Deep Inelastic Scattering

- At large q^2, elastic scattering becomes relatively unlikely :

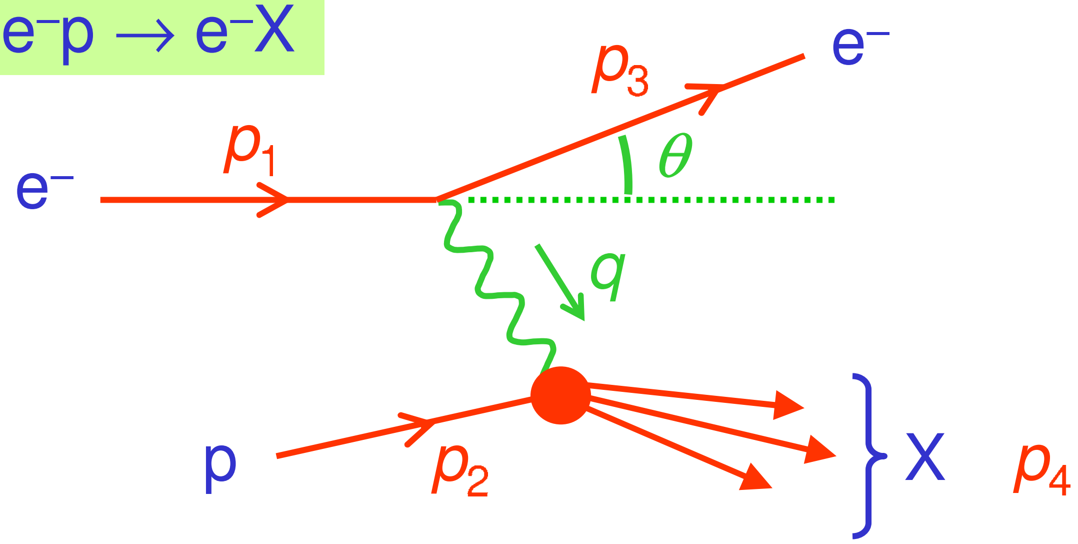
$$G_M(q^2) \sim \frac{1}{q^4} \quad \Rightarrow \quad \frac{d\sigma}{dq^2} \sim \frac{1}{q^{12}}$$

$\Rightarrow$ proton unlikely to recoil intact

Instead: proton breaks up into hadrons …

… inelastic scattering more likely

$e^-p \rightarrow e^-X$

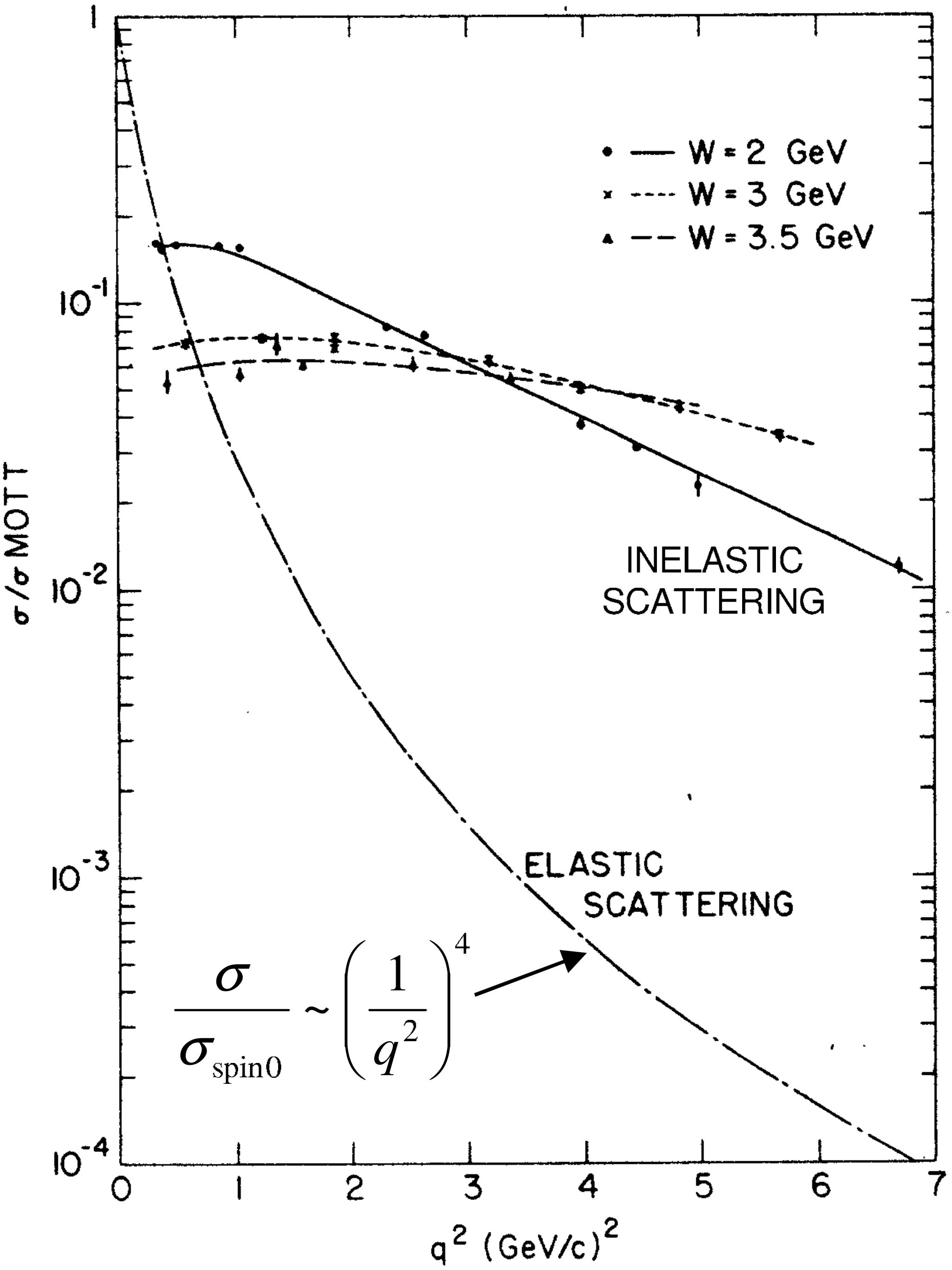


⇨ "Deep Inelastic Scattering" (DIS)

$q^2 >> M^2$ proton breaks up

- probe proton structure up to $q^2 \sim 10000\ \mathrm{GeV}^2$ (i.e. down to $\sim 10^{-18}$ m)

Comparison of elastic and inelastic cross sections :

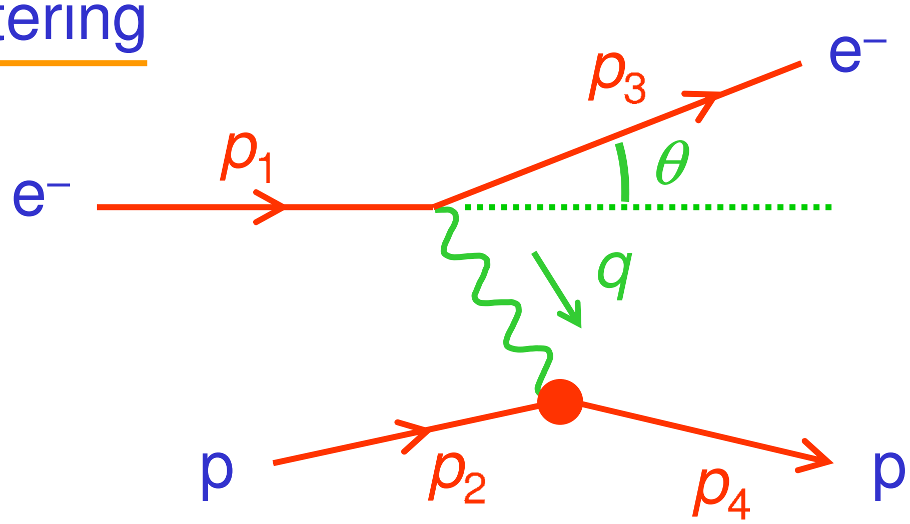


M.Breidenbach et al.,
Phys. Rev. Lett. **23** (1969) 935

W ≡ mass of system X (target fragments)

(cf. proton mass = 0.938 GeV)

Elastic scattering



$e^-p \to e^-p$

$$(q+p_2)^2 = p_4^2 \quad \Rightarrow \quad q^2 + M^2 + 2p_2 \cdot q = M^2$$

$$\Rightarrow \quad q^2 = -2p_2 \cdot q$$

$\Rightarrow$ E_3 and θ are related :

(lab frame)

$$q^2 = -4E_1E_3\sin^2\frac{\theta}{2} = -2M(E_1 - E_3)$$

Define

$$x \equiv \frac{Q^2}{2p_2.q}$$

Bjorken x

(Lorentz invariant)

where

$$Q^2 \equiv -q^2 > 0$$

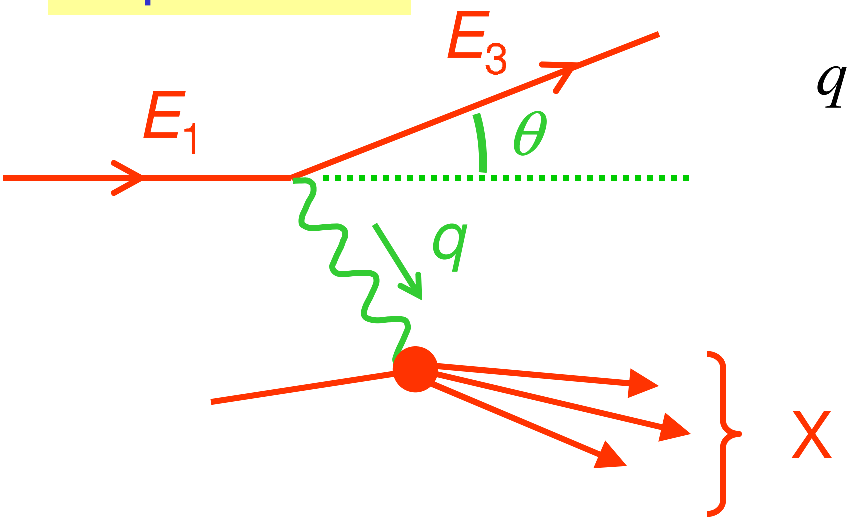
Elastic scattering constraint is then just

$$x = 1$$

$\Rightarrow$ only one independent variable (Q^2, say)

Inelastic scattering

$e^- p \to e^- X$



mass of system X not fixed :

$$q^2 + M^2 + 2p_2.q = M_X^2$$

$$M_X > M$$

(final state contains at least one baryon)

$$\Rightarrow \quad Q^2 < 2p_2.q \quad \Rightarrow \quad \boxed{0 < x < 1}$$

Now two independent variables (x and Q^2, say)

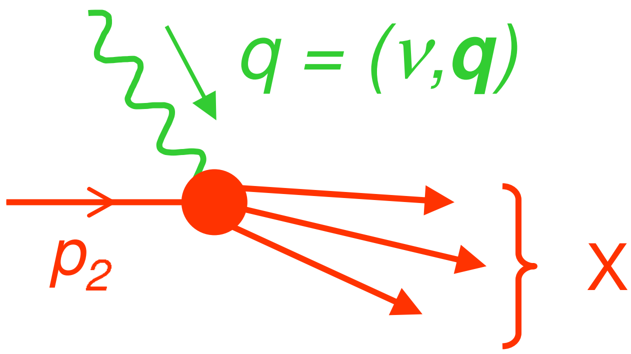
Equivalently : E_3 and θ no longer related

Can also define

$$\boxed{\nu \equiv \frac{p_2 \cdot q}{M}}$$

(Lorentz invariant)

$$\Rightarrow \quad x \equiv \frac{Q^2}{2M\nu}$$



In lab frame : $\nu = E_1 - E_3$

DIS kinematics

Have now introduced **four** Lorentz invariant variables to describe inelastic scattering:

$$Q^2 = -q^2 = -(p_1 - p_3)^2 > 0$$

$$x \equiv \frac{Q^2}{2p_2.q} \qquad y \equiv \frac{p_2.q}{p_2.p_1} \qquad \nu \equiv \frac{p_2.q}{M}$$

plus $s = (p_1 + p_2)^2$ (regarded as fixed)

- Only **two** of Q^2,ν,x,y are independent

$$Q^2 = \left(s - M^2\right)xy \qquad x = \frac{Q^2}{2M\nu} \qquad y = \frac{2M\nu}{s - M^2}$$

- Any pair of them can be used

[except (y,ν)]

In proton rest frame:

$$Q^2 = 4E_1E_3\sin^2\frac{\theta}{2} \qquad y = \frac{E_1 - E_3}{E_1}$$

$$\nu = E_1 - E_3 \qquad s = M\left(2E_1 + M\right)$$

In e⁻q centre of mass frame:

$$y = \frac{1}{2}(1 - \cos\theta^*)$$

- Elastic scattering : $G_E(Q^2)$, $G_M(Q^2)$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(\frac{G_E^2+\tau G_M^2}{1+\tau}\left(1\text{-}y\text{-}\frac{M^2y^2}{Q^2}\right)+\frac{y^2}{2}G_M^2\right)$$

(Rosenbluth) $\tau \equiv Q^2/4M^2$

- Inelastic scattering : $G(Q^2) \rightarrow F(x,Q^2)$

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{F_2}{x}\left(1\text{-}y\text{-}\frac{M^2x^2y^2}{Q^2}\right)+\frac{y^2}{2}\frac{2xF_1}{x}\right]$$

- F_1, F_2 known as structure functions:
  - not directly related to FT of charge or magnetic moment distribution (because also depend on *x*)
  - provide info on number and properties of quark and gluon constituents in proton

    ⇨ "quark-parton model"
  - cannot (yet) be predicted from first principles (QCD)

    ⇨ must be measured ….

Bjorken Scaling

- To good approximation, F_1 and F_2 are found to depend only on x at fixed Q^2 :

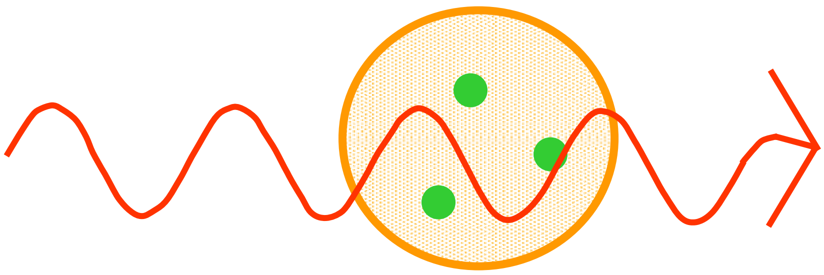
$$F(x, Q^2) \rightarrow F(x)$$

"Bjorken scaling"

At fixed x, the scattering is independent of Q^2

- Suggests that the virtual photon is scattering off pointlike constituents within the nucleon:

$$\lambda \sim \frac{2\pi}{|\mathbf{q}|} \sim \frac{4\pi M x}{Q^2}$$



Probing something pointlike

$\Rightarrow$ cross section independent of λ

(a point looks like a point whatever the wavelength)

$\Rightarrow$ cross section independent of Q^2

But substructure with length scale $1/Q_0$ (say)

$\Rightarrow$ cross section would depend on Q^2/Q_0^2

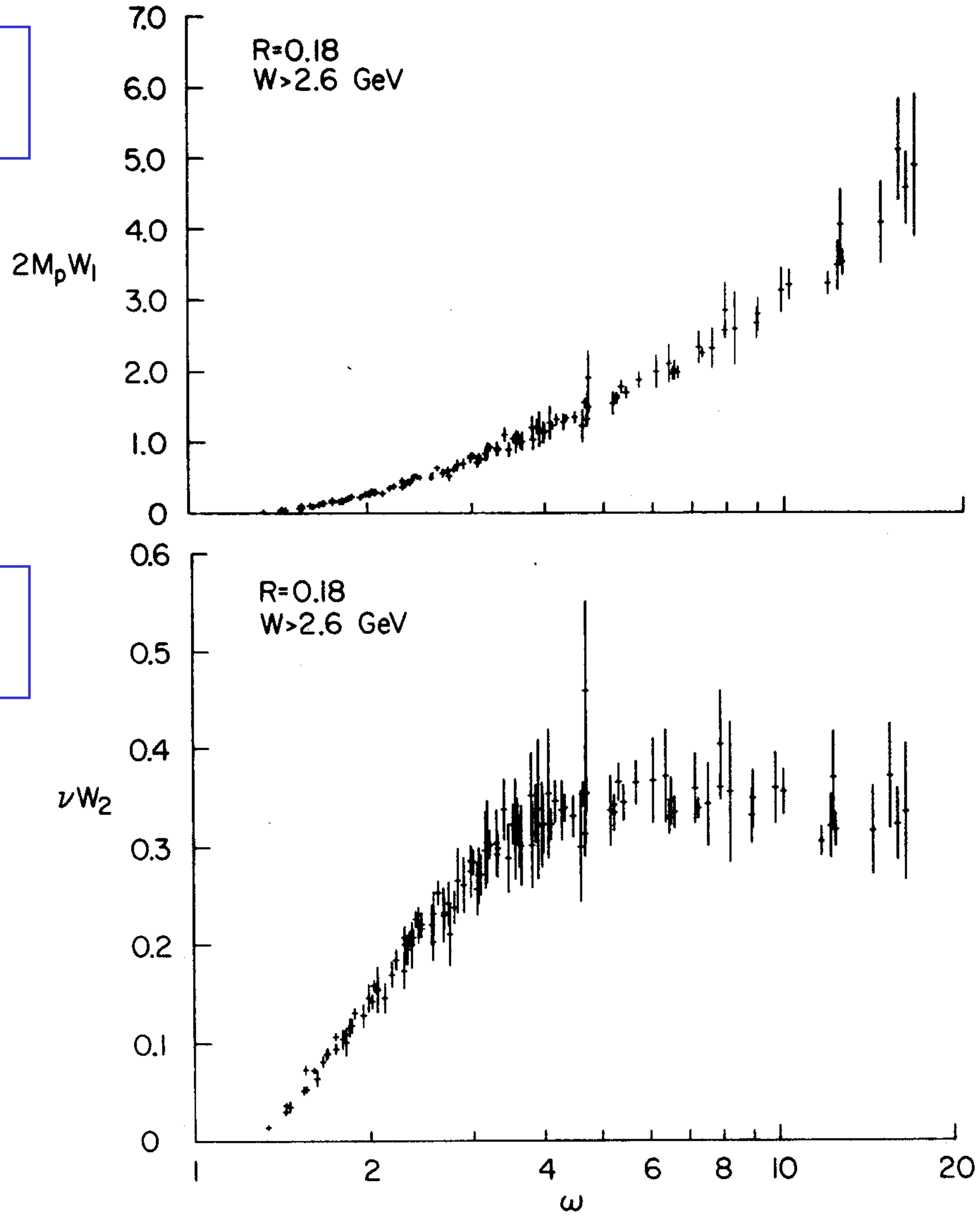
Measurements of F_1, F_2

Data from SLAC 8 GeV spectrometer :

(with several Q^2 values in range Q^2 = 1.5-11 GeV^2 superimposed)

$2F_1^{ep}$

F_2^{ep}



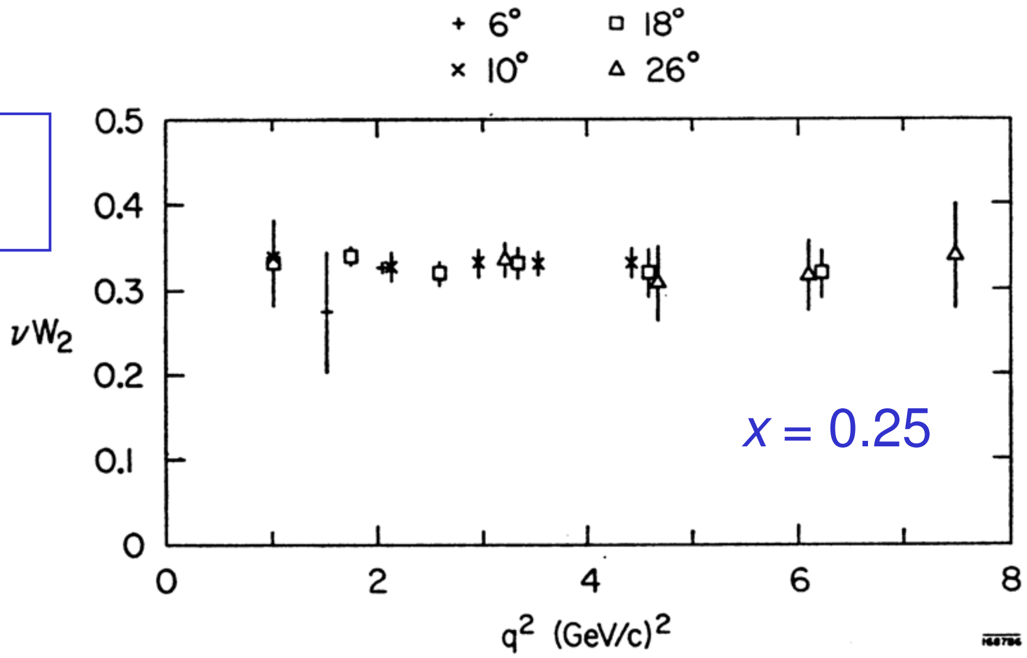
(1 / x) ⟶

G.Miller et al, Phys. Rev. **D5** (1972) 528

⇒ F_2 depends principally on x , approximately independent of Q^2

F_2 vs Q^2 at fixed x :

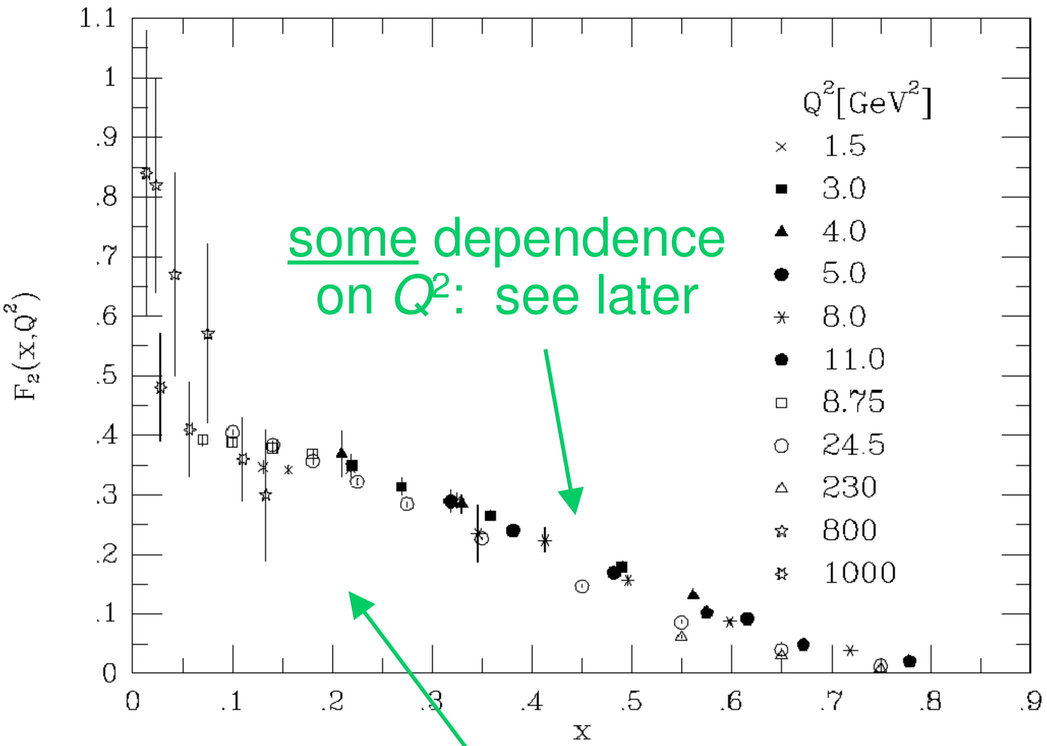
F_2^{ep}



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. **22** (1972) 203

Note contrast with elastic form factors, which fall rapidly with Q^2

F_2 vs x at fixed Q^2 : (recent compilation)

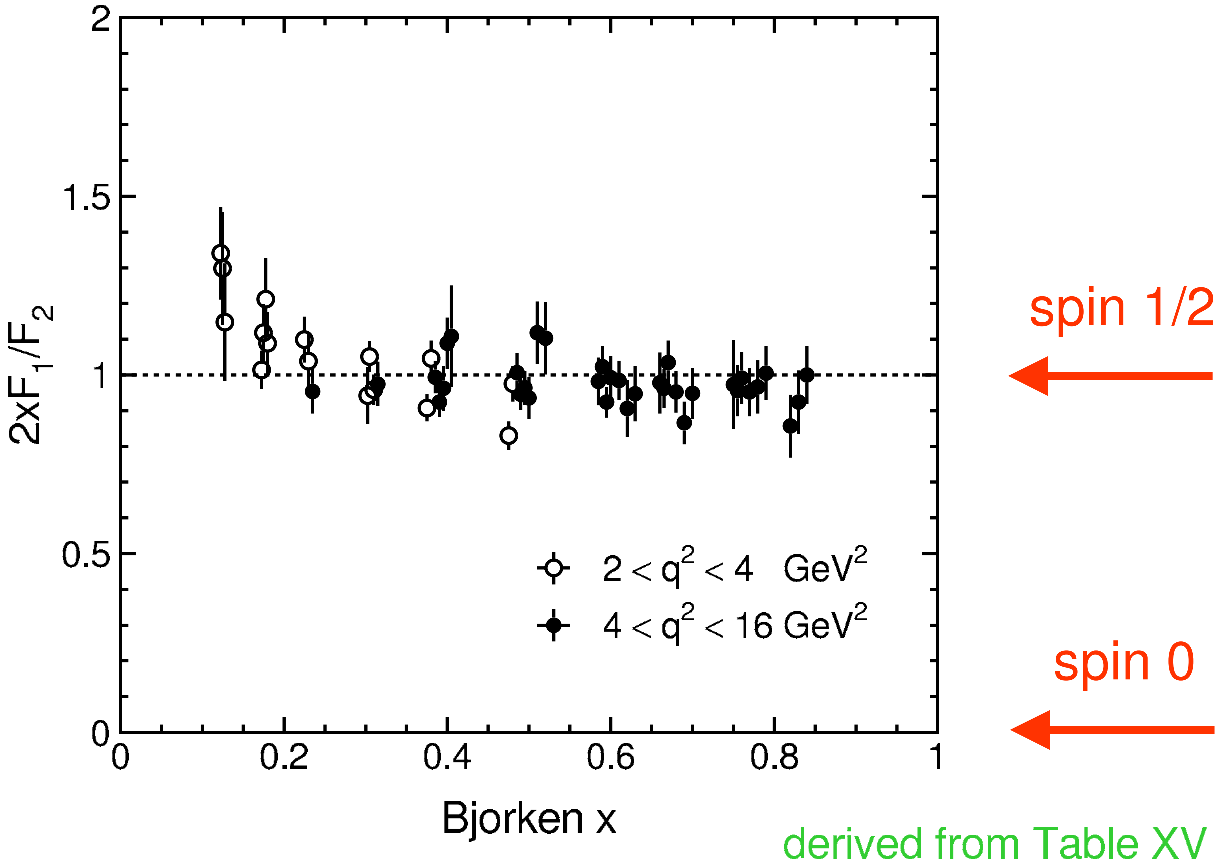


Ellis, Stirling + Webber

For future reference: area = $\int_0^1 F_2^{ep} dx \approx 0.18$

Callan-Gross Relation

- F_1 and F_2 are found to be closely related :



derived from Table XV of
A.Bodek et al, Phys. Rev. **D20** (1979) 1471

Data consistent with
Callan-Gross relation :

$F_2 = 2xF_1$

In lab frame : (examples sheet)

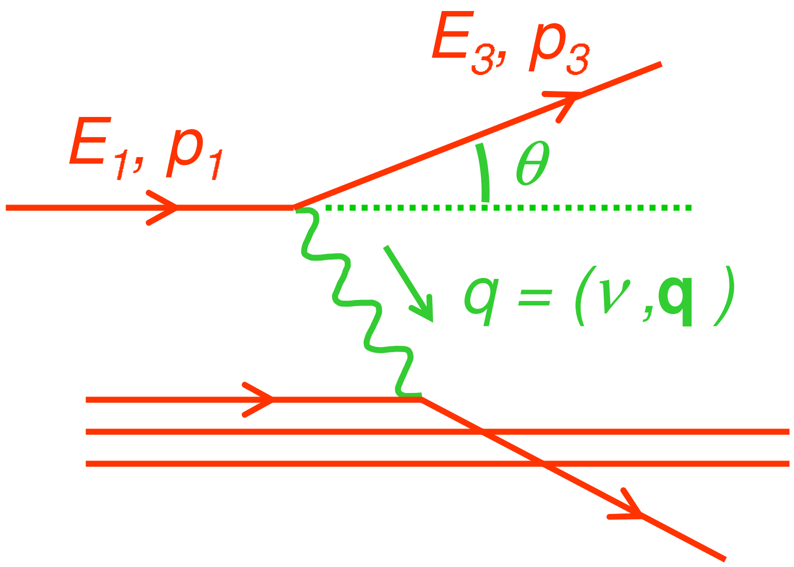
$$\frac{d^2\sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2}\left[\frac{F_2}{\nu}\cos^2\frac{\theta}{2} + \frac{2F_1}{M}\sin^2\frac{\theta}{2}\right]$$

Spin 0 quarks would give no $\sin^2\theta/2$ term
and hence would give $F_1 = 0$

The Parton Model

(The putative pointlike constituents of the nucleon were termed "partons" by Feynman, before quarks and gluons became firmly established.)

- Can obtain expressions for F_1 and F_2 by assuming underlying process is elastic scattering from pointlike quarks:



$$\nu = E_1 - E_3$$

$$q^2 = -4E_1E_3\sin^2\frac{\theta}{2}$$

(lab frame)

- Assume that the interaction with the quark takes place sufficiently fast that the quark can be treated as a free particle

But : quarks are believed to be permanently confined within hadrons, and therefore presumably very tightly bound

→ Asymptotic freedom in QCD (see later)

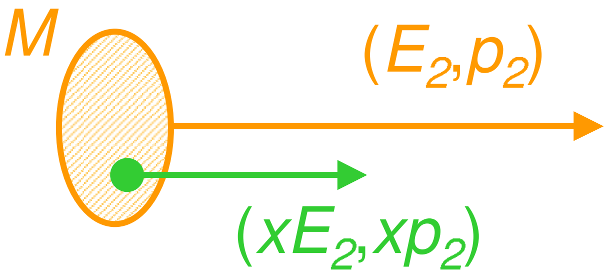
Bjorken x

- Parton model is easiest formulated in "infinite momentum frame" ($E_2 >> M$):

  Proton $\rightarrow$ "infinite" momentum

  $\rightarrow$ quark momentum parallel to proton momentum

  x = fraction of proton's momentum carried by quark:



$$\hat{p}_2 = x p_2$$

- In lab frame, for example :

$$x = \frac{Q^2}{2M\nu} = \frac{4E_1 E_3 \sin^2\theta/2}{2M(E_1 - E_3)}$$

$\Rightarrow$ can measure quark's momentum from scattered electron alone: (E_3, θ)

(don't need to measure hadronic system X)

(though often do !)

Parton momentum distributions

- Define quark distribution functions for proton :

$u^{\mathrm{p}}(x)\mathrm{d}x =$ number of u quarks in proton with momentum fraction *x* to *x*+d*x*

plus similar for other (anti)quark flavours :

$$d^{\mathrm{p}}(x),\ \bar{u}^{\mathrm{p}}(x),\ \bar{d}^{\mathrm{p}}(x),\ \ldots$$

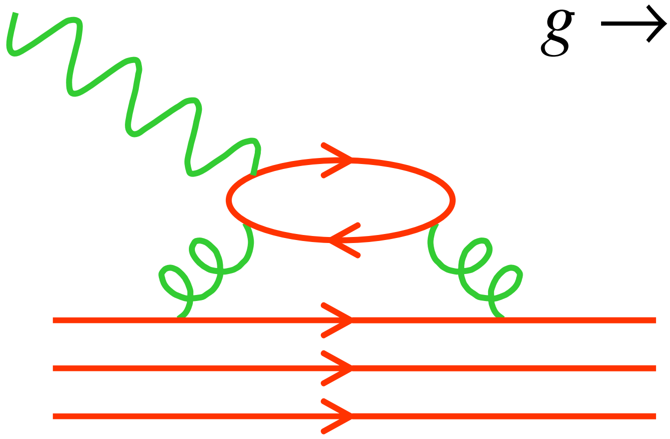
plus similar for neutron :

$$u^{\mathrm{n}}(x),\ d^{\mathrm{n}}(x),\ \bar{u}^{\mathrm{n}}(x),\ \bar{d}^{\mathrm{n}}(x),\ \ldots$$

- Antiquark content of nucleon arises from higher order processes :

$$g \rightarrow q\bar{q}$$

e.g.



$\Longrightarrow$ nucleon contains a "sea" of quark-antiquark pairs as well as the 3 valence quarks

(p=uud, n=ddu)

Parton model predictions for F_1, F_2

- Electron-quark scattering :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{Q^4} e_{\mathrm{q}}^2 \left[1 + (1-y)^2\right]$$

- Electron-nucleon scattering :

  sum over all quark and antiquark flavours in the nucleon :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{2\pi\alpha^2}{Q^4} \left[1 + (1-y)^2\right] \sum_{\mathrm{q}} e_{\mathrm{q}}^2 q(x)$$

- Compare with general form of cross section :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}Q^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y)\frac{F_2}{x} + \frac{y^2}{2}\frac{2xF_1}{x} \right]$$

$$\left(Q^2 >> M^2\right)$$

Gives Callan-Gross relation :

$$F_2 = 2xF_1 \qquad \left(= x\sum_{\mathrm{q}} e_{\mathrm{q}}^2 q(x) \right)$$

Consequence of quarks being spin 1/2

(spin 0 quarks would give $F_1 = 0$)

Predictions for F_2^{ep} and F_2^{en}

- Neglecting $s\bar{s}$, $c\bar{c}$, … contributions to the sea :

$$\left.\begin{aligned} F_2^{ep}(x) &= \frac{4}{9}xu^{p}(x)+\frac{1}{9}xd^{p}(x)+\frac{4}{9}x\bar{u}^{p}(x)+\frac{1}{9}x\bar{d}^{p}(x) \\ F_2^{en}(x) &= \frac{4}{9}xu^{n}(x)+\frac{1}{9}xd^{n}(x)+\frac{4}{9}x\bar{u}^{n}(x)+\frac{1}{9}x\bar{d}^{n}(x) \end{aligned}\right\}$$

- Isospin symmetry relates p and n :

p=(uud)
n=(ddu)

$$u(x)\equiv u^{p}(x)=d^{n}(x) \qquad \bar{u}(x)\equiv \bar{u}^{p}(x)=\bar{d}^{n}(x)$$

$$d(x)\equiv d^{p}(x)=u^{n}(x) \qquad \bar{d}(x)\equiv \bar{d}^{p}(x)=\bar{u}^{n}(x)$$

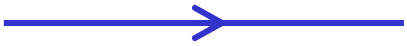
$\Rightarrow$ can avoid “p” and “n” suffices :

$$\left.\begin{aligned} F_2^{ep}(x) &= \frac{4}{9}xu(x)+\frac{1}{9}xd(x)+\frac{4}{9}x\bar{u}(x)+\frac{1}{9}x\bar{d}(x) \\ F_2^{en}(x) &= \frac{4}{9}xd(x)+\frac{1}{9}xu(x)+\frac{4}{9}x\bar{d}(x)+\frac{1}{9}x\bar{u}(x) \end{aligned}\right\}$$

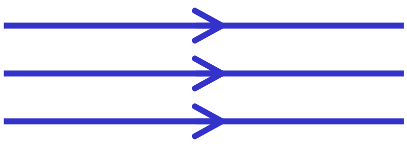
$\Rightarrow$ can extract $u(x)+\bar{u}(x)$ and $d(x)+\bar{d}(x)$ from measurements of $F_2^{ep}(x)$ and $F_2^{en}(x)$

Predictions for F_2

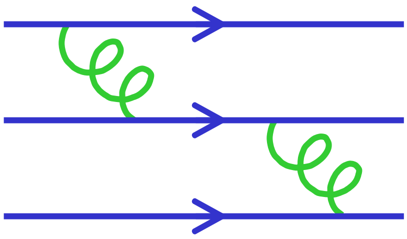
Dirac proton

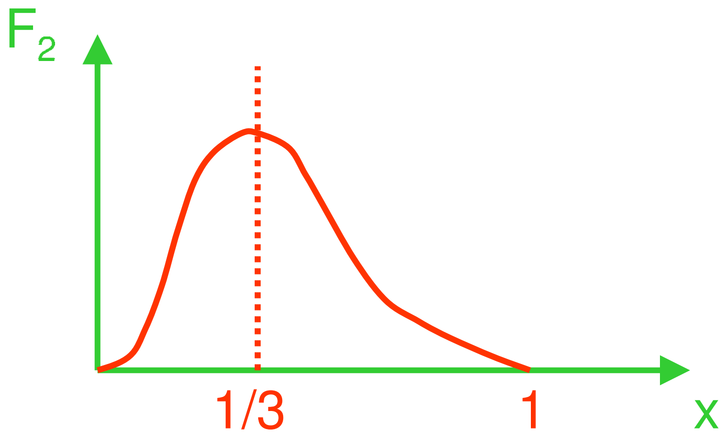


3 free quarks

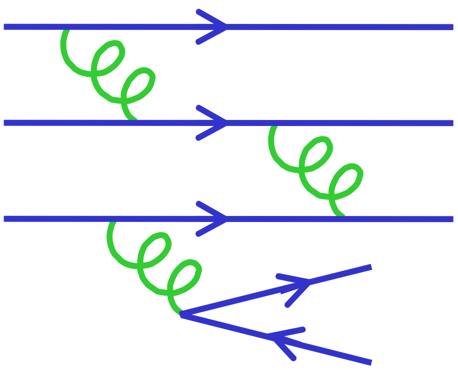


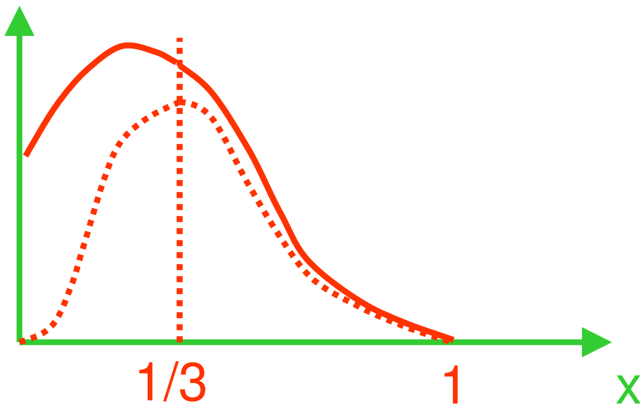
3 bound quarks





3 bound quarks plus "stuff"





Structure Function Integrals

- Area under F_2 :

proton :

$$\int_0^1 F_2^{\text{ep}} \mathrm{d}x = \frac{4}{9}\int_0^1 x(u+\bar{u})\mathrm{d}x + \frac{1}{9}\int_0^1 x(d+\bar{d})\mathrm{d}x$$

$$\equiv \frac{4}{9} f_{\text{u}} + \frac{1}{9} f_{\text{d}}$$

neutron :

$$\int_0^1 F_2^{\text{en}} \mathrm{d}x = \frac{4}{9}\int_0^1 x(d+\bar{d})\mathrm{d}x + \frac{1}{9}\int_0^1 x(u+\bar{u})\mathrm{d}x$$

$$\equiv \frac{4}{9} f_{\text{d}} + \frac{1}{9} f_{\text{u}}$$

where f_u = fraction of proton's momentum carried by all u and ū (anti)quarks

- Measurements give

$$\int_0^1 F_2^{\text{ep}} \mathrm{d}x = \frac{4}{9} f_{\text{u}} + \frac{1}{9} f_{\text{d}} \approx 0.18$$

$$\int_0^1 F_2^{\text{en}} \mathrm{d}x = \frac{4}{9} f_{\text{d}} + \frac{1}{9} f_{\text{u}} \approx 0.12$$

$\Rightarrow$

$$f_{\text{u}} \approx 0.36$$
$$f_{\text{d}} \approx 0.18$$

$\Rightarrow$ only ~ 50% of proton's momentum is carried by quarks or antiquarks

rest is carried by gluons

- In summary, quark-parton model predicts :

1) Bjorken scaling : $F(x,Q^2) \to F(x)$

F_1, F_2 are functions of one variable, not two, because underlying scattering is pointlike

→ no dependence on photon λ

→ no dependence on Q^2 at fixed x

2) Callan-Gross relation : $F_2 = 2xF_1$

→ quarks are spin 1/2

Only one unknown function because charge and μ related for Dirac particles $\left(\mu = \frac{e}{2m} \right)$

3) Parton momentum fractions :

Total momentum fraction carried by quarks plus antiquarks is

$$f_u + f_d = \frac{9}{5}\left[\int_0^1 F_2^{ep} dx + \int_0^1 F_2^{en} dx \right] \approx 0.5$$

Remainder is carried by gluons

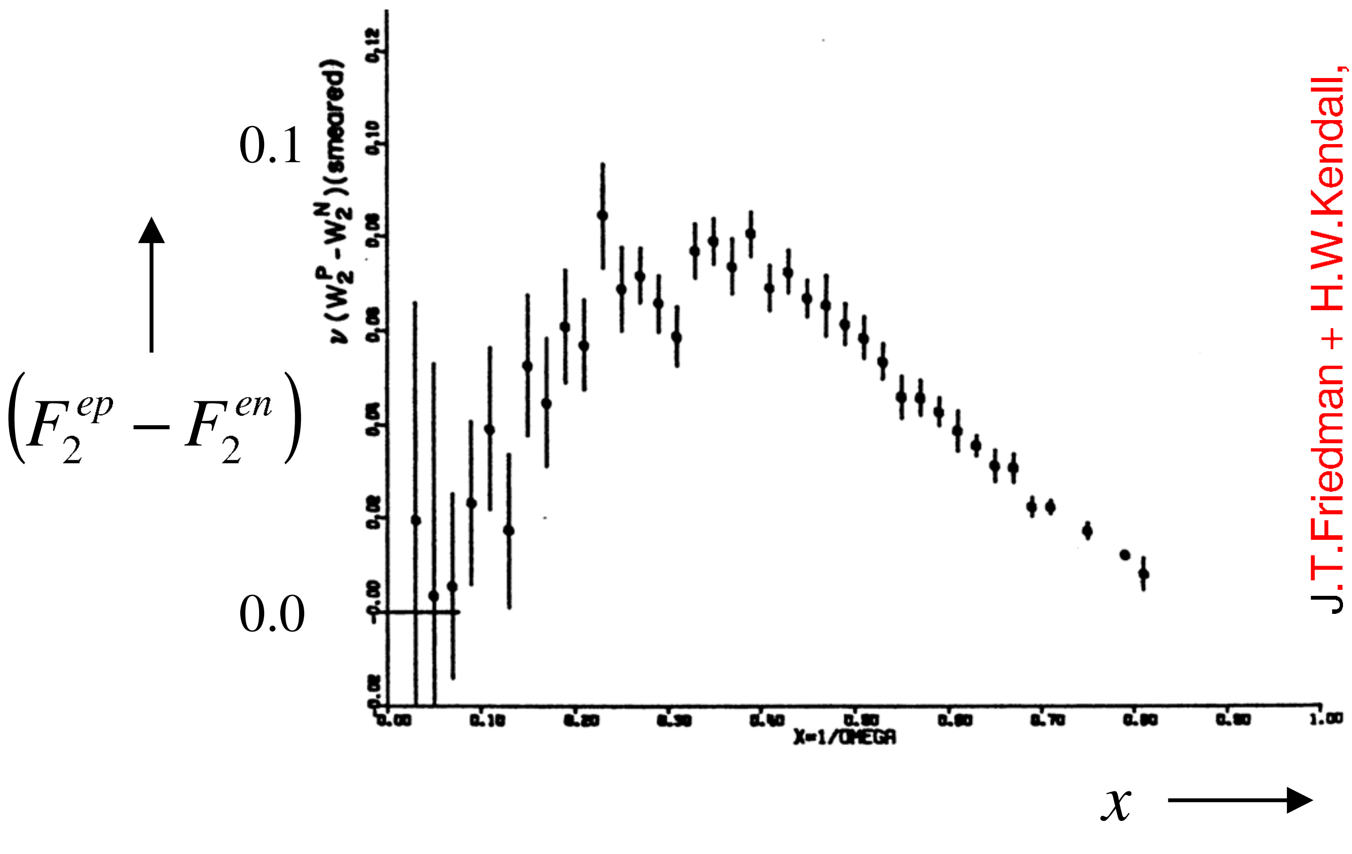
“Valence” and “Sea” Components

$$u(x) = u_V(x) + S(x)$$
$$d(x) = d_V(x) + S(x)$$
$$\bar{u}(x) = \bar{d}(x) = S(x)$$

- Structure function difference :

$$F_2^{ep} - F_2^{en} = x\left[\frac{1}{3}u_V - \frac{1}{3}d_V\right]$$

$\Rightarrow$ can extract valence component alone



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. **22** (1972) 203

Naively, expect $u_V \approx 2d_V$ (since p=uud)

$$\Rightarrow \quad F_2^{ep} - F_2^{en} \approx \frac{1}{3}x\,d_V$$

(but only holds very approximately – see next slide)

Structure function ratio :

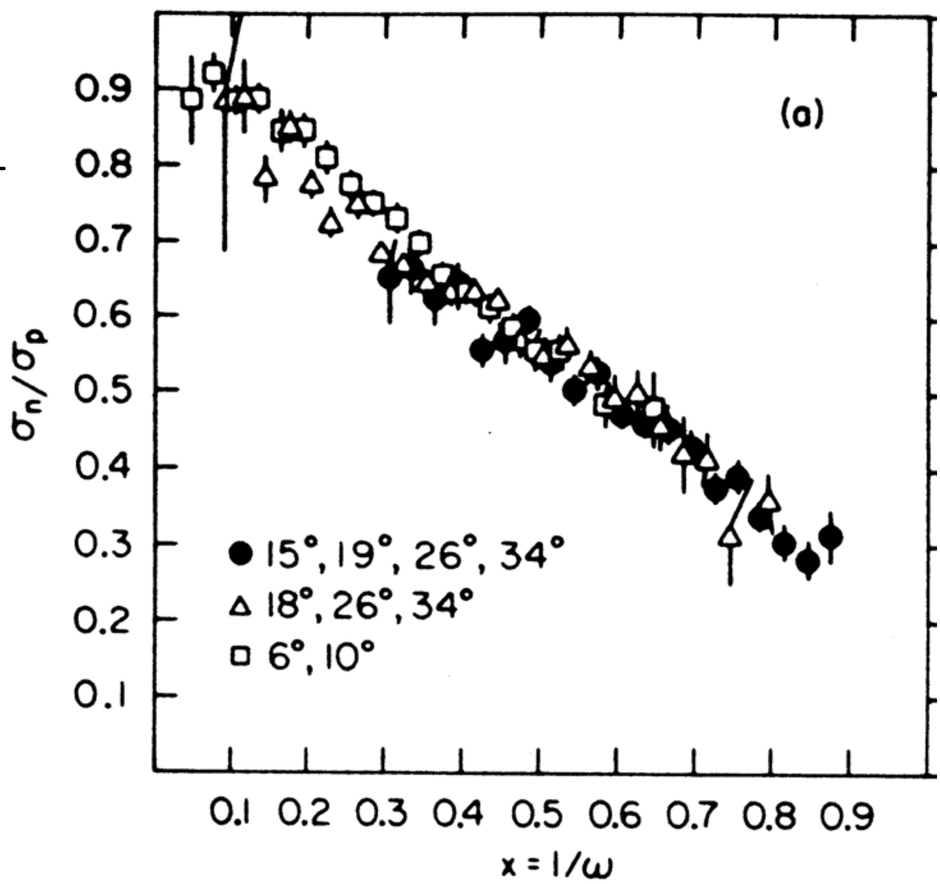
- For small *x*, expect sea to dominate:

$$\frac{F_2^{en}}{F_2^{ep}} \to 1 \qquad \text{as} \quad x \to 0$$

- For large *x*, expect sea to be negligible:

$$\frac{F_2^{en}}{F_2^{ep}} \to \frac{4d_V + u_V}{4u_V + d_V} \qquad \text{as} \quad x \to 1$$

$$\frac{F_2^{en}}{F_2^{ep}}$$



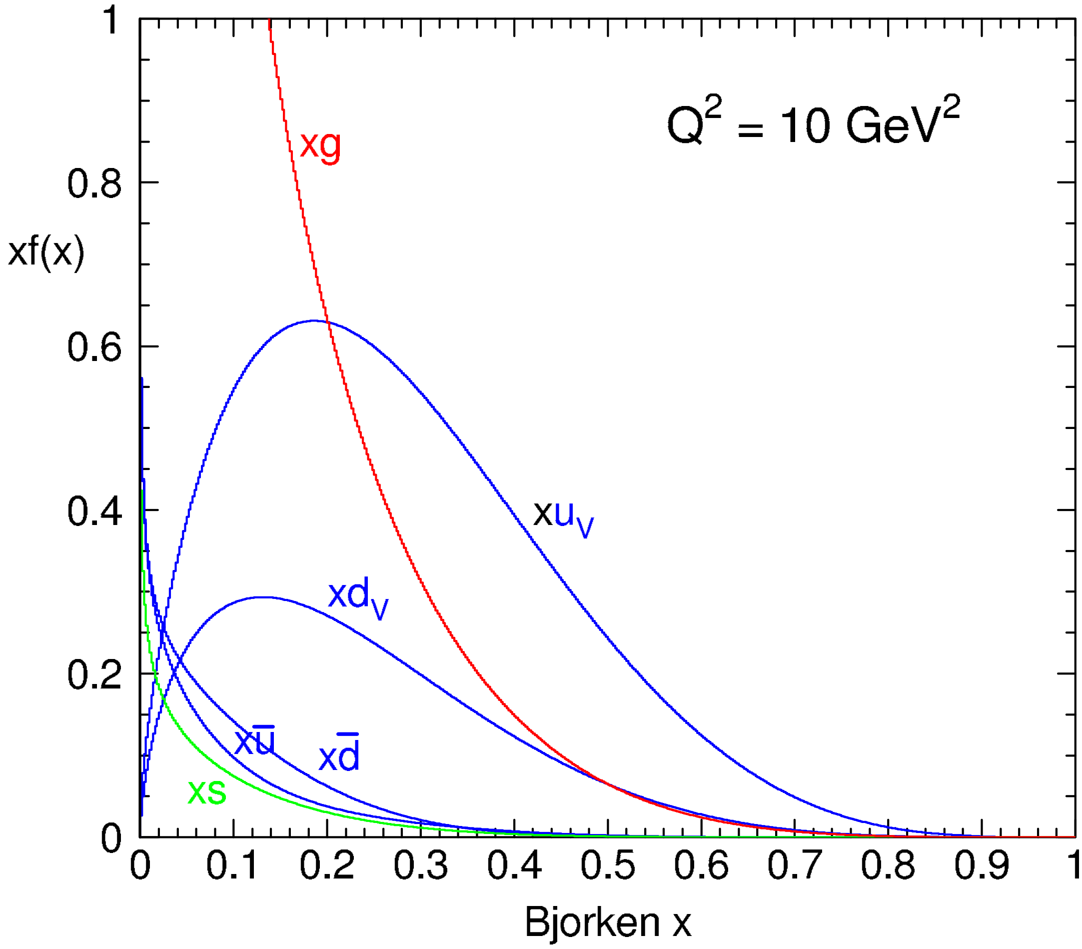
$$\Rightarrow \quad d_V << u_V \quad \text{as} \quad x \to 1$$

Note: $u_V = 2d_V$ would give ratio 2/3 as $x \to 1$

Parton Distributions

- From a fit to all current data :

  (including neutrino scattering - see later)



- $u_V \approx 2d_V$ (except at large *x*)
- $\bar{u}(x) \neq \bar{d}(x)$
- Gluon is dominant constituent for $x < 0.2$

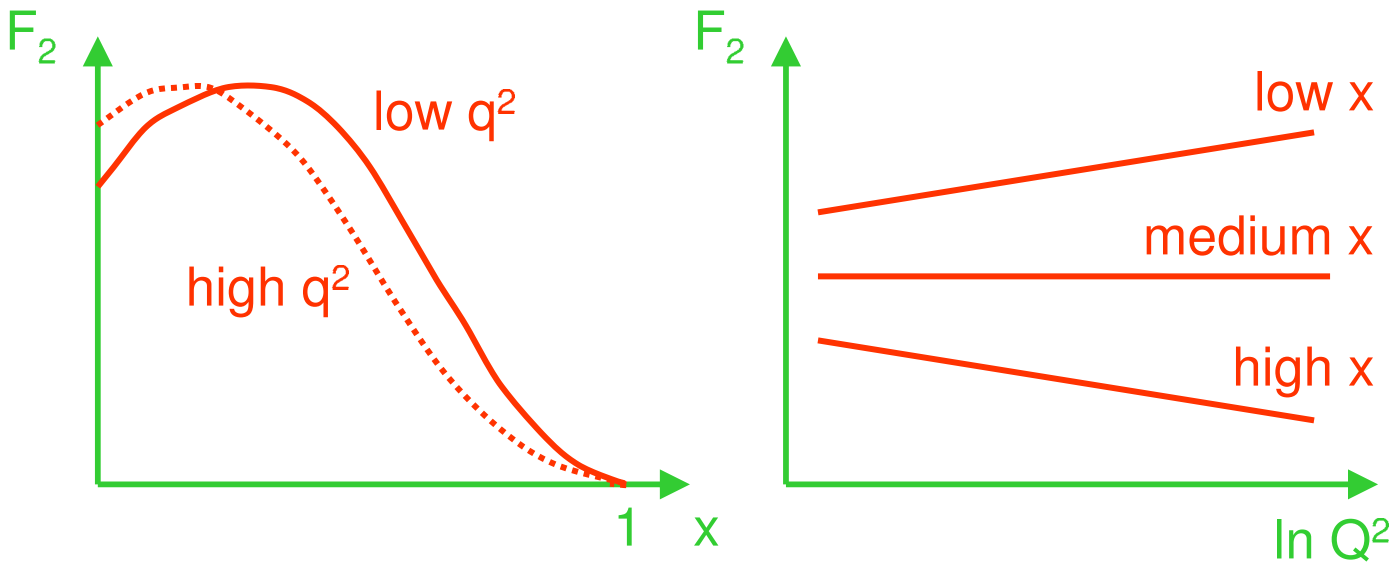
- *u*(*x*), *d*(*x*) cannot (yet) be predicted from QCD

  (requires understanding of QCD in a non-perturbative regime)

Scaling Violations

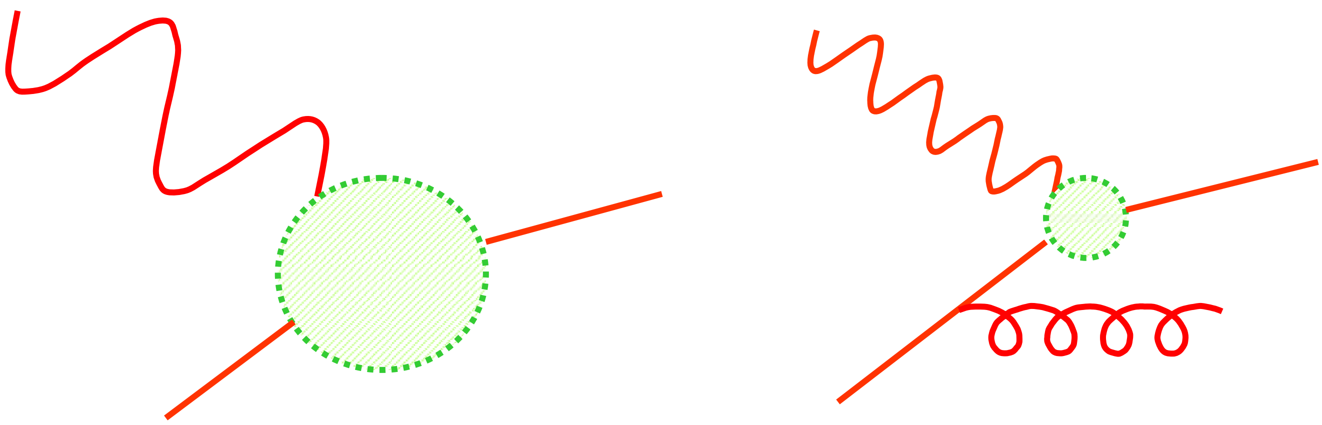
- Precision experiments show small deviations from exact Bjorken scaling :

$$F_2(x) \to F_2(x, Q^2)$$



- This is expected in QCD:

  large Q^2 reveals that a quark is sharing its momentum with gluons:

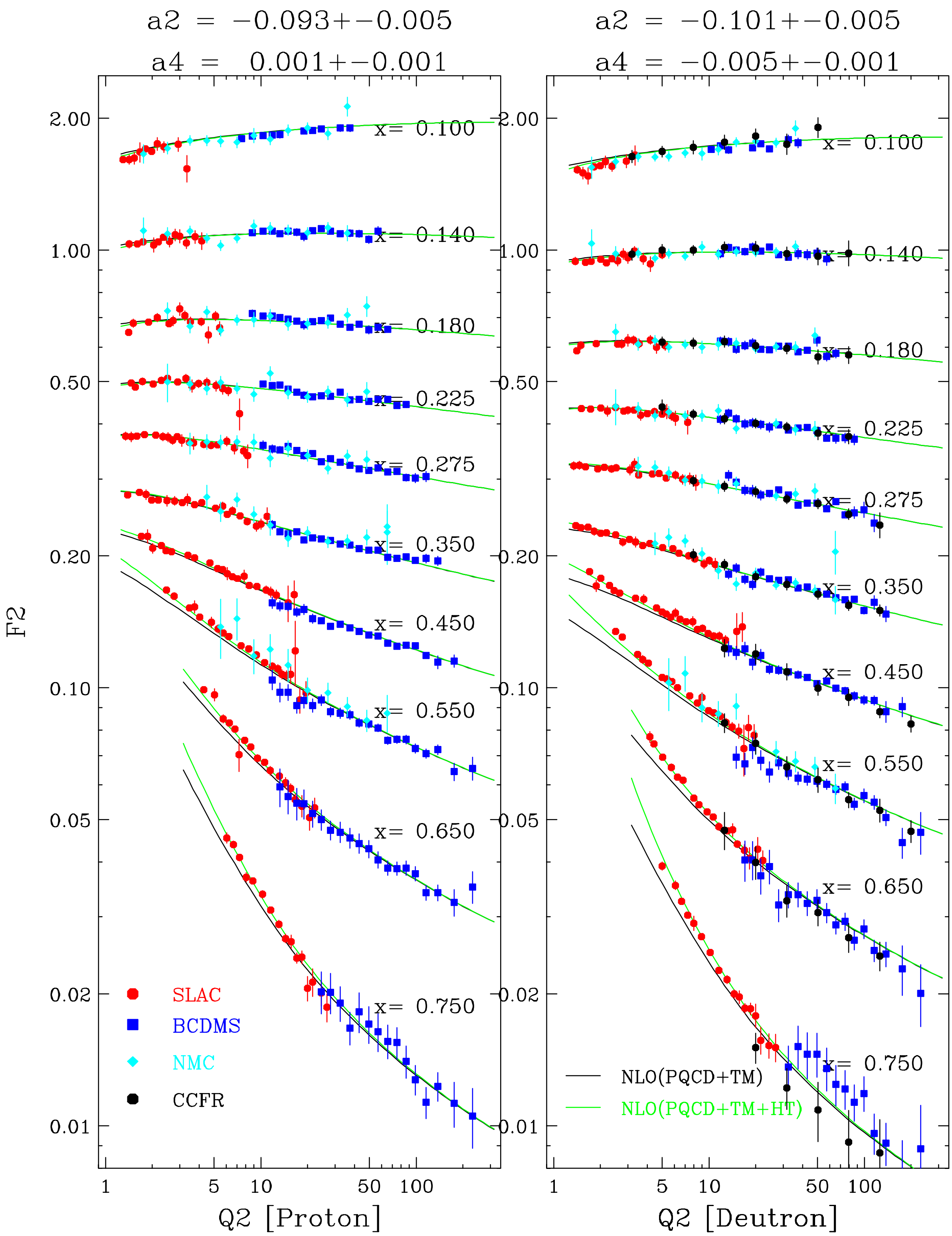


$\Rightarrow$ average x of quarks is reduced at high Q^2

QCD can predict the Q^2 dependence of $F_2(x, Q^2)$

Scaling violations: $F_2\,(x,Q^2)$

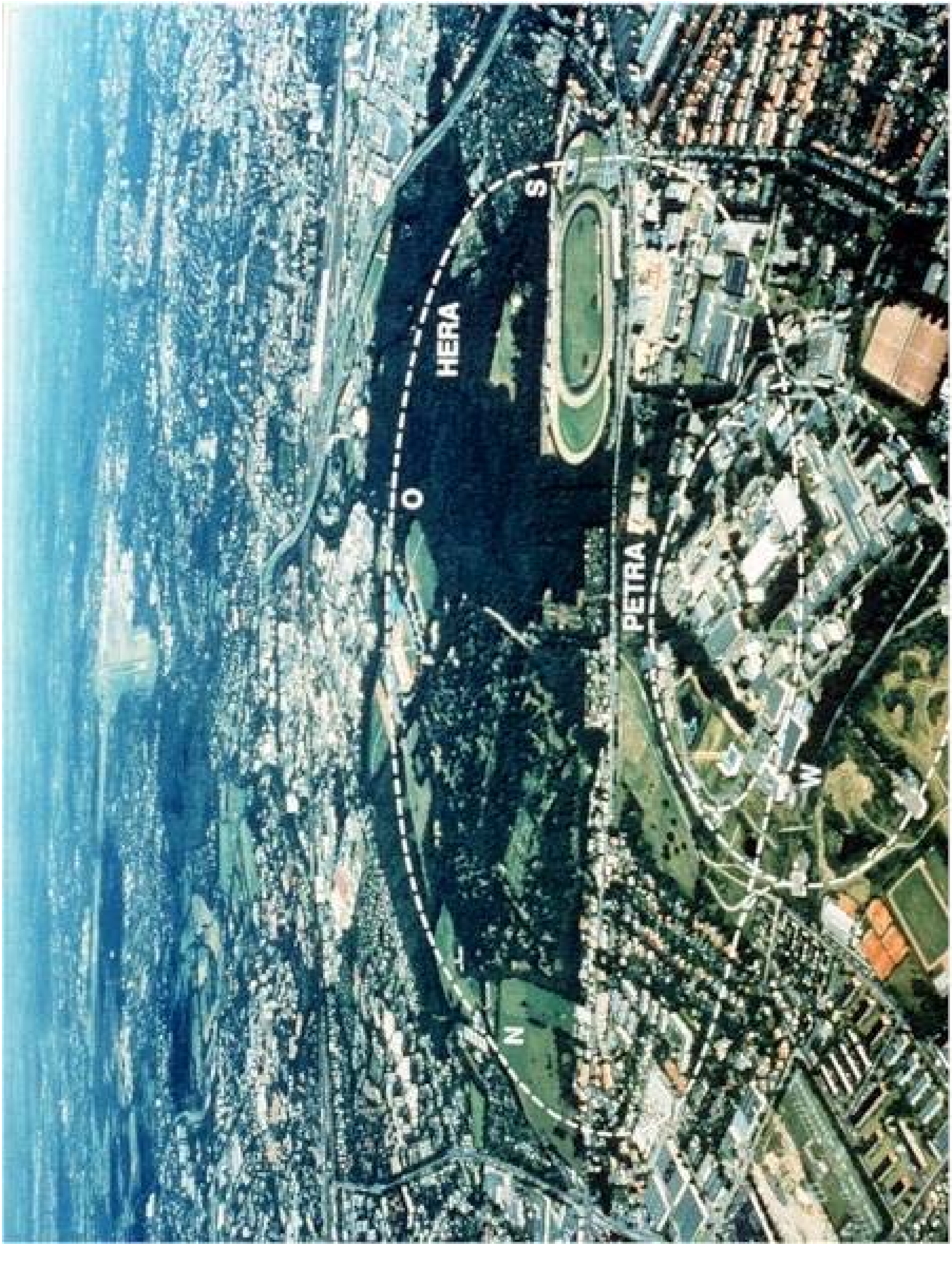
Summary of fixed target data:



e^-p, μ^-p

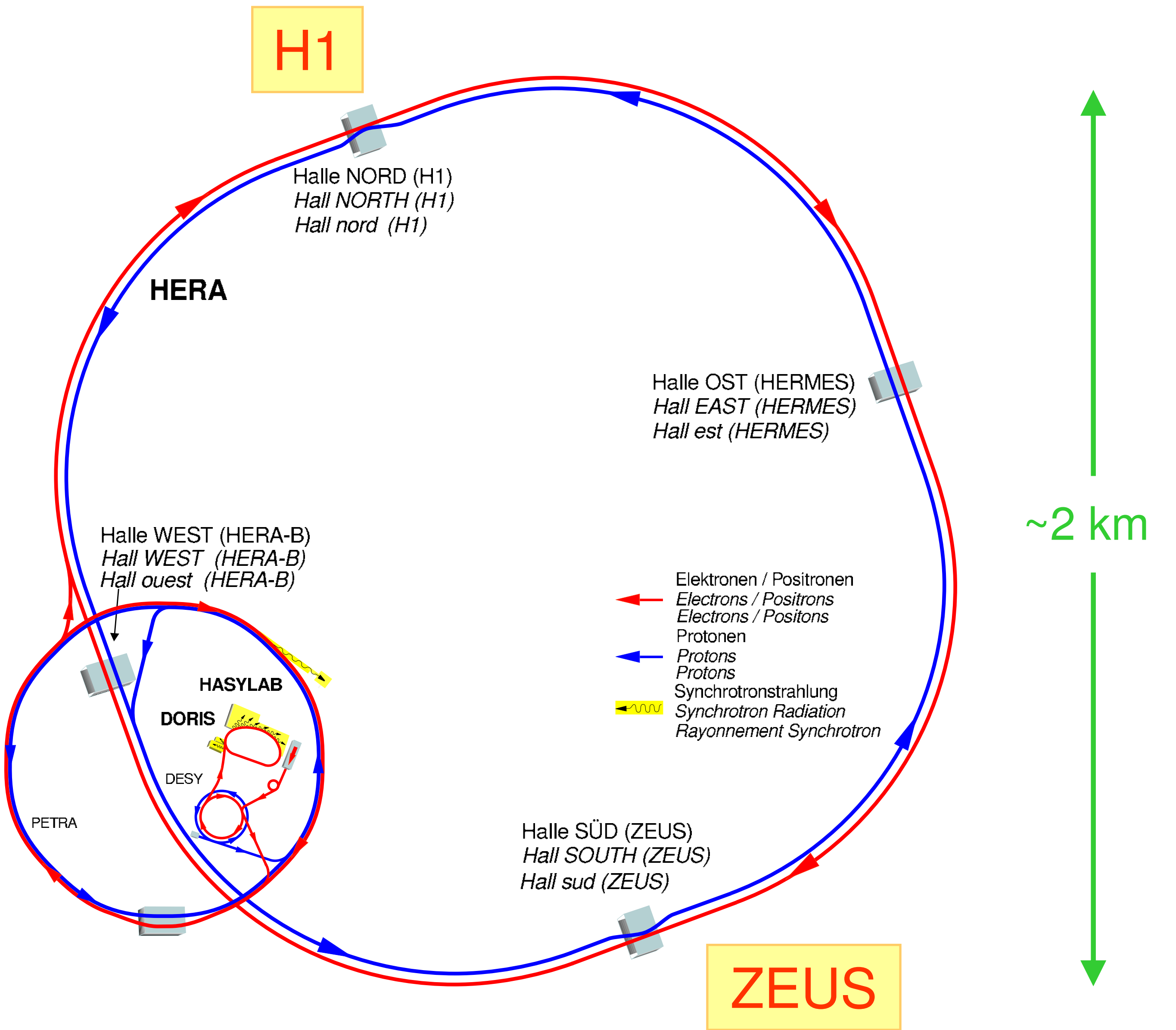
e^-d, μ^-d, (νd)

DESY Laboratory, Hamburg



HERA $e^{\pm}p$ Collider

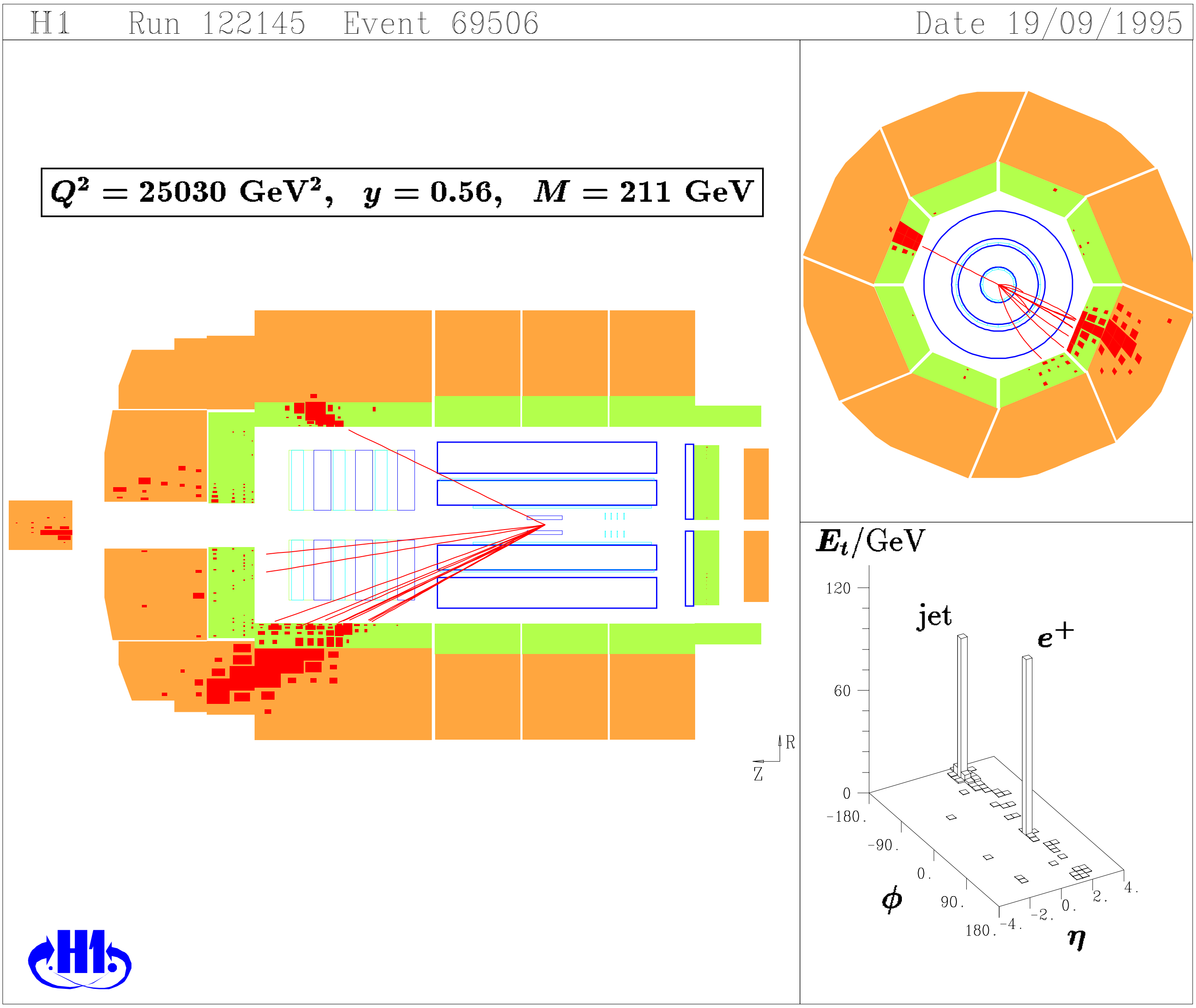
(DESY lab, Hamburg, Germany)

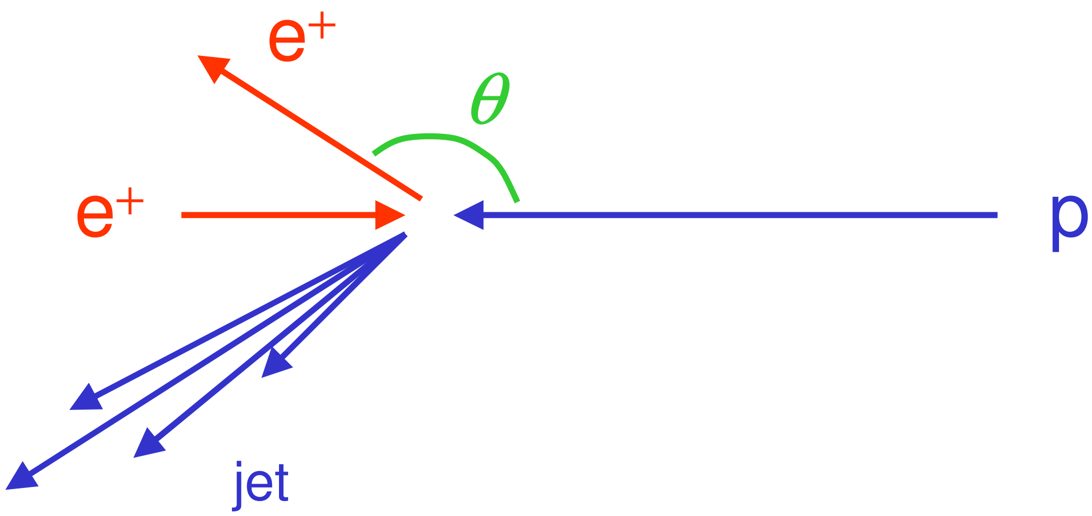


$\sqrt{s} = 300$ GeV

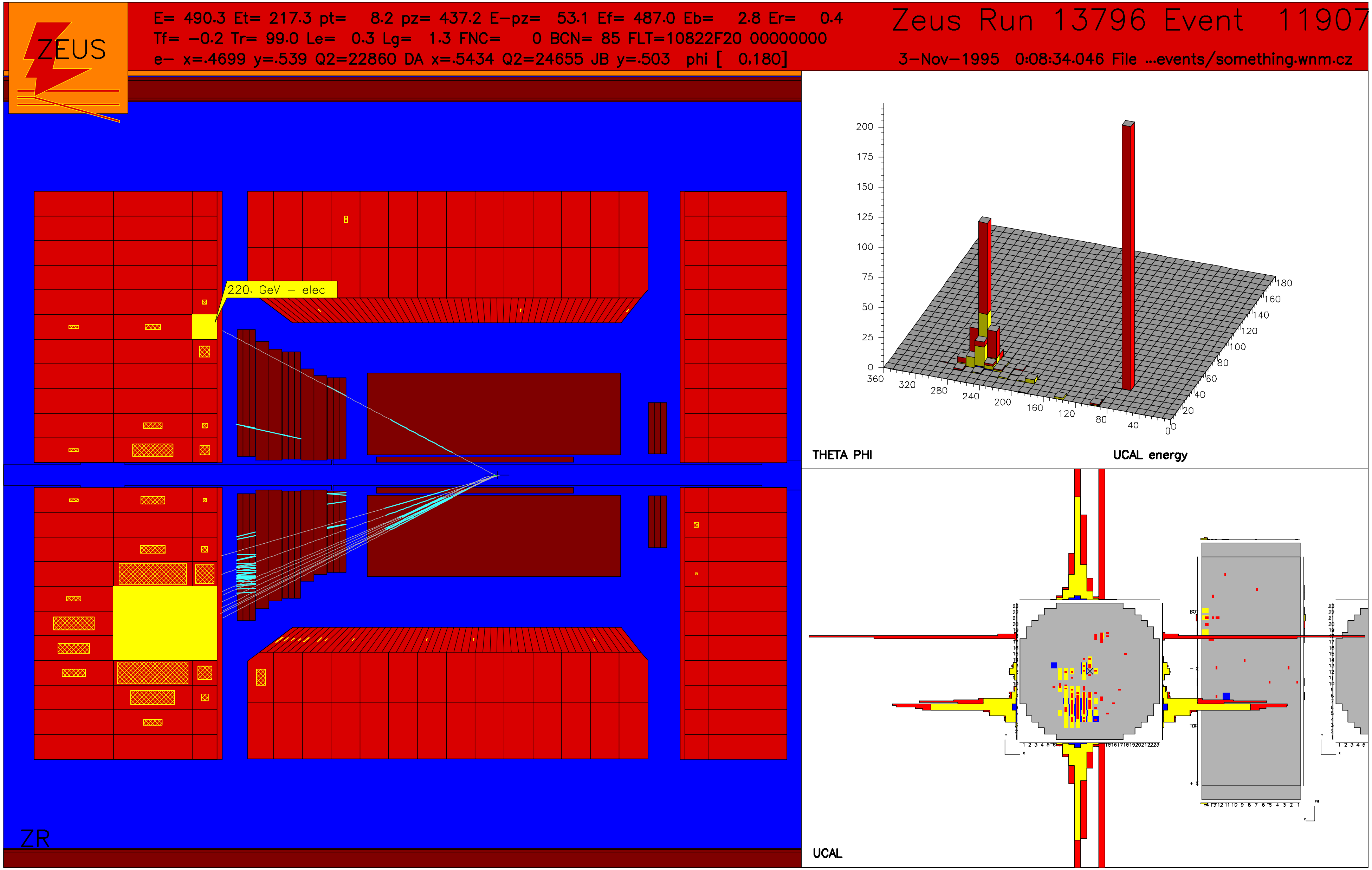
⇨ explore higher Q^2 and lower x

High Q^2 Event in H1

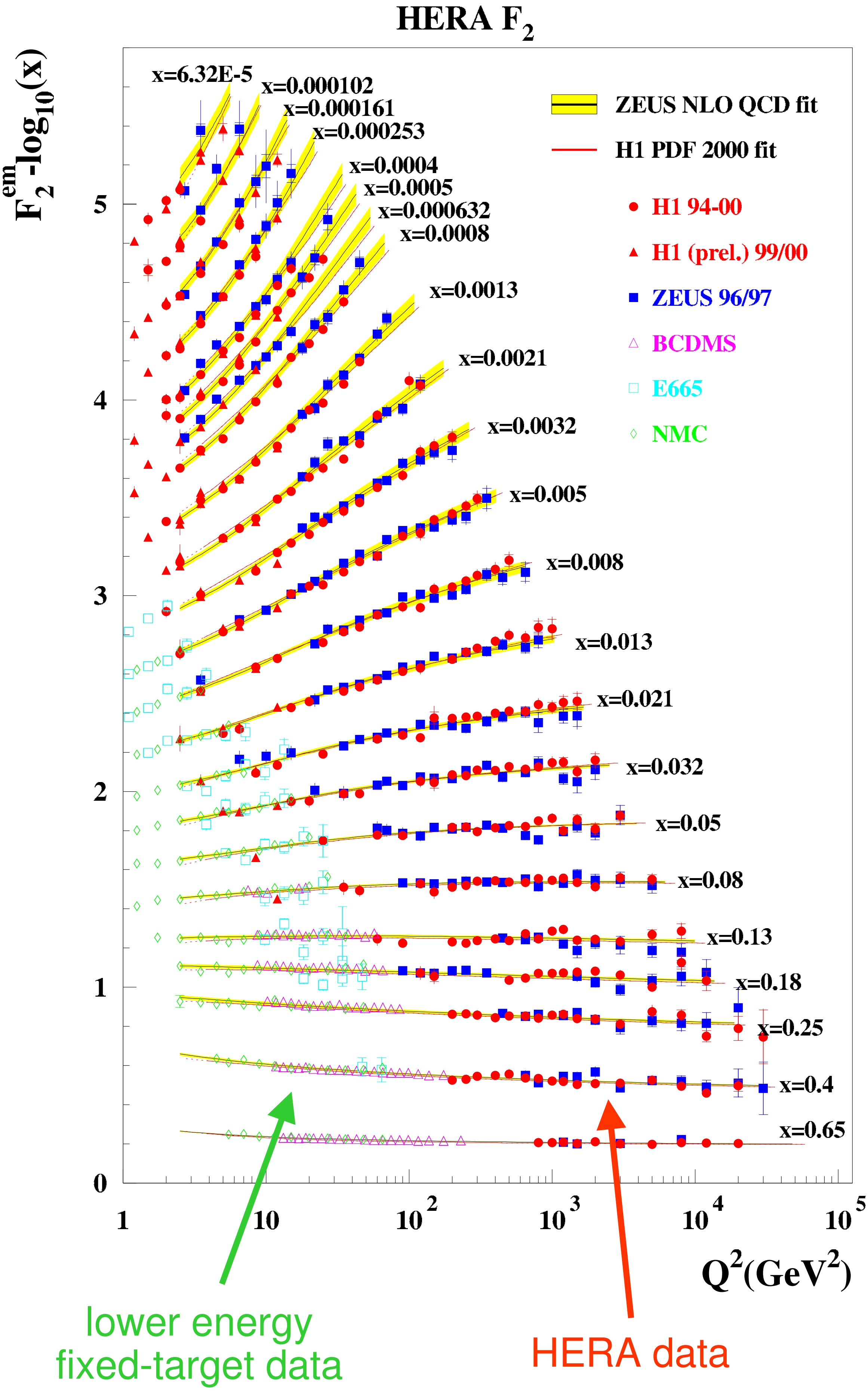




High Q^2 Event in ZEUS

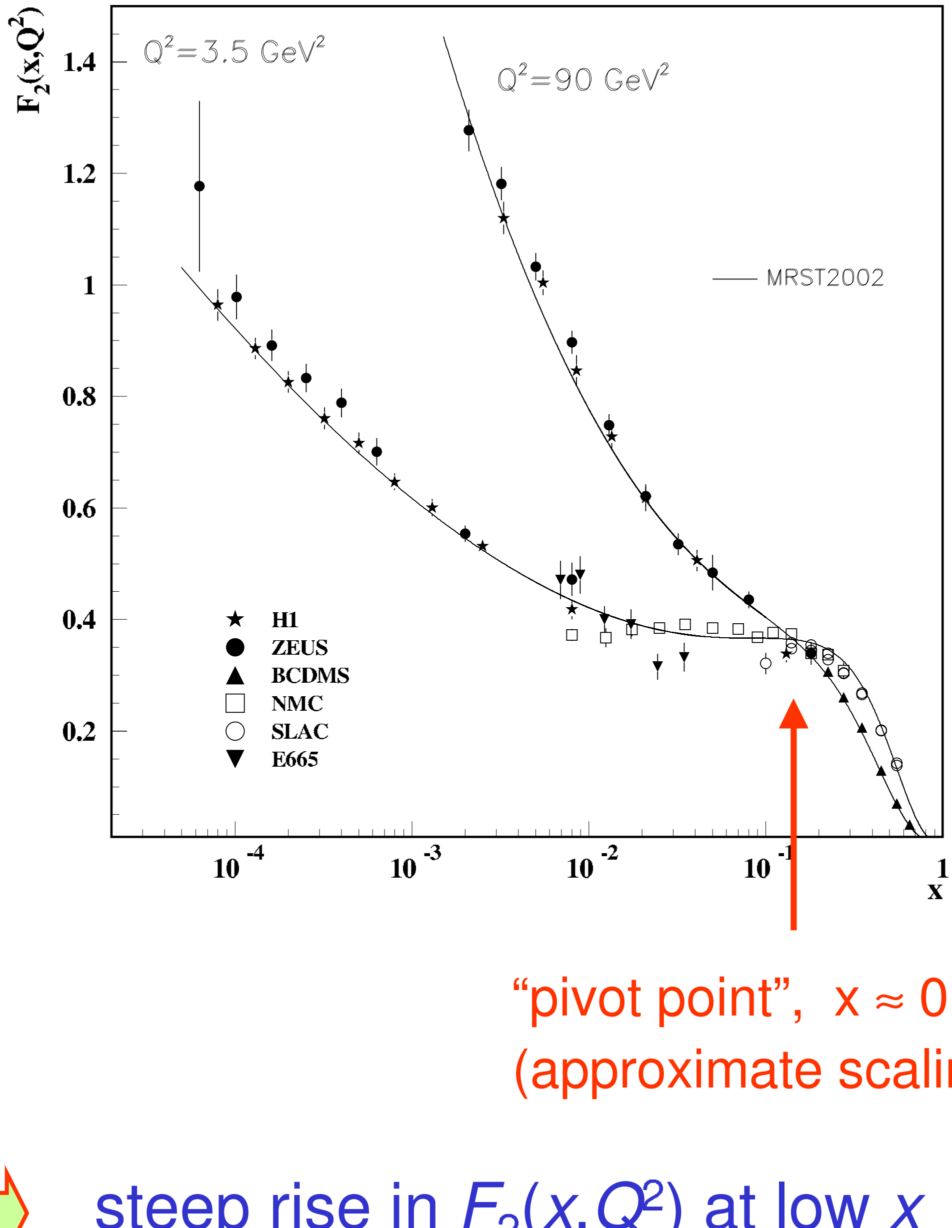


$F_2(x,Q^2)$ from HERA



$F_2(x,Q^2)$ at small x

As well as high Q^2, HERA can explore the region of very low values of x :



"pivot point", x ≈ 0.15
(approximate scaling)

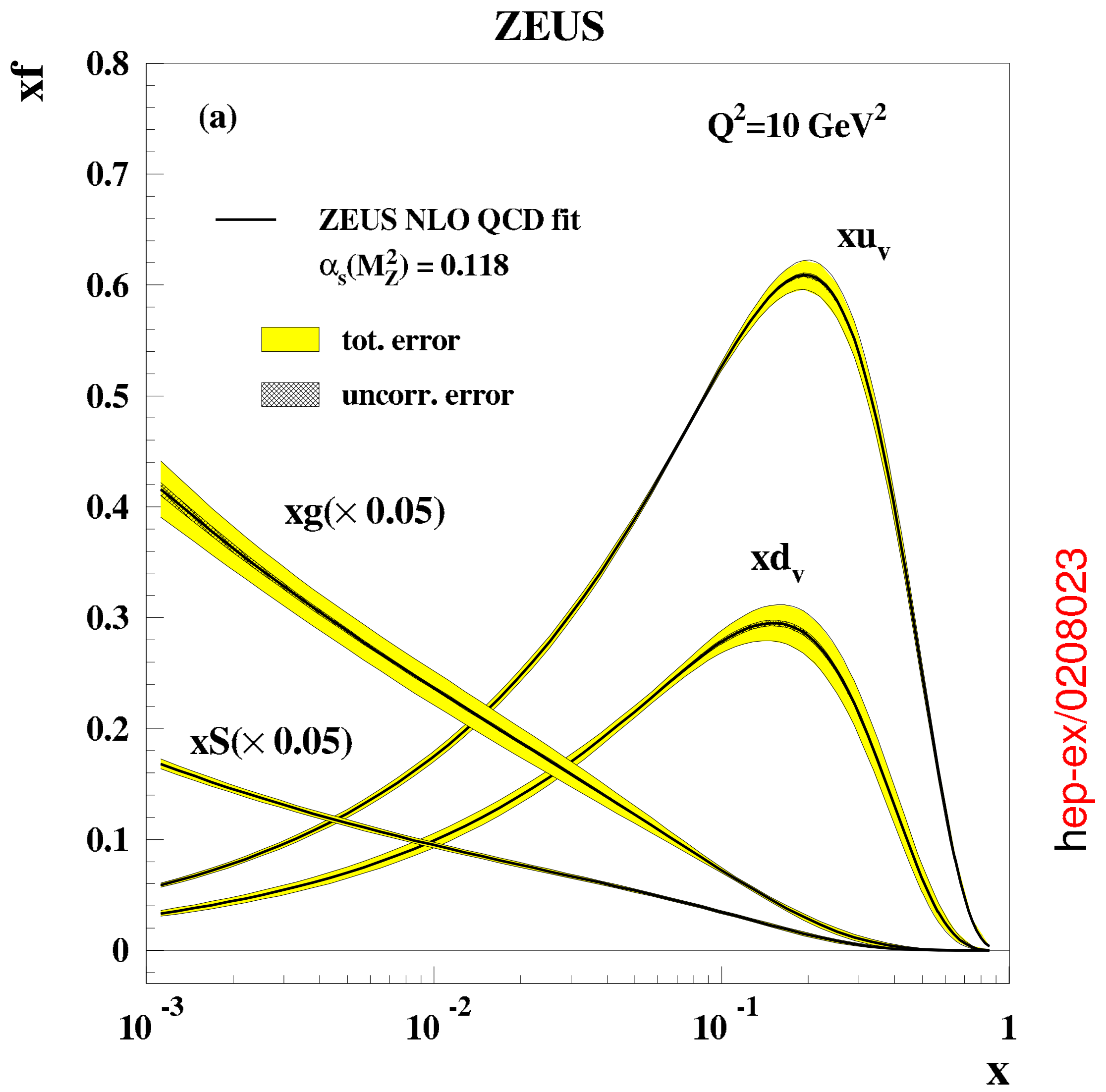
⇨ steep rise in $F_2(x,Q^2)$ at low x

(initially unexpected)

QCD successfully accounts for the variation of $F_2(x,Q^2)$ with Q^2, and for the low x behaviour

Parton Distribution Functions (PDF's)

… with log *x*-axis to emphasise small *x*



N.B. *g*(*x*) and *S*(*x*) have been scaled down by factor 20

Shaded bands indicate current uncertainty on PDF measurements

To be continued ….

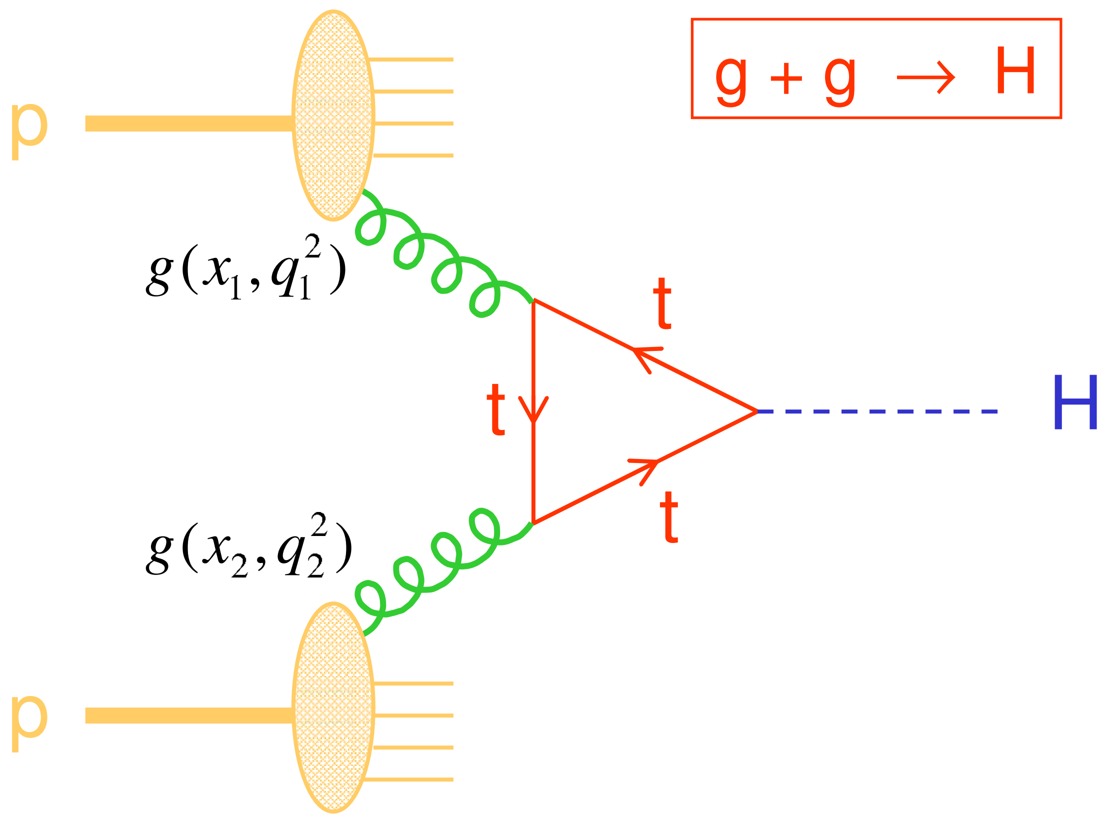
…. deep-inelastic neutrino scattering

Hadron-hadron collisions

- PDF measurements are an essential input to calculations of pp, $p\bar{p}$ cross sections

e.g. Higgs production at the LHC

Dominated by "gluon-gluon fusion" :



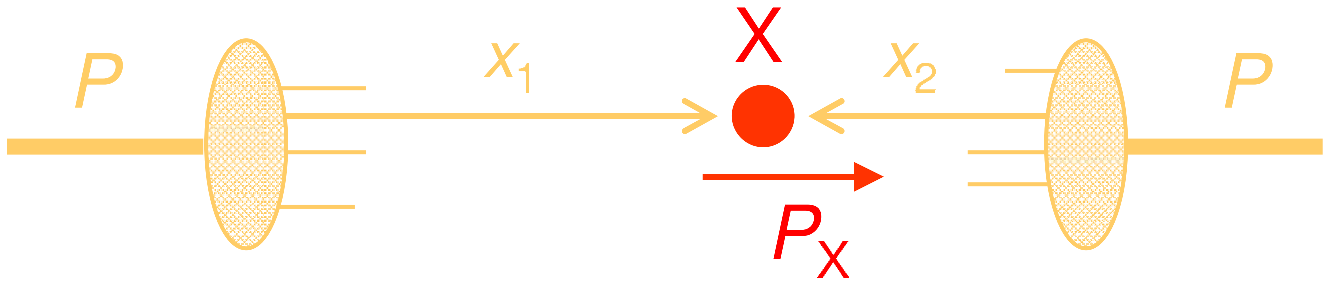
$$\sigma\left(pp \rightarrow H + X\right) \sim \int_0^1 dx_1 \int_0^1 dx_2 g(x_1) g(x_2) \sigma\left(gg \rightarrow H\right)$$

Uncertainty in gluon distribution $g(x,q^2)$

⇒ ±5% uncertainty in Higgs cross section

(was ±25% before HERA data)

- e.g. new particle X of mass M_X produced by partons with momentum fractions x_1, x_2



initial parton 4-momenta :

$$(x_1 P, 0, 0, x_1 P) \qquad (x_2 P, 0, 0, -x_2 P)$$

final 4-momentum of particle X :

$$((x_1 + x_2)P, 0, 0, (x_1 - x_2)P) = (E_X, 0, 0, P_X)$$

$$\Rightarrow \quad \begin{array}{l} x_1 = (E_X + P_X)/2P \\ x_2 = (E_X - P_X)/2P \end{array} \qquad E_X^2 = P_X^2 + M_X^2$$

e.g. M_X = 120 GeV Higgs produced at LHC, moving with P_X = 30 GeV

(pp collisions with P = 7 TeV = 7000 GeV)

$$\Rightarrow \quad x_1 = 0.011 \qquad x_2 = 0.007$$

- Similarly for production of particle pairs :

e.g. $\mathrm{u\bar{u}} \to \mathrm{t\bar{t}} \qquad \mathrm{gg} \to \mathrm{t\bar{t}} \qquad \mathrm{gg} \to \tilde{\mathrm{g}}\tilde{\mathrm{g}}$