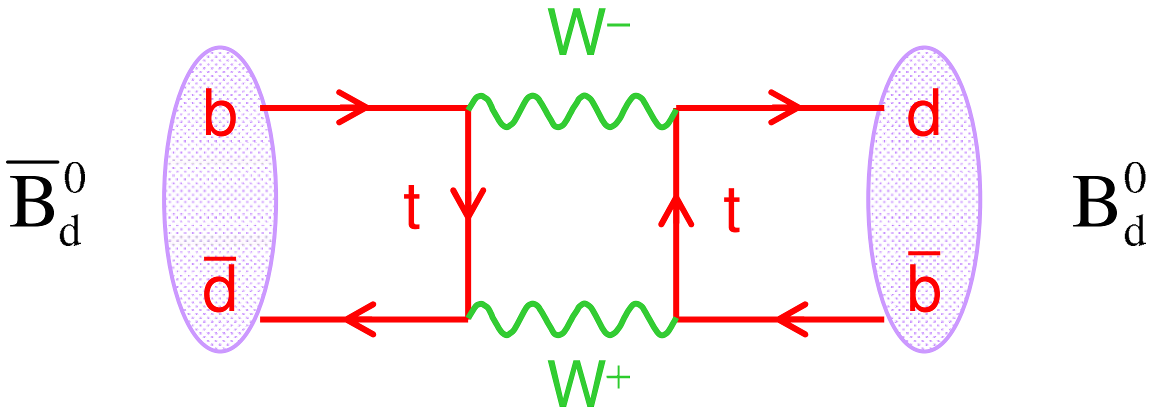
So, at the moment :

the CKM elements V_{tb} V_{ts} V_{td} involving top are best constrained using unitarity

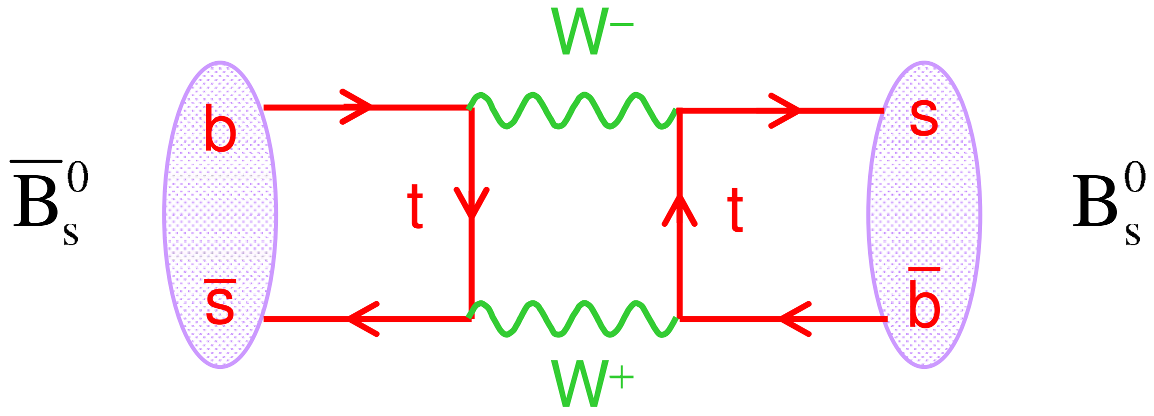
(but really, we would like to test unitarity)

The $B^0 - \bar{B}^0$ Systems

- Similar to K^0–$\bar{K}^0$ system:



Diagrams involving t quark give dominant contribution since V_{tb} much greater than V_{cb} and V_{ub}



- Expect B^0–$\bar{B}^0$ oscillations with frequency determined by mass differences:

$$\Delta m_{\mathrm{d}} \propto |M_{\mathrm{fi}}| \propto |V_{\mathrm{tb}} V_{\mathrm{td}}|^2$$

$$\Delta m_{\mathrm{s}} \propto |M_{\mathrm{fi}}| \propto |V_{\mathrm{tb}} V_{\mathrm{ts}}|^2$$

(Handout 8.12)

e.g. B_d mixing signal from OPAL at LEP:

e.g. at LEP: $e^+e^- \to Z^0 \to b\,\bar{b}$

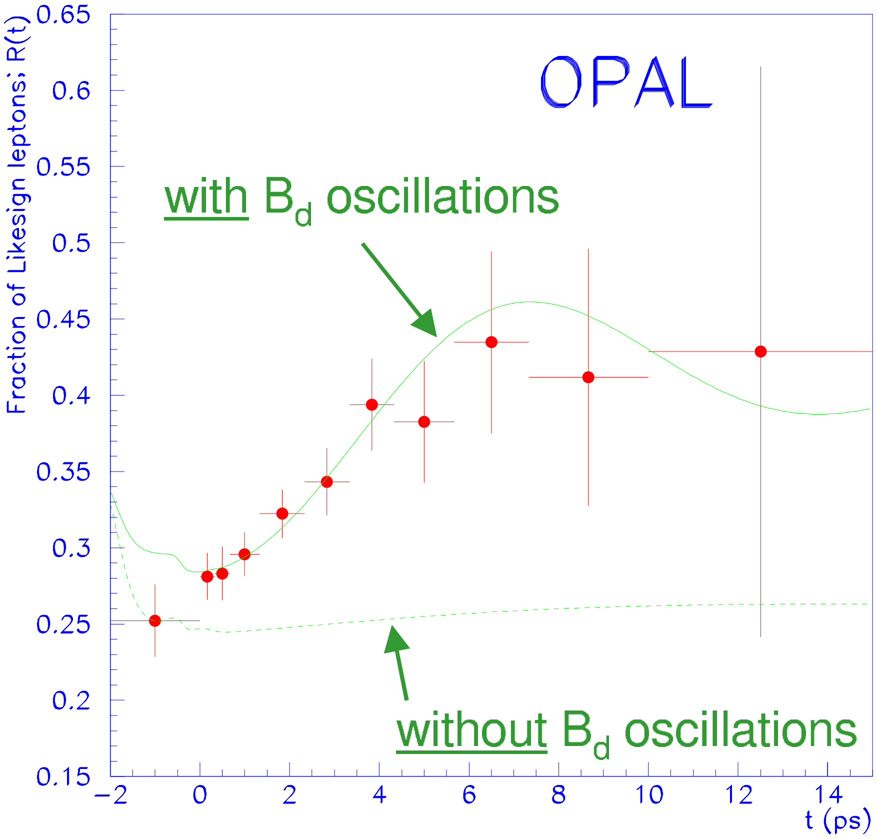
mixing

$\bar{b} \to b \to c l^- \bar{\nu}_l$

$b \to c l^- \bar{\nu}_l$

look for $l^- l^-$ events

$$R(t) = \frac{N(l^+l^+) + N(l^-l^-)}{N(l^+l^+) + N(l^-l^-) + N(l^+l^-) + N(l^-l^+)}$$



Current world average:

$$\Delta m_{\mathrm{d}} = 0.464 \pm 0.018\ \mathrm{ps}^{-1}$$
$$= (3.05 \pm 0.12) \times 10^{-13}\ \mathrm{GeV}$$

B_s oscillations are too rapid to be resolved (yet)

$$\Longrightarrow \quad \Delta m_{\mathrm{s}} > 59.9 \times 10^{-13}\ \mathrm{GeV}$$

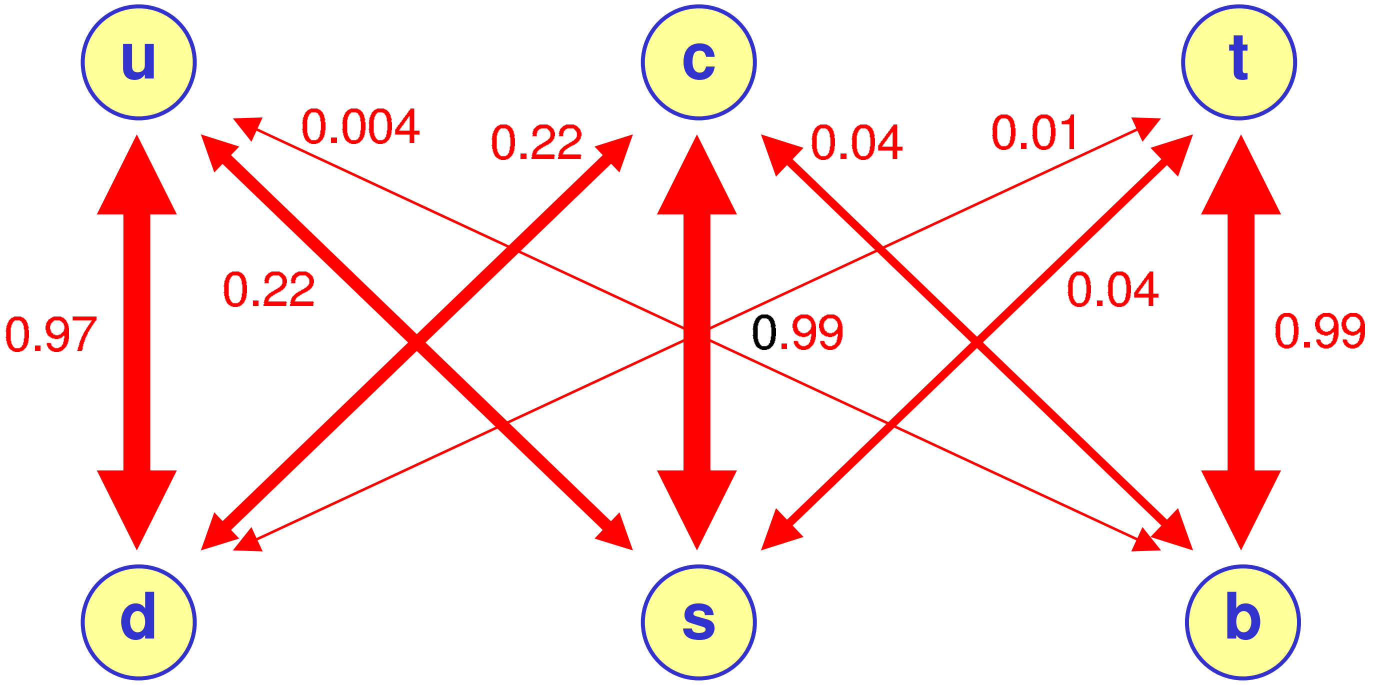
Putting it all together ….

≈ Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.97 & 0.22 & 0.004 \\ 0.22 & 0.99 & 0.04 \\ 0.01 & 0.04 & 0.99 \end{pmatrix}$$

⇒ CKM matrix approximately diagonal

(elements get smaller further from diagonal)



⇒ each generation is only loosely coupled to the others

W± interactions are :

- the only way to change quark flavour
- the only way to change from one generation of quarks and leptons to another

CP Violation in the SM

(Handout 8.11)

- A standard parameterisation of the CKM matrix is the Wolfenstein parameterisation :

$$\begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$V_{\mathrm{CKM}}^{+} V_{\mathrm{CKM}} = 1 + \mathrm{O}(\lambda^4)$$

(reflects hierarchy of observed values)

- λ and A are quite well measured :

$$\lambda = 0.2265 \pm 0.0020$$ (from $\sin\theta_{\mathrm{C}}$)

$$A = 0.81 \pm 0.02$$ (from V_{cb})

- The CKM matrix contains a complex phase (the phase of $\rho + i\eta$) which allows CP violation within the SM :

$$\boxed{\eta \neq 0 \quad \Longrightarrow \quad \text{CP is violated}}$$

N.B. needs 3 (or more) generations of quarks
(2 x 2 unitary matrices are purely real)

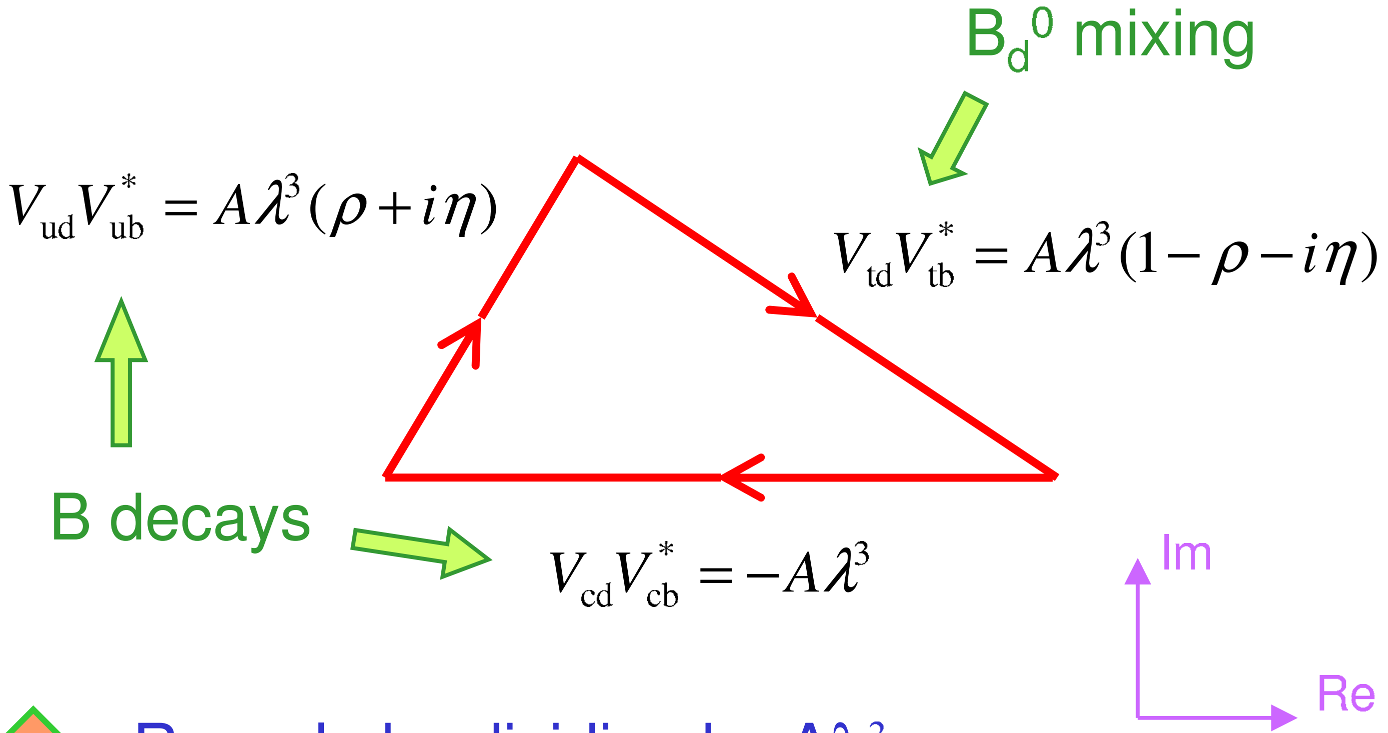
The Unitarity Triangle

- Unitarity of CKM matrix gives

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0$$

(plus five other similar equations)

Can represent this in the complex plane as the unitarity triangle (UT)



B$_d{}^0$ mixing

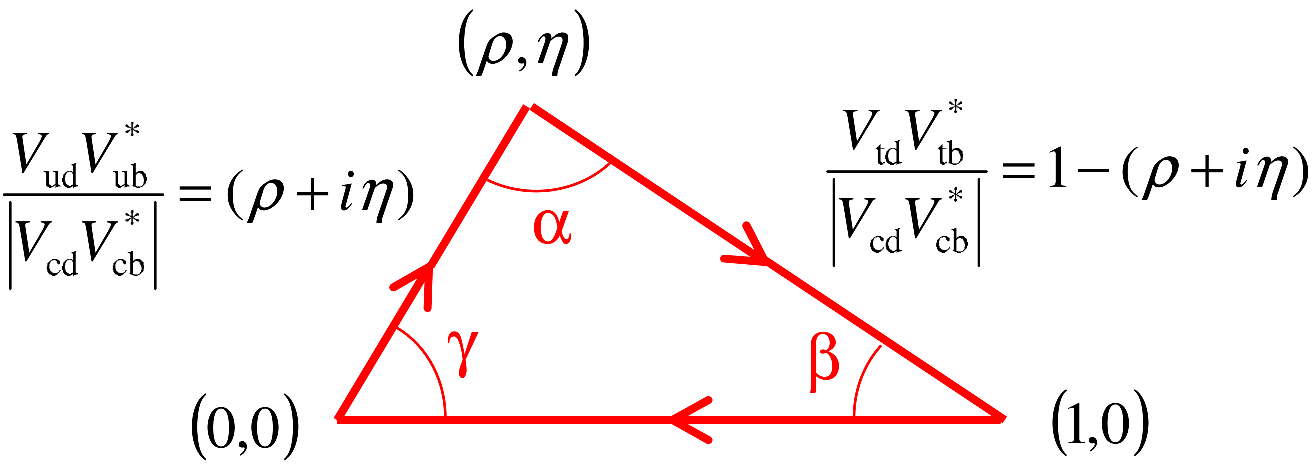
$$V_{ud}V_{ub}^{*}=A\lambda^{3}(\rho+i\eta)$$

$$V_{td}V_{tb}^{*}=A\lambda^{3}(1-\rho-i\eta)$$

B decays

$$V_{cd}V_{cb}^{*}=-A\lambda^{3}$$

- Rescale by dividing by Aλ^3 :



$$\frac{V_{ud}V_{ub}^{*}}{|V_{cd}V_{cb}^{*}|}=(\rho+i\eta)$$

$$\frac{V_{td}V_{tb}^{*}}{|V_{cd}V_{cb}^{*}|}=1-(\rho+i\eta)$$

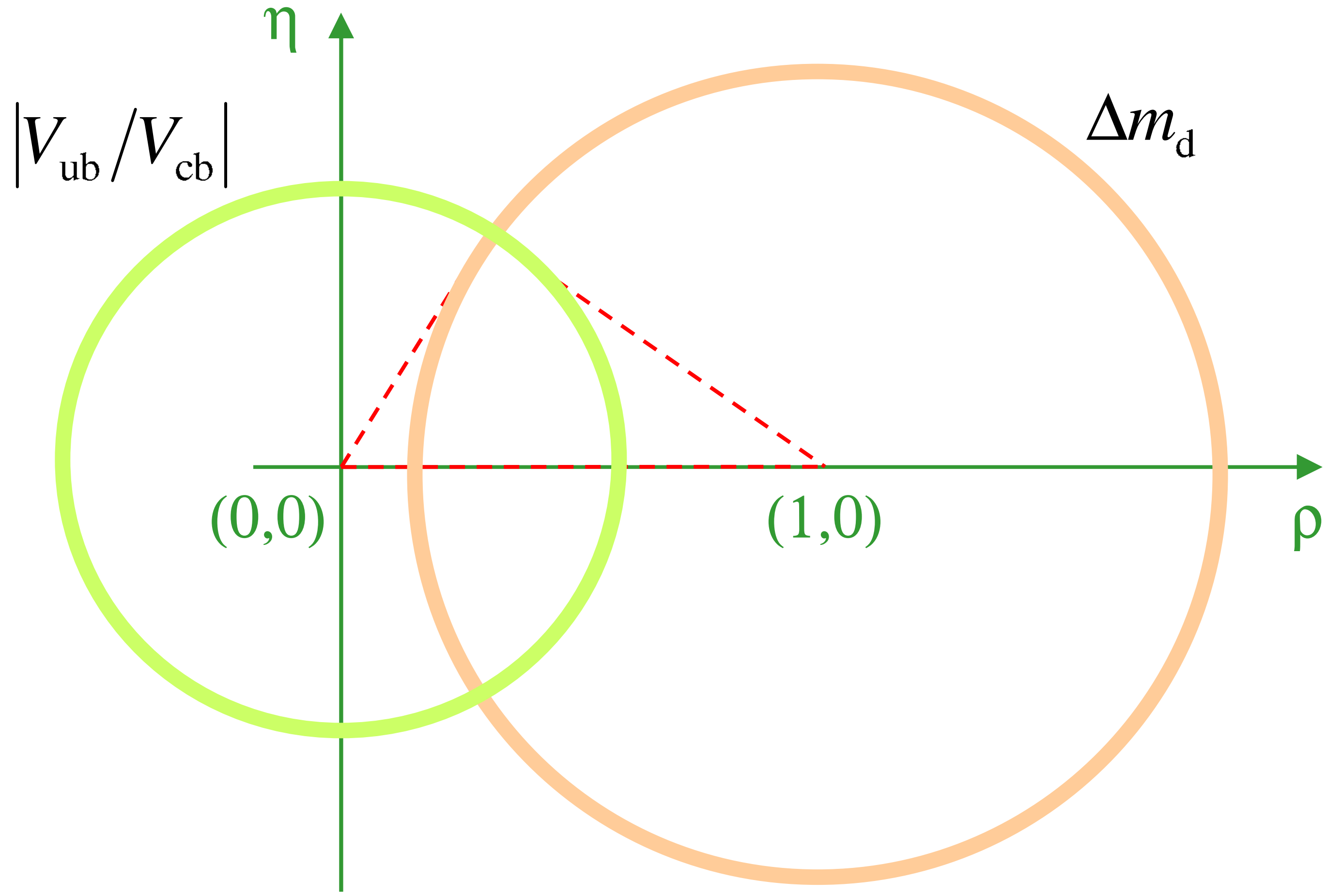
Experimental constraints on ρ and η

- B_d mixing:

$$\Delta m_{\mathrm{d}} \propto \left|V_{\mathrm{td}} V_{\mathrm{tb}}^*\right|^2$$

$$\left|V_{\mathrm{td}} V_{\mathrm{tb}}^*\right| / A\lambda^3 = \sqrt{(1-\rho)^2 + \eta^2}$$

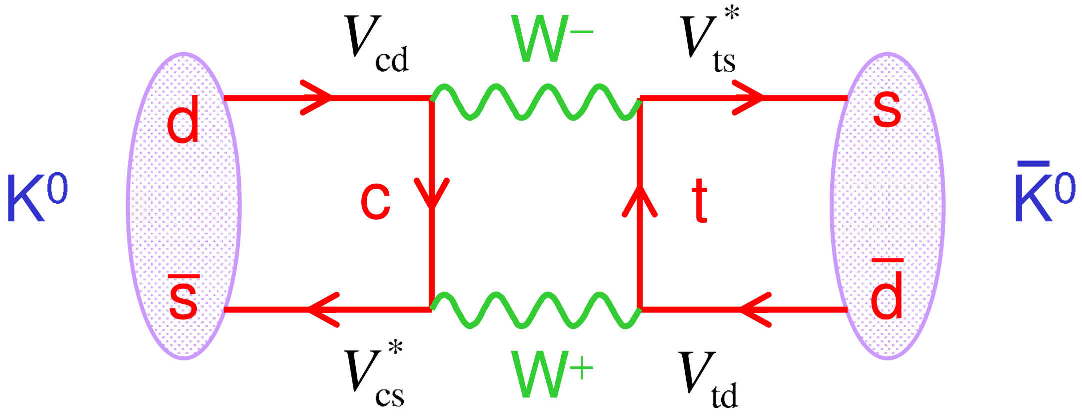
⇒ A measurement of Δm_d fixes the radius of a circle centred on (1,0)



- b → u decays : $\left|V_{\mathrm{ub}}\right| / \left|V_{\mathrm{cb}}\right| = \lambda\sqrt{\rho^2 + \eta^2}$

⇒ A measurement of BR(b→u) fixes the radius of a circle centred on (0,0)

CP violation in neutral kaons :



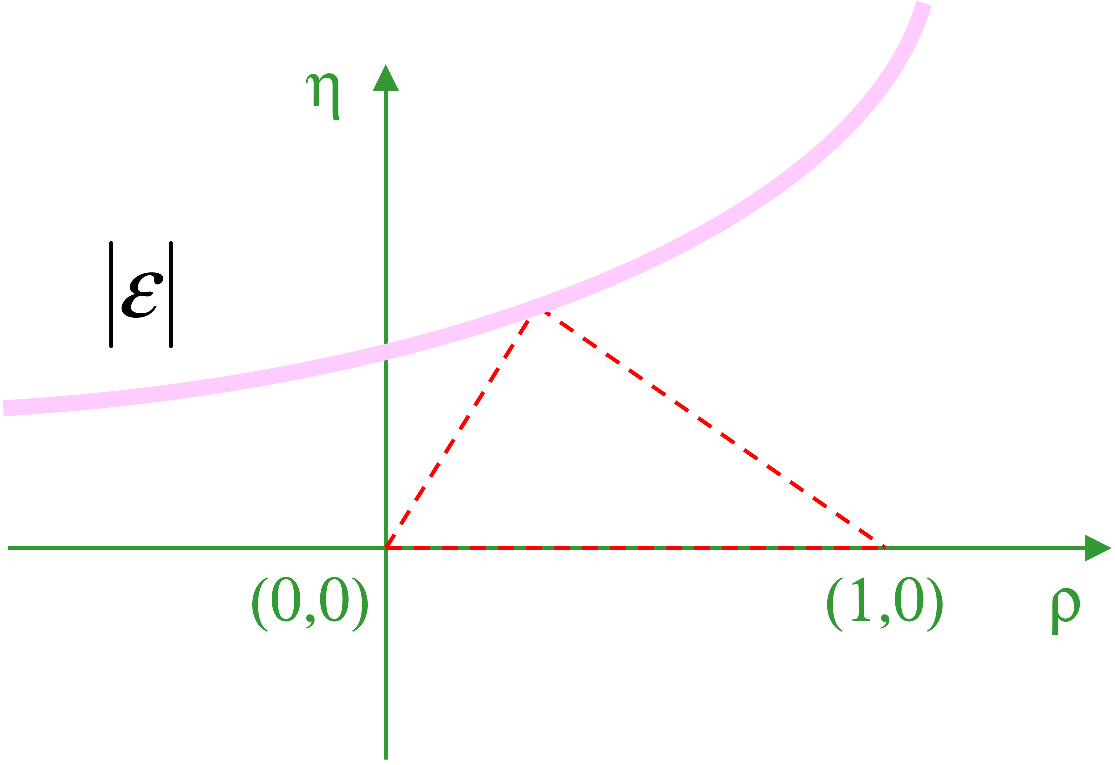
$$|\varepsilon| \propto \mathrm{Im}\left(M_{fi}\right)$$

$$|\varepsilon| \sim A_{ut}.\mathrm{Im}\left(V_{ud}V_{us}^{*}V_{td}V_{ts}^{*}\right) + A_{ct}.\mathrm{Im}\left(V_{cd}V_{cs}^{*}V_{td}V_{ts}^{*}\right) + A_{tt}.\mathrm{Im}\left(V_{td}V_{ts}^{*}\right)^{2}$$

$$\sim \eta[1-\rho+\mathrm{const}]$$

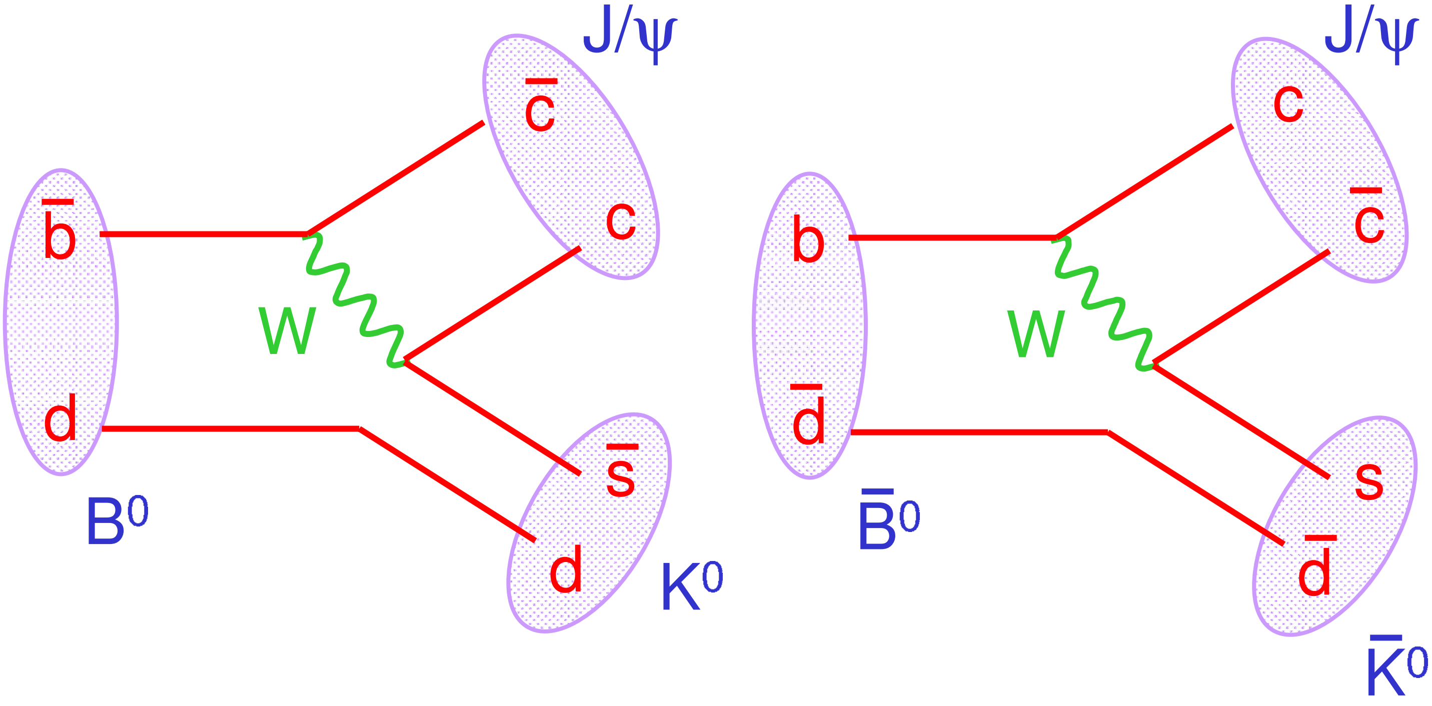
(examples sheet)

⇒ A measurement of $|\varepsilon|$ determines a hyperbola in the (ρ,η) plane :



(η ≠ 0 always: CP violating)

- 2001: CP violation is observed outside the K^0 system
  in $B_d^0\left(\overline{B}_d^0\right) \to J/\psi + K_{S,L}$ decays



- Measures angle β of unitarity triangle :

(Handout 8.10)

$$\Gamma\left(B^0_{t=0} \to \psi K_{S,L}\right) \propto e^{-\Gamma t}\left(1 - \sin 2\beta \sin \Delta m_d t\right)$$

$$\Gamma\left(\overline{B}^0_{t=0} \to \psi K_{S,L}\right) \propto e^{-\Gamma t}\left(1 + \sin 2\beta \sin \Delta m_d t\right)$$

$$A_{\psi K_{S,L}} \equiv \frac{\Gamma\left(\overline{B}^0_{t=0} \to \psi K_{S,L}\right) - \Gamma\left(B^0_{t=0} \to \psi K_{S,L}\right)}{\Gamma\left(\overline{B}^0_{t=0} \to \psi K_{S,L}\right) + \Gamma\left(B^0_{t=0} \to \psi K_{S,L}\right)}$$

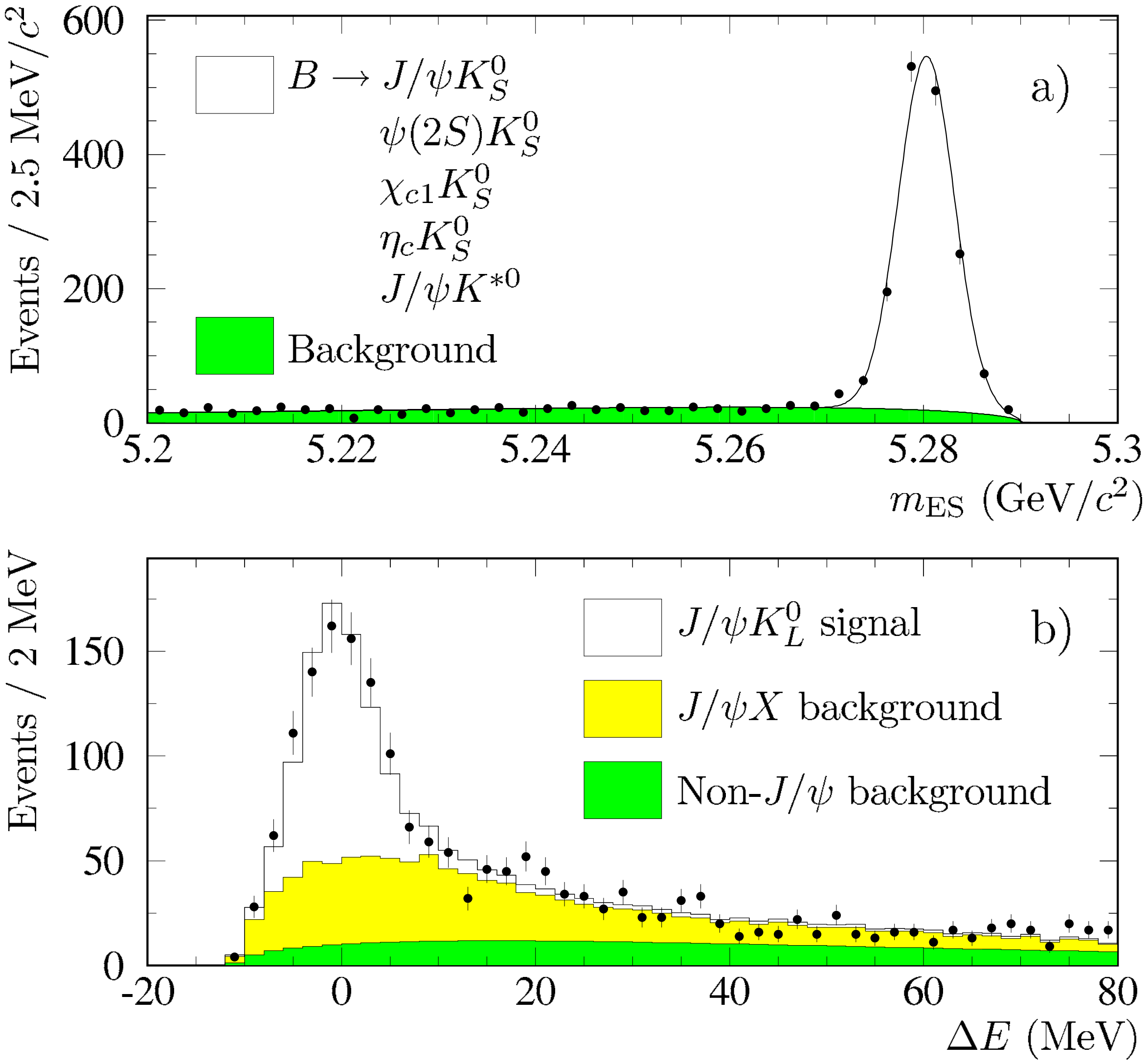
$= +\sin 2\beta \sin \Delta m_d t$ (K_S) $CP(\psi K_S) = -1$

$= -\sin 2\beta \sin \Delta m_d t$ (K_L) $CP(\psi K_L) = +1$

e.g. BABAR experiment at SLAC "B Factory"

From 88 million $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ decays

(bottomonium $b\bar{b}$)



B.Aubert et al., Phys. Rev. Lett. 89 (2002) 201802-1

~ 1500 B^0 (or $\bar{B}^0$) → $J/\psi.K_S$ decays

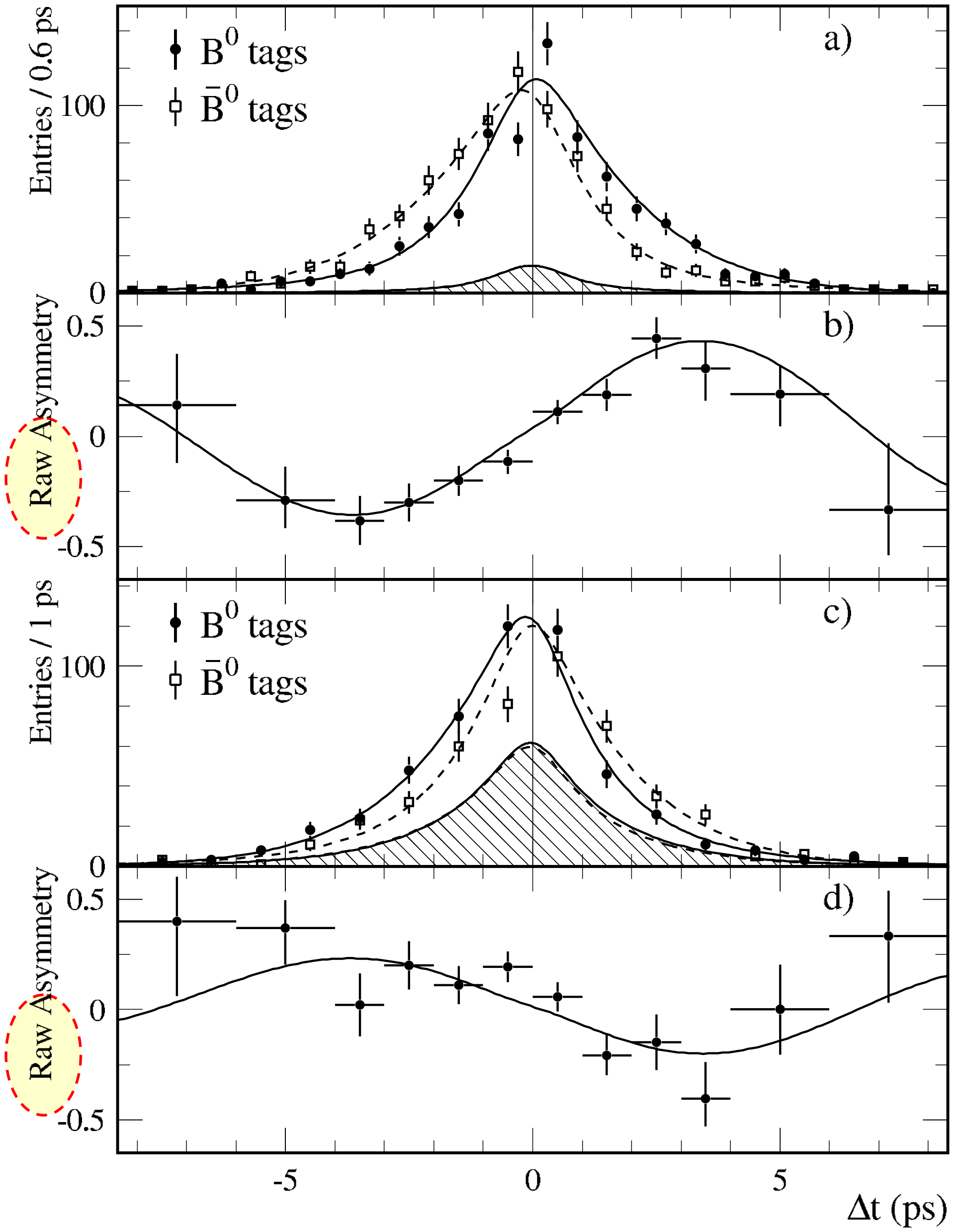
~ 900 B^0 (or $\bar{B}^0$) → $J/\psi.K_L$ decays

$$\mathrm{BR}\left(B^0 \rightarrow J/\psi + K_{S,L}\right) \sim 8\times10^{-4}$$

Asymmetry :

$$A_{\psi K_{S,L}} \equiv \frac{\Gamma\left(\overline{B}^0_{t=0} \to \psi K_{S,L}\right) - \Gamma\left(B^0_{t=0} \to \psi K_{S,L}\right)}{\Gamma\left(\overline{B}^0_{t=0} \to \psi K_{S,L}\right) + \Gamma\left(B^0_{t=0} \to \psi K_{S,L}\right)}$$

$$= \pm \sin 2\beta \sin \Delta m_d t$$



J/ψ.K$_S$

J/ψ.K$_L$

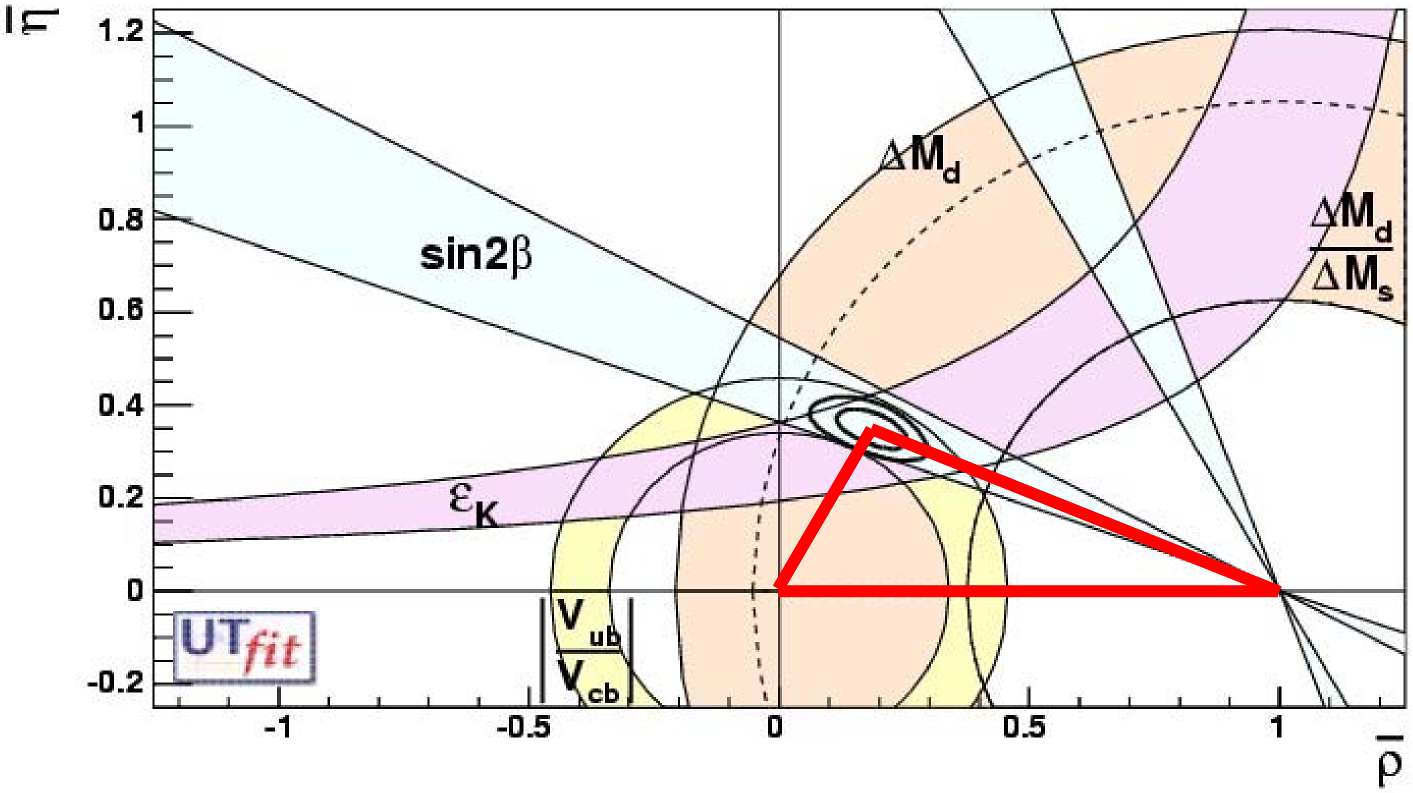
Amplitude measures sin2β :

BABAR : $\sin 2\beta = 0.741 \pm 0.075$

BELLE: $\sin 2\beta = 0.719 \pm 0.082$

Summary of current measurements of the unitarity triangle (UT) :

(hep-ph/0408079)



- Measurements have a common overlap at

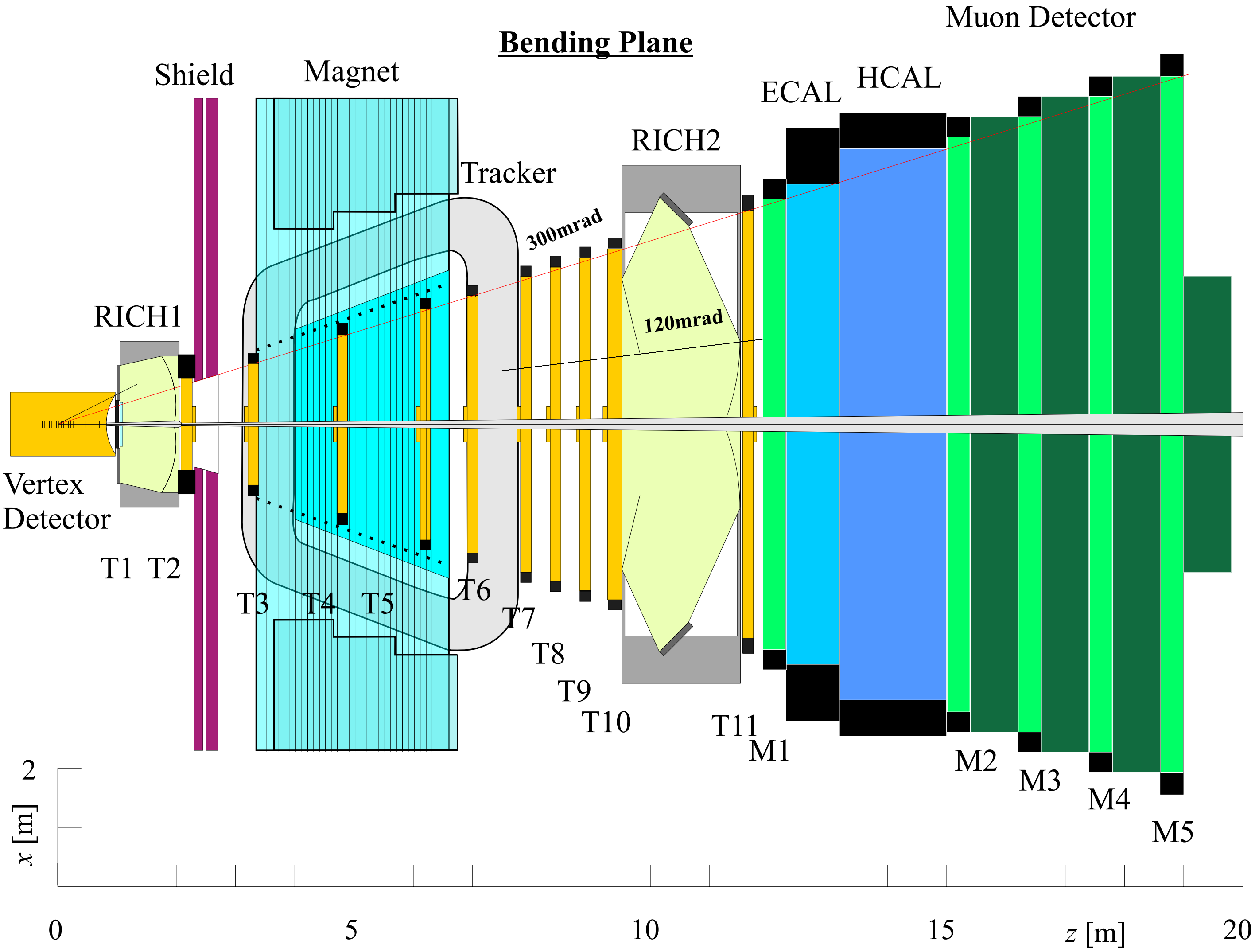
$$\boxed{\begin{aligned}\rho &= 0.172 \pm 0.047 \\ \eta &= 0.348 \pm 0.028\end{aligned}}$$

⇒ data is consistent with Standard Model explanation of origin of CP violation

(but errors still quite large)

- CP violation is **<u>not</u>** an intrinsically small effect : η is a parameter of order unity

- Programme of precision measurements :
  - “B-factories” at SLAC and KEK (Japan)
  - LHCb experiment at LHC (2007)



→ accurately measure sides and angles of UT :

- will they be consistent ?
- will angles add up to 180° ?

- Is the CP violation in the SM enough for the Big Bang ?