$e^-p \to e^-p$ Elastic Scattering

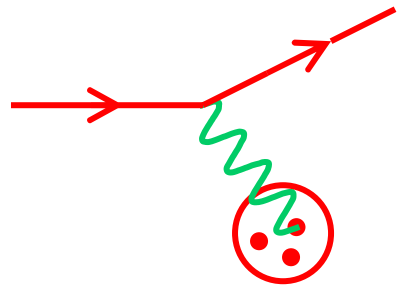
- Aiming towards study of

  $e^-p \to e^-p$ elastic scattering

  followed by

  $e^-p \to e^-X$

  deep-inelastic scattering

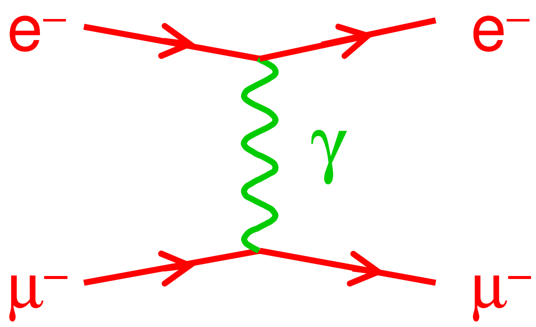


  ➡ probe structure of proton

- Begin with a pointlike equivalent:

  $$e^-\mu^- \to e^-\mu^-$$

  $(e^-q \to e^-q)$

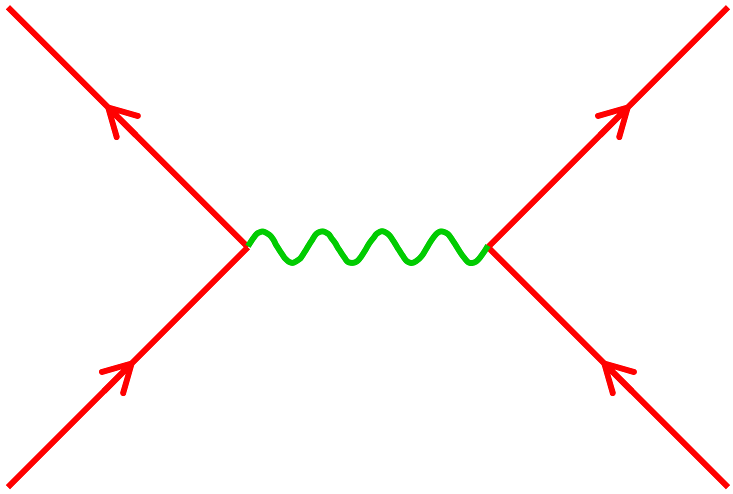


- Two ways to proceed:

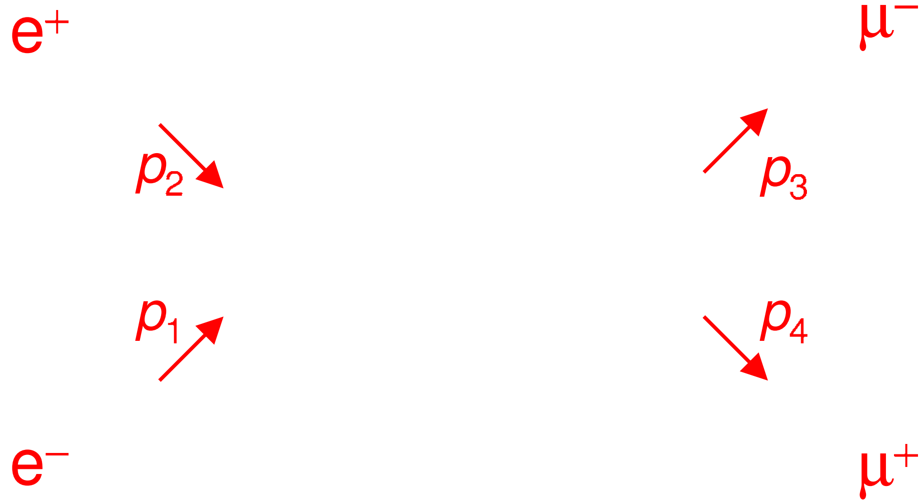
  1) carry out similar matrix element calculation to that of $e^+e^- \to \mu^+\mu^-$

     ➡ question 10 on examples sheet

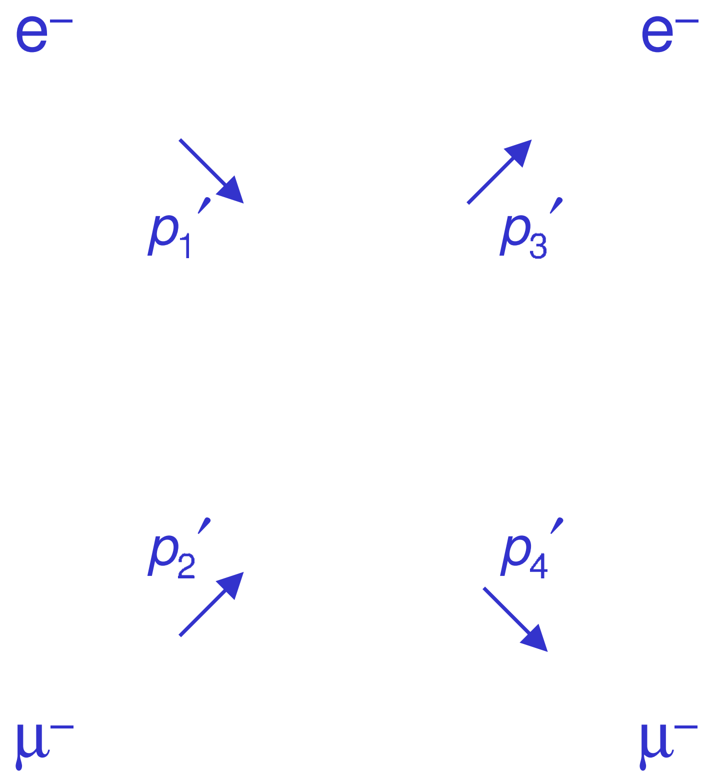
  2) use crossing symmetry ....



$e^+ e^- \to \mu^+ \mu^-$

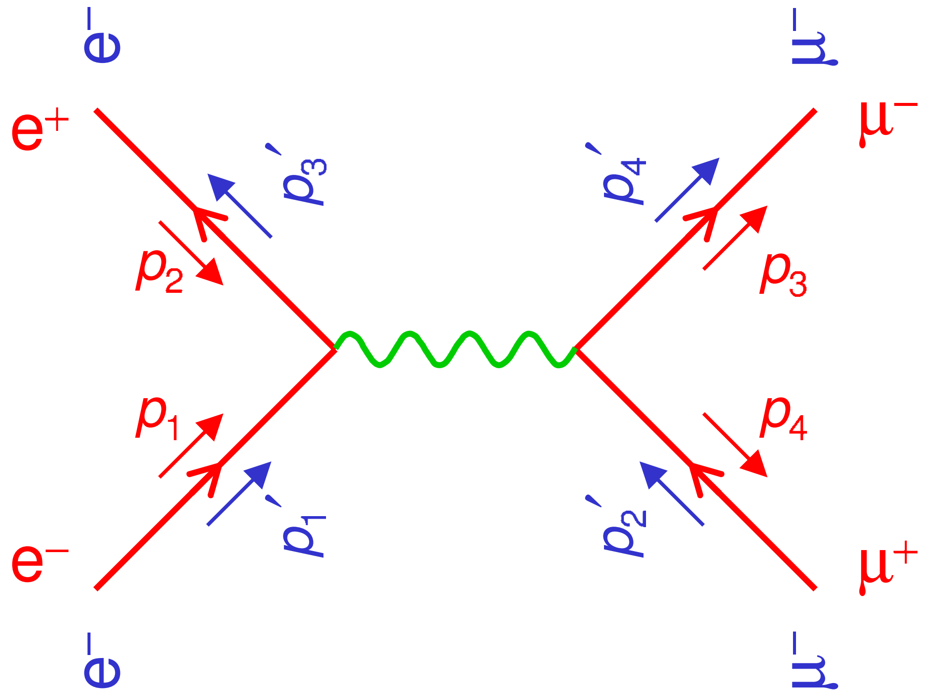


$e^- \mu^- \rightarrow e^- \mu^-$



$$e^+ e^- \rightarrow \mu^+ \mu^-$$

$$e^- \mu^- \rightarrow e^- \mu^-$$



The transformation

$$\begin{aligned} p_1 &\rightarrow p_1' \\ p_2 &\rightarrow -p_3' \\ p_3 &\rightarrow p_4' \\ p_4 &\rightarrow -p_2' \end{aligned}$$

changes

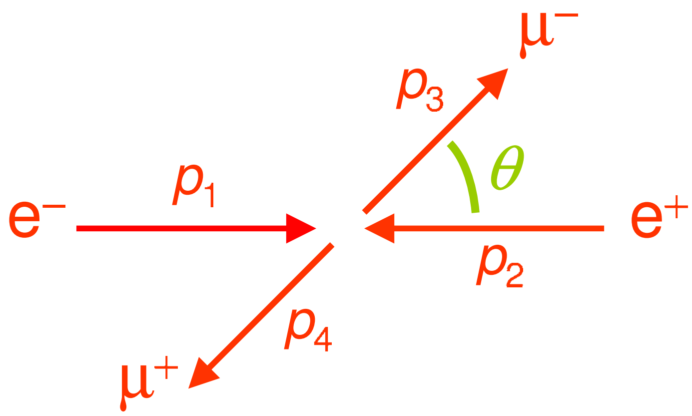
$$(e^+ e^- \rightarrow \mu^+ \mu^-) \;\rightarrow\; (e^- \mu^- \rightarrow e^- \mu^-)$$

$p_2 \quad p_1 \quad\quad p_4 \quad p_3 \quad\quad\quad p_1' \quad p_2' \quad\quad p_3' \quad p_4'$

Known as crossing symmetry

$e^+ e^- \rightarrow \mu^+ \mu^-$

From Handout 4:



$$\langle |M_{fi}|^2 \rangle = e^4 (1 + \cos^2\theta)$$

$$p_1 = (E, 0, 0, E) \qquad p_3 = (E, E\sin\theta, 0, E\cos\theta)$$

$$p_2 = (E, 0, 0, -E) \qquad p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

Convert to Lorentz invariant form :

$$p_1.p_2 = 2E^2$$

$$p_1.p_3 = E^2(1 - \cos\theta)$$

$$p_1.p_4 = E^2(1 + \cos\theta)$$

$$\boxed{\langle |M_{fi}|^2 \rangle = 2e^4 \frac{(p_1.p_3)^2 + (p_1.p_4)^2}{(p_1.p_2)^2}}$$

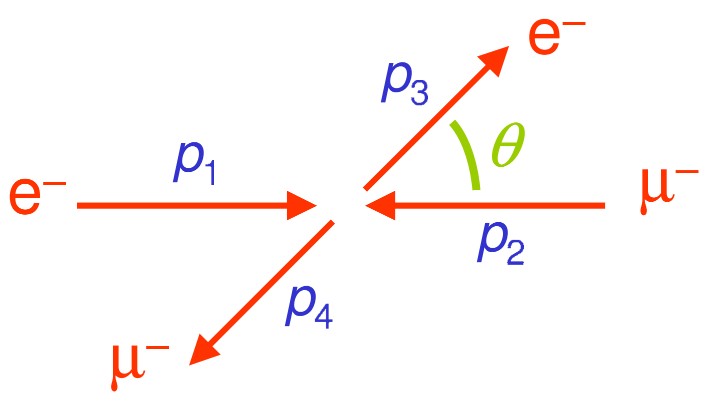
Apply crossing symmetry:

$$p_1 \rightarrow p_1' \qquad p_2 \rightarrow -p_3' \qquad p_3 \rightarrow p_4' \qquad p_4 \rightarrow -p_2'$$

Gives $e^- \mu^- \rightarrow e^- \mu^-$:

$$\boxed{\langle |M_{fi}|^2 \rangle = 2e^4 \frac{(p_1'.p_4')^2 + (p_1'.p_2')^2}{(p_1'.p_3')^2}}$$

$e^- \mu^- \rightarrow e^- \mu^-$



From now on, drop the primes:

$$\langle |M_{fi}|^2 \rangle = 2e^4 \frac{(p_1.p_4)^2 + (p_1.p_2)^2}{(p_1.p_3)^2}$$

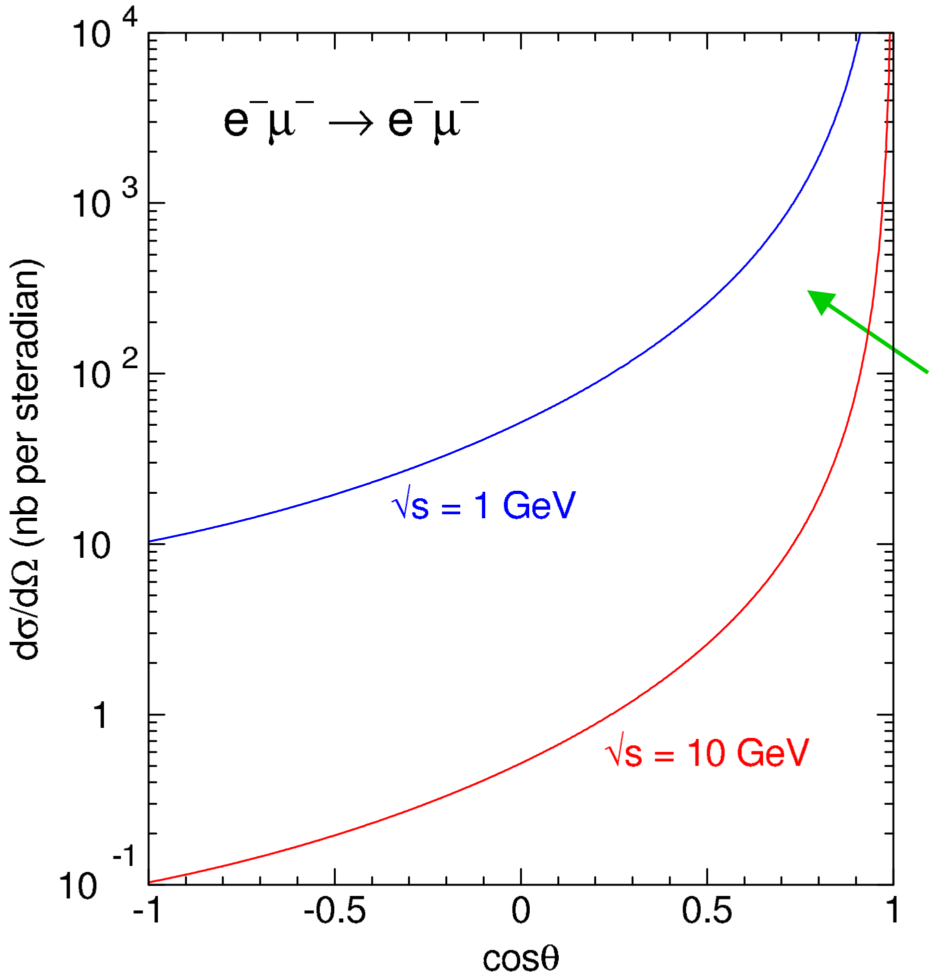
In centre of mass frame :

$p_1.p_2 = 2E^2$

$p_1.p_3 = E^2(1 - \cos\theta)$

$p_1.p_4 = E^2(1 + \cos\theta)$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\langle |M_{fi}|^2 \rangle = \frac{e^4}{8\pi^2 s} \cdot \frac{1 + \frac{1}{4}(1+\cos\theta)^2}{(1-\cos\theta)^2}$$

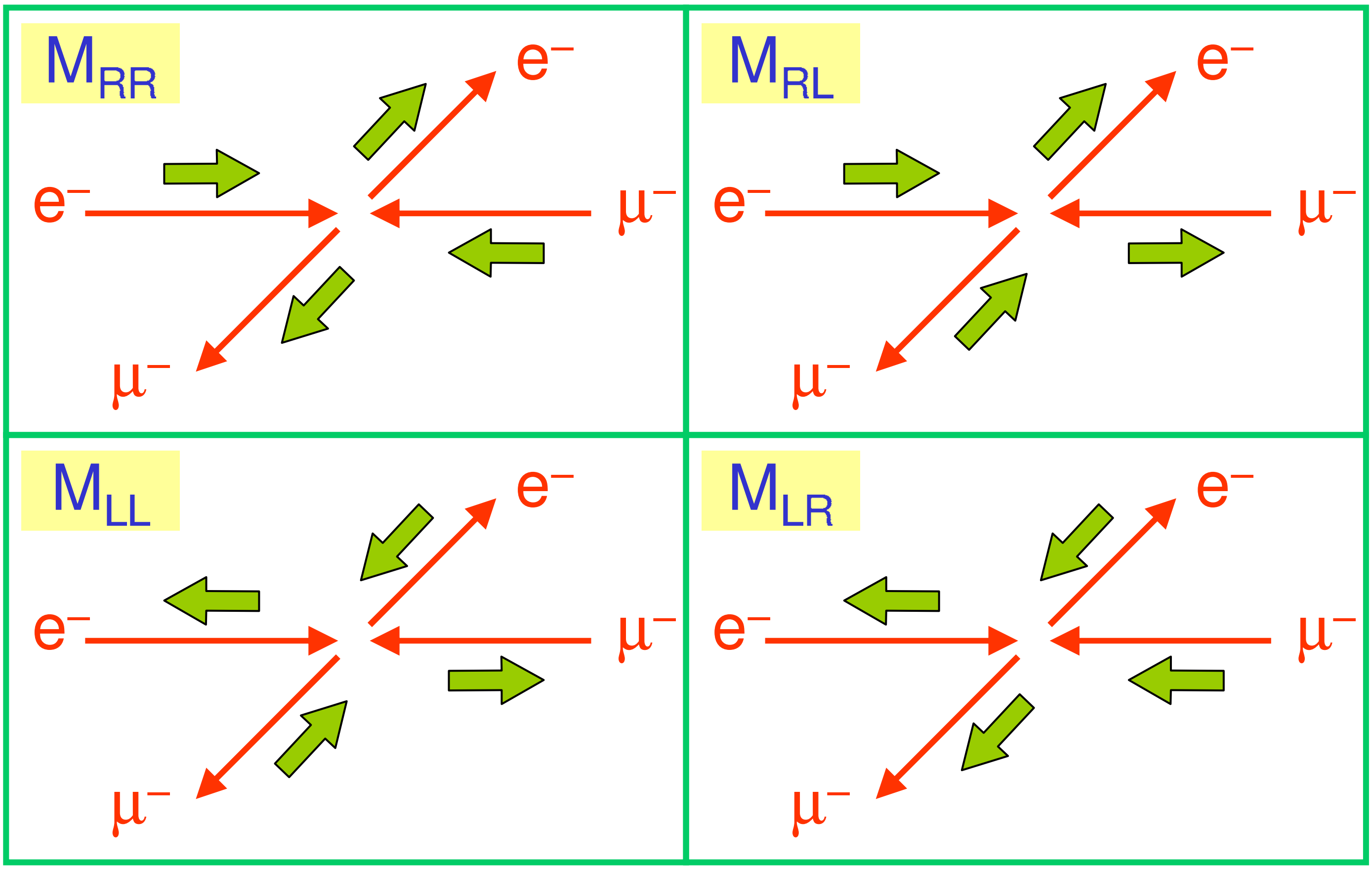


large forward peak due to photon propagator

$$1/q^2 \propto 1/(1-\cos\theta)$$

- $1+\frac{1}{4}(1+\cos\theta)^2$ term in numerator is due to helicity conservation :

  → only 4 allowed spin configurations (out of 16)



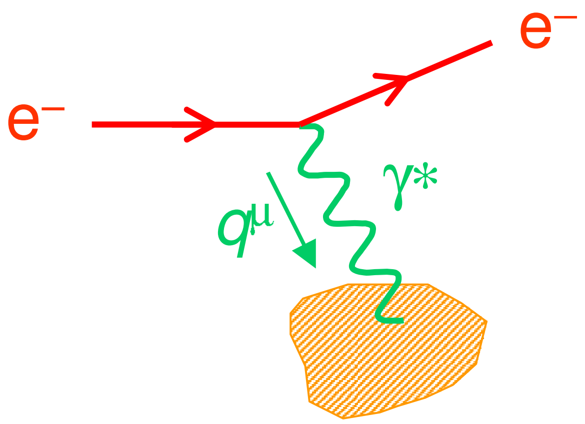
$S_z = 0$	$S_z = \pm 1$
$\frac{d\sigma}{d\Omega} \propto 1$ (isotropic)	$\frac{d\sigma}{d\Omega} \propto \frac{1}{4}(1+\cos\theta)^2$

- Denominator comes from photon propagator:

$$\left(\frac{1}{q^2}\right)^2 = \left(\frac{1}{-2E^2(1-\cos\theta)}\right)^2 \propto \frac{1}{(1-\cos\theta)^2}$$

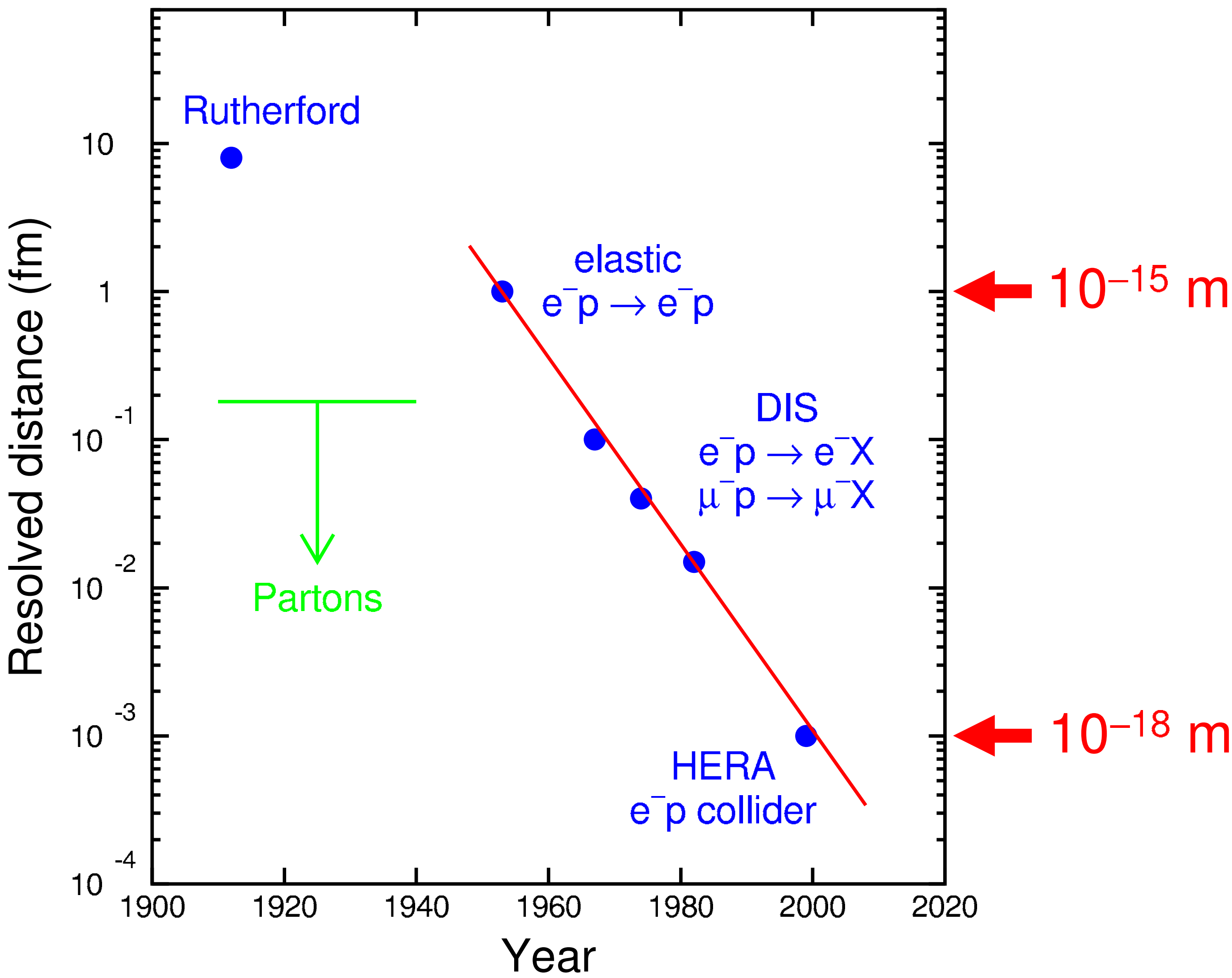
Nucleon Structure

- Use virtual photons to probe the internal structure of the proton and neutron



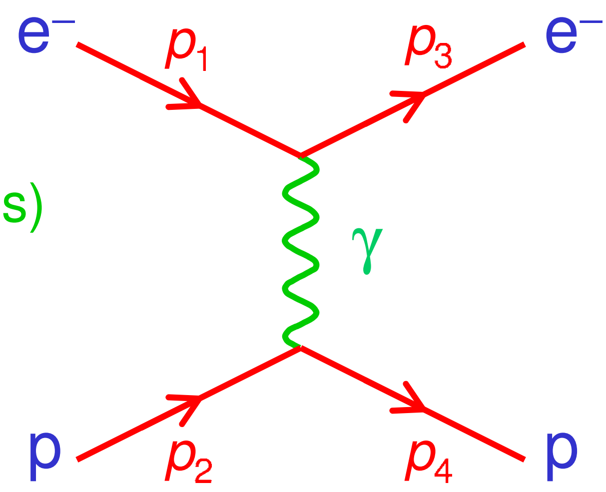
photon wavelength :

$$\lambda \sim \frac{h}{|\mathbf{q}|} \sim \frac{1\,\mathrm{GeV.fm}}{|\mathbf{q}|(\mathrm{GeV})}$$



$e^-p \to e^-p$

- Begin by treating proton as pointlike spin ½ Dirac particle:



At very high energies:

(i.e. neglecting particle masses)

$$p_1.p_2 = p_3.p_4$$
$$p_1.p_4 = p_2.p_3$$

$$\left\langle |M_{fi}|^2 \right\rangle = \frac{8e^4}{(p_1 - p_3)^4} \cdot \left[(p_1.p_2)(p_3.p_4) + (p_1.p_4)(p_2.p_3)\right]$$

- Complete expression valid at all energies:

$$\left\langle |M_{fi}|^2 \right\rangle = \frac{8e^4}{(p_1 - p_3)^4} \cdot \left[(p_1.p_2)(p_3.p_4) + (p_1.p_4)(p_2.p_3) - (p_1.p_3)M^2 - (p_2.p_4)m^2 + 2m^2M^2\right]$$

(examples sheet)

- Consider two cases:

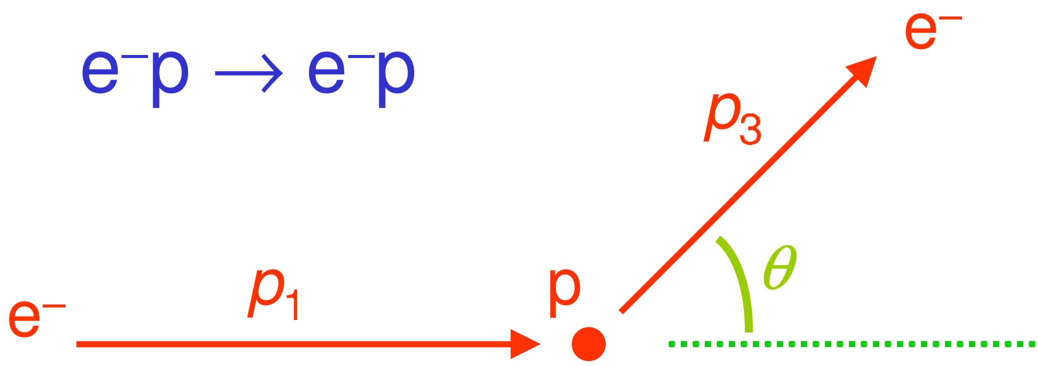
  1) lab frame at very low energy

     (neglect proton recoil)

  2) lab frame at very high energy

Low energy limit, lab frame: (Handout 5.2.1)

$e^-p \to e^-p$



(neglect proton recoil)

$$p_1 = (E,0,0,p) \qquad p_3 = (E, p\sin\theta, 0, p\cos\theta)$$
$$p_2 = (M,0,0,0) \qquad p_4 = (M,0,0,0)$$

with: $p << m, \quad E \approx m, \quad E << M$

Obtain Rutherford scattering cross section:

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4E'\sin^2\theta/2}\right)^2} \qquad \left(E' = \tfrac{1}{2}mv^2\right)$$

➡ Spin of particles has no effect at low energy

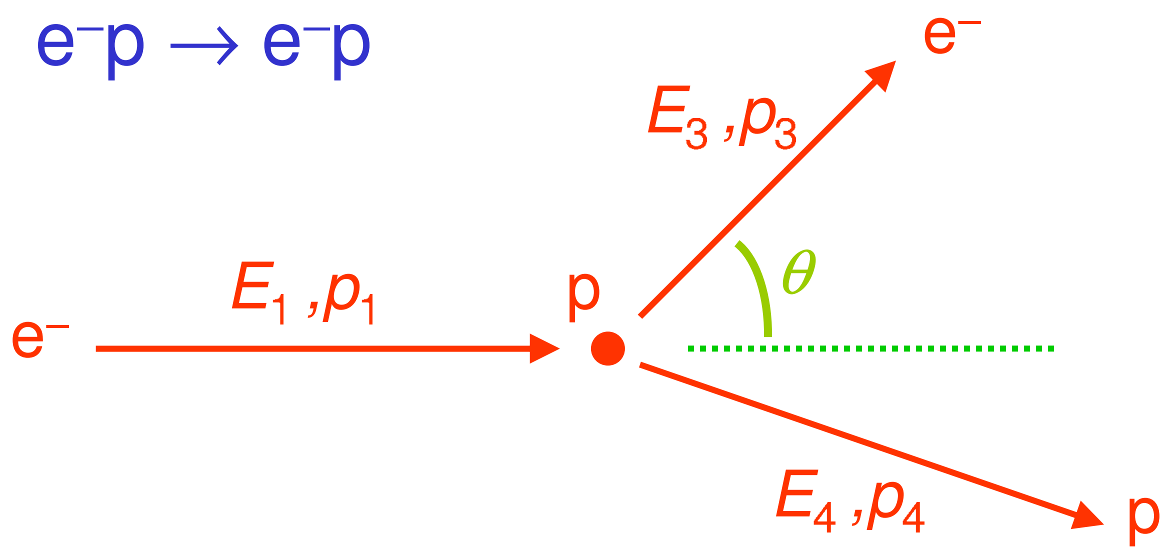
lower energy ⇒ only probing relatively large r

⇒ magnetic moment unimportant (~ $1/r^3$)

only charge matters (~ $1/r^2$)

High energy limit, lab frame: (Handout 5.2.2)

$e^-p \rightarrow e^-p$



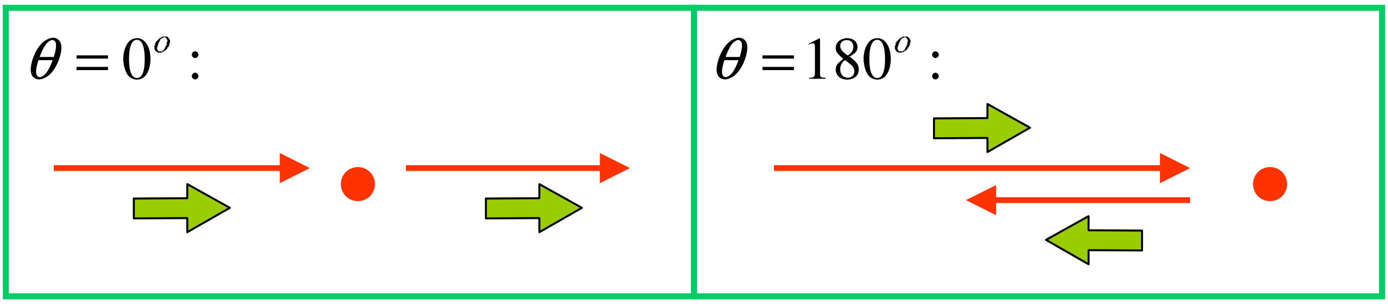
$$p_1 = (E_1, 0, 0, E_1) \qquad p_3 = (E_3, E_3\sin\theta, 0, E_3\cos\theta)$$

$$p_2 = (M, 0, 0, 0) \qquad p_4 = (E_4, \mathbf{p}_4)$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2} \cdot \frac{E_3}{E_1} \cdot \left(\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

Rutherford | recoil | electric scattering | magnetic scattering

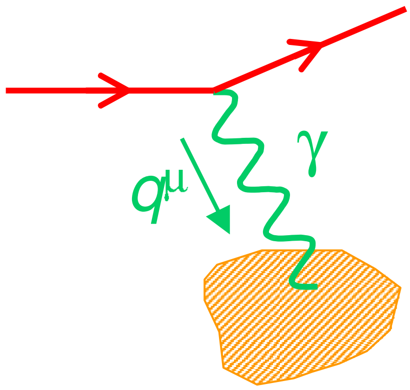
$\cos^2\theta/2$ reflects helicity conservation :



favoured *suppressed*

Nucleon Form Factors

- To take into account finite size of proton:



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{pointlike}} \times \left|F(q^2)\right|^2$$

$F(q^2)$ = Form Factor

$q^2=0$: $F(q^2)$ = F.T. of charge distribution

- Spin 1/2 target $\Longrightarrow$ need two form factors :

  $G_E(q^2)$: charge distribution

  $G_M(q^2)$: magnetic moment distribution

Rosenbluth Formula:

$$\tau \equiv -q^2/4M^2$$

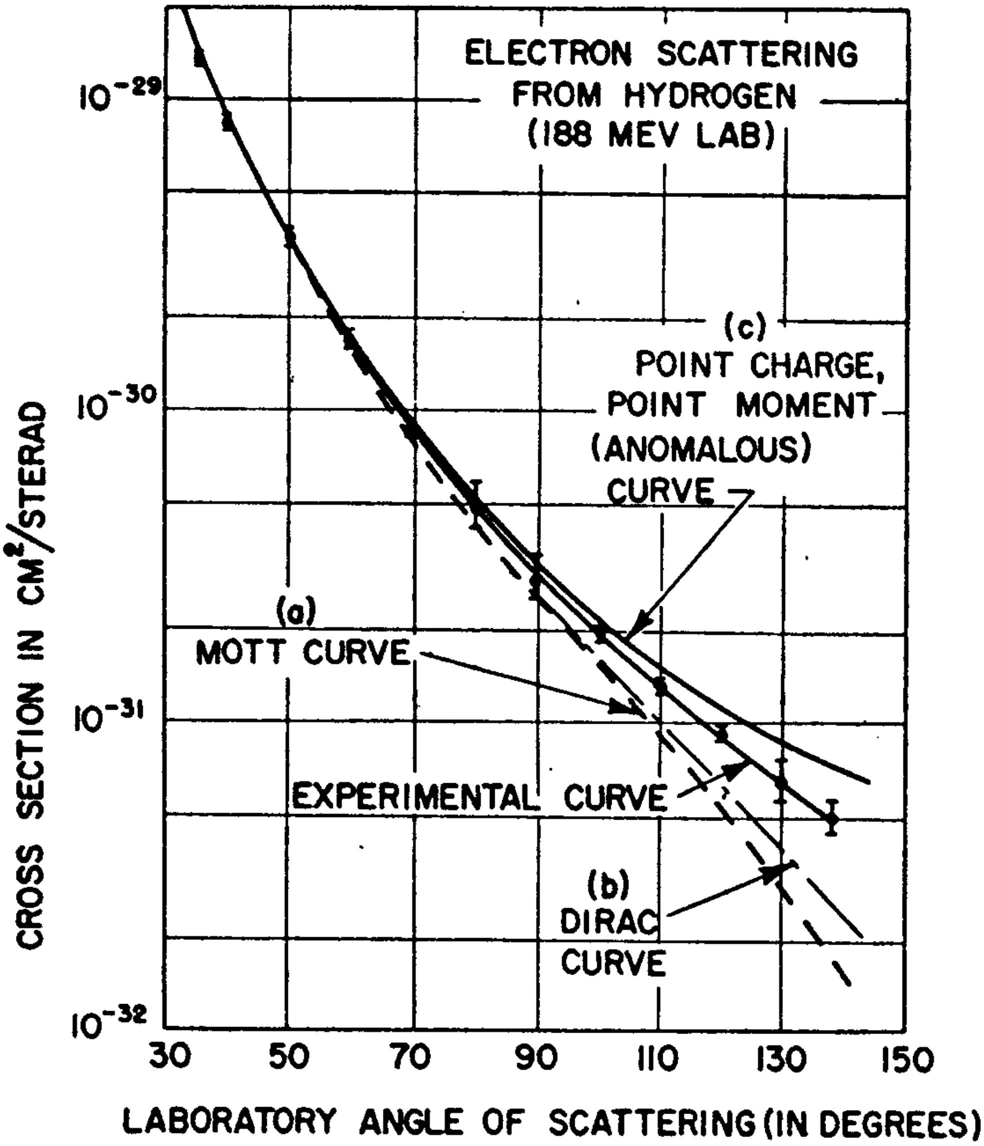
$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{spin0}} \times \left[\frac{G_E^2 + \tau \cdot G_M^2}{1+\tau} + 2\tau \cdot G_M^2 \tan^2\frac{\theta}{2}\right]$$

$G_E(0)$ = total charge = 1 (p)
= 0 (n)

$G_M(0)$ = magnetic moment = +2.79 (p)
= −1.91 (n)

First evidence for an extended proton

Using 188 MeV e⁻ beam:



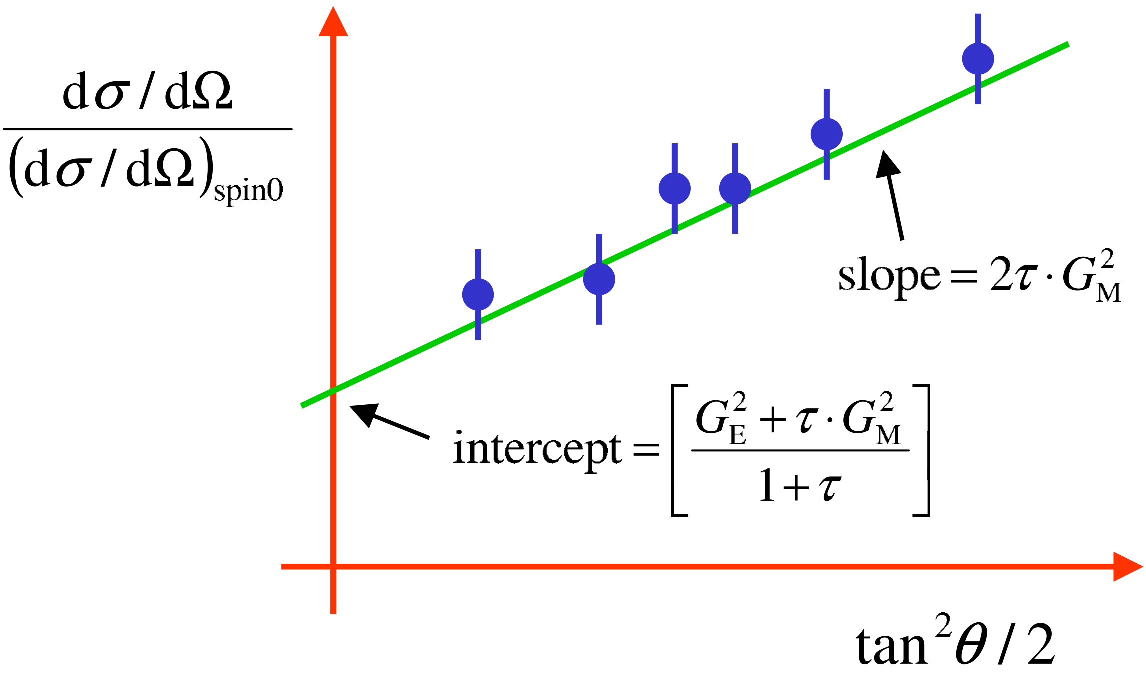
R.Hofstadter + R.W.McAllister, Phys. Rev. **98** (1955) 217

R.W.McAllister + R.Hofstadter, Phys. Rev. **102** (1956) 851

Data lies below expectation for a pointlike proton with $\mu = 2.79$

(upper solid curve : $G_E(q^2) = 1$, $G_M(q^2) = 2.79$)

- To extract $G_E(q^2)$ and $G_M(q^2)$, plot ….



…. at a fixed value of q^2

- Once q^2 and θ have been chosen :

$$E_1 - E_3 = \frac{-q^2}{2M} \qquad E_1 E_3 = \frac{-q^2}{2(1-\cos\theta)}$$

$\Rightarrow$ E_1 and E_3 are then determined

(and hence vary along the straight line above)

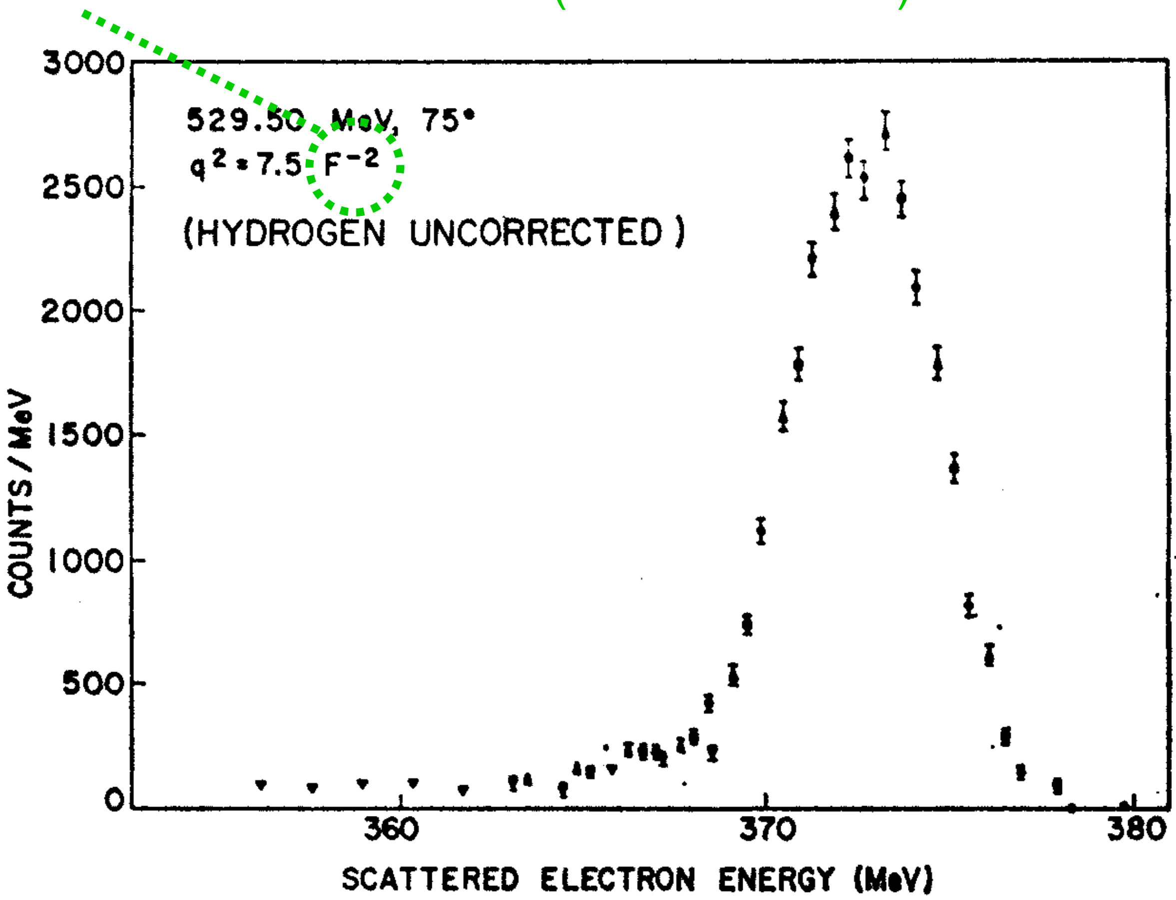
E_1 = incident e⁻ beam energy
E_3 = scattered e⁻ energy

Distribution of scattered energies should show an elastic peak at a well-defined energy (E_3)

◇ e.g. $e^- p \rightarrow e^- p$ at E_{beam} = 529.5 MeV and θ = 75°

Note olden-days units :

1 F^{-2} = 1 fm^{-2} = 1 fm^{-2} x (0.197 GeV.fm)^2 = 0.039 GeV^2



T.Janssens et al., Phys. Rev. **142** (1966) 922
E.B.Hughes et al., Phys. Rev. **139** (1965) B458

Expect elastic peak at scattered energy

$$E_3 = \frac{ME_1}{M + E_1(1-\cos\vartheta)}$$

M_p = 0.938 GeV

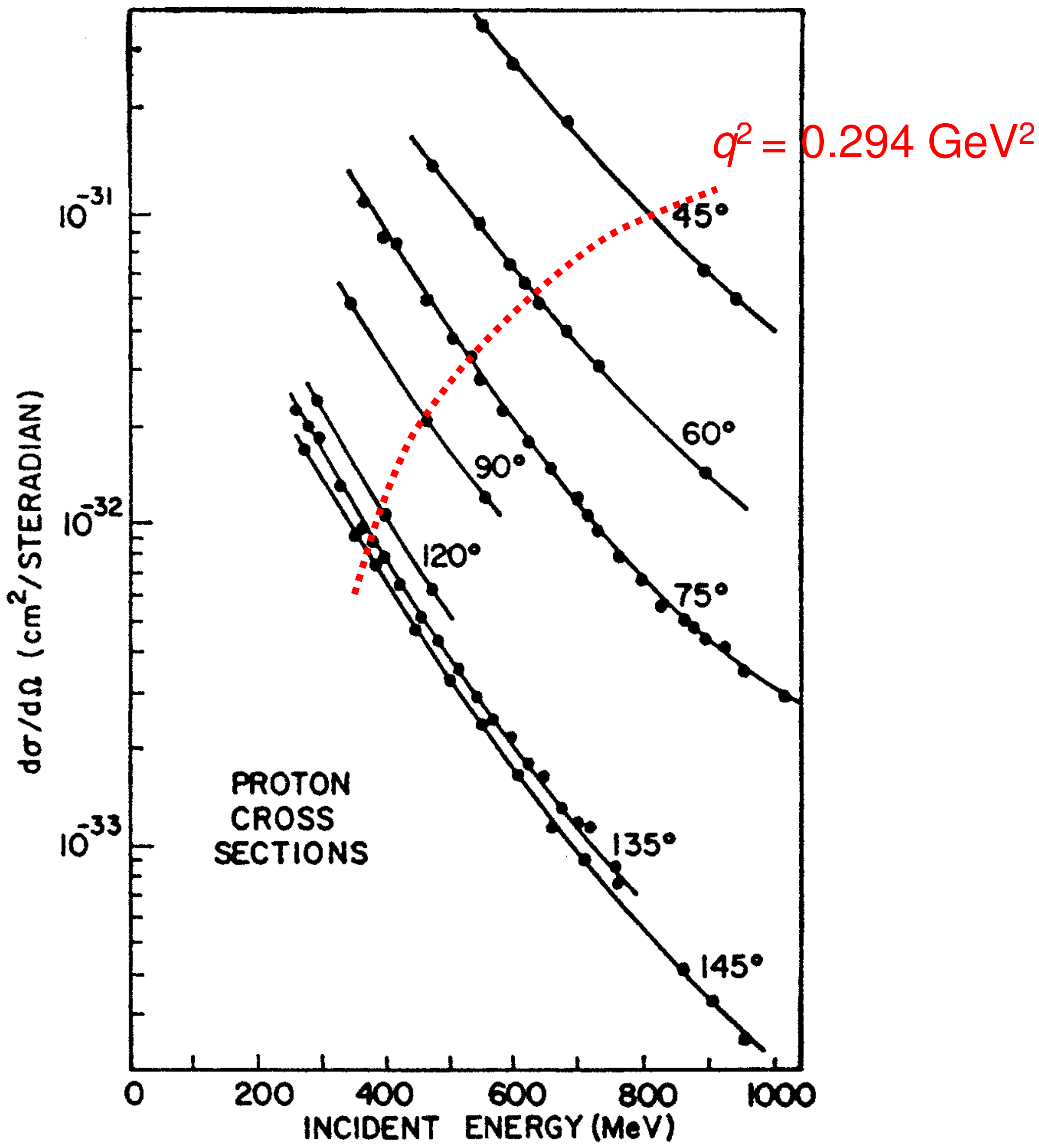
$$= \frac{0.938 \times 0.5295}{0.938 + 0.5295(1-\cos 75^\circ)} = 0.373\,\text{GeV}$$

(as observed)

$$|q^2| = 2M(E_1 - E_3)$$

$$= 2 \times 0.938(0.5295 - 0.373) = 0.294\,\text{GeV}^2 = 7.5\,\text{F}^{-2}$$

◇ Measured $e^-p \rightarrow e^-p$ cross sections :

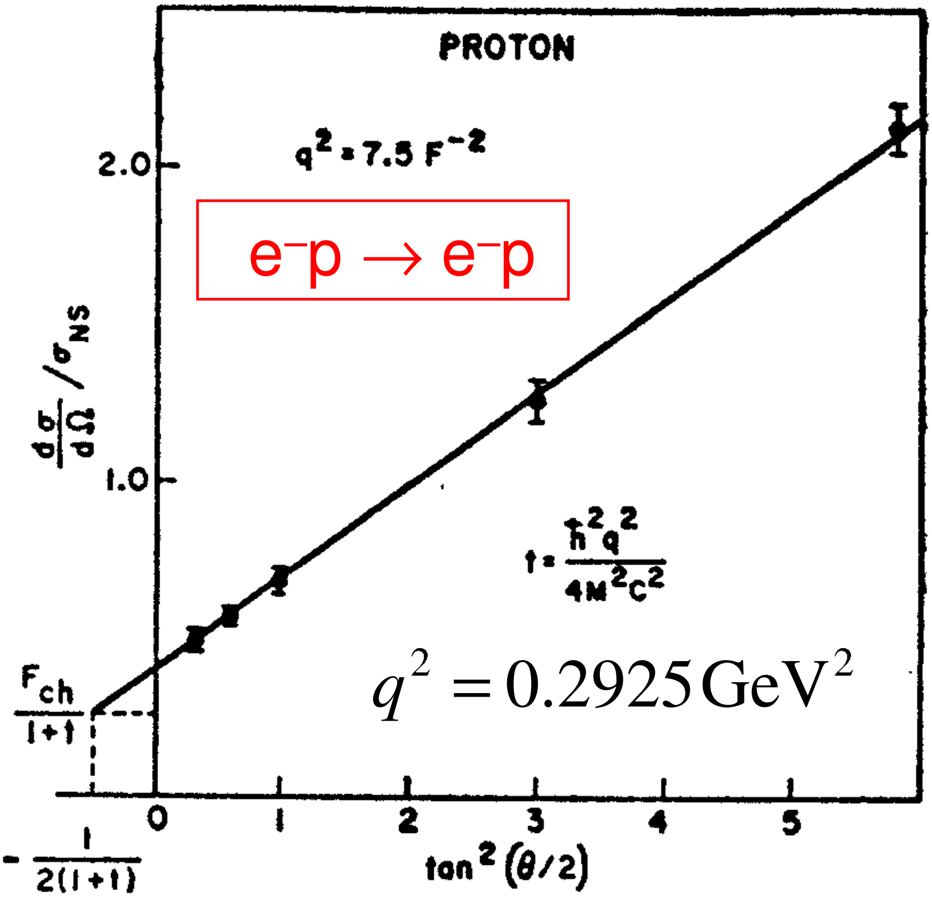


- Beam energies chosen to give particular values of q^2
- Cross sections measured with typical precision ± (2-3)%

(more recent experiments approach ±1%)

Example of Rosenbluth plot :

$$\uparrow \quad \frac{d\sigma}{d\Omega} \Big/ \left(\frac{d\sigma}{d\Omega} \right)_{\text{spin}\,0}$$



$$\tan^2\theta/2 \longrightarrow$$

$\Rightarrow$ consistent with Rosenbluth prediction

(i.e. with single photon exchange)

$\Rightarrow$ can extract $G_E^p(q^2), G_M^p(q^2)$

To extract $G_E^n(q^2), G_M^n(q^2)$ for neutron:

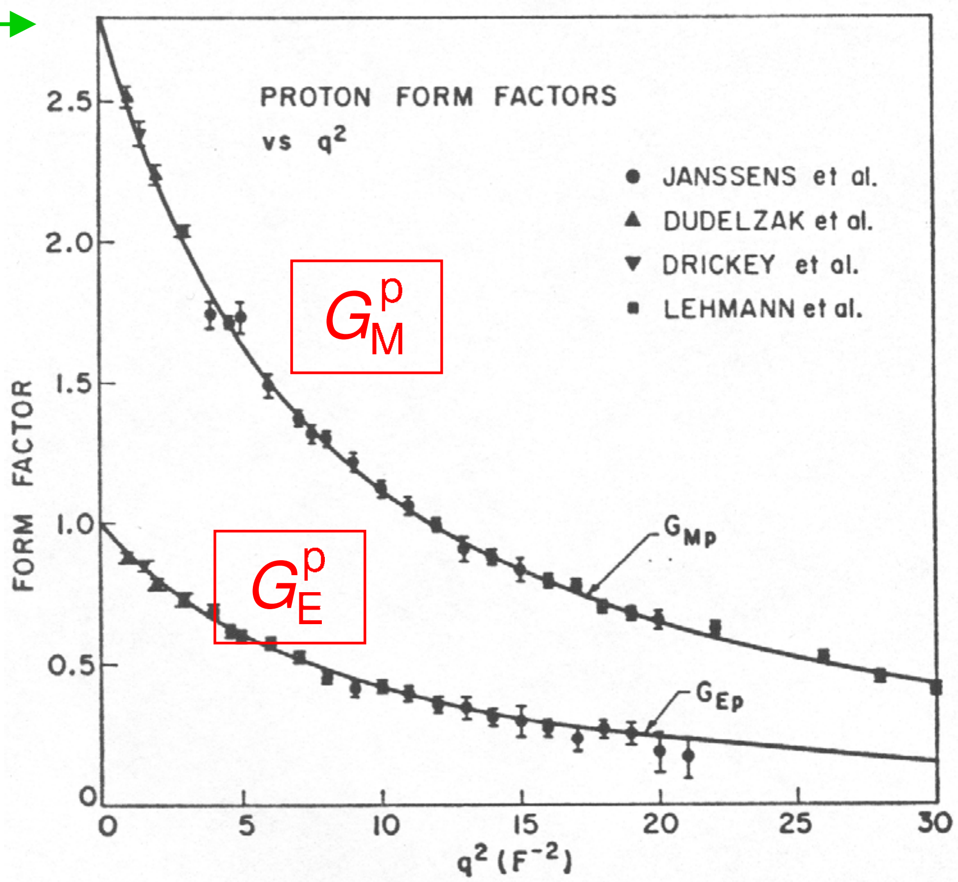
use cross section <u>difference</u> between hydrogen and deuterium targets

$G_E(q^2)$ and $G_M(q^2)$, up to $q^2 \approx 1.2$ (GeV)^2 :

$\mu_p = +2.79\ \mu_N$ →

proton

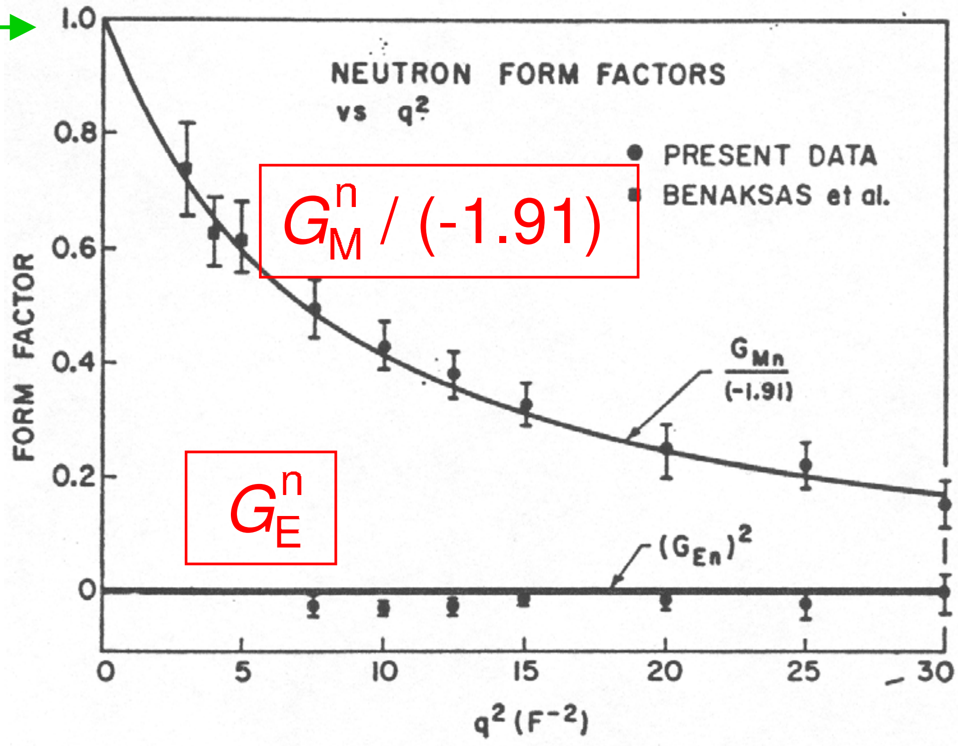
$Q_p = +1$ →



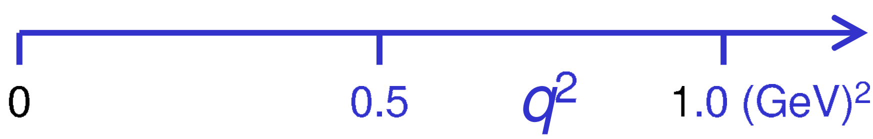
$\mu_n = -1.91\ \mu_N$ →

neutron

$Q_n = 0$ →

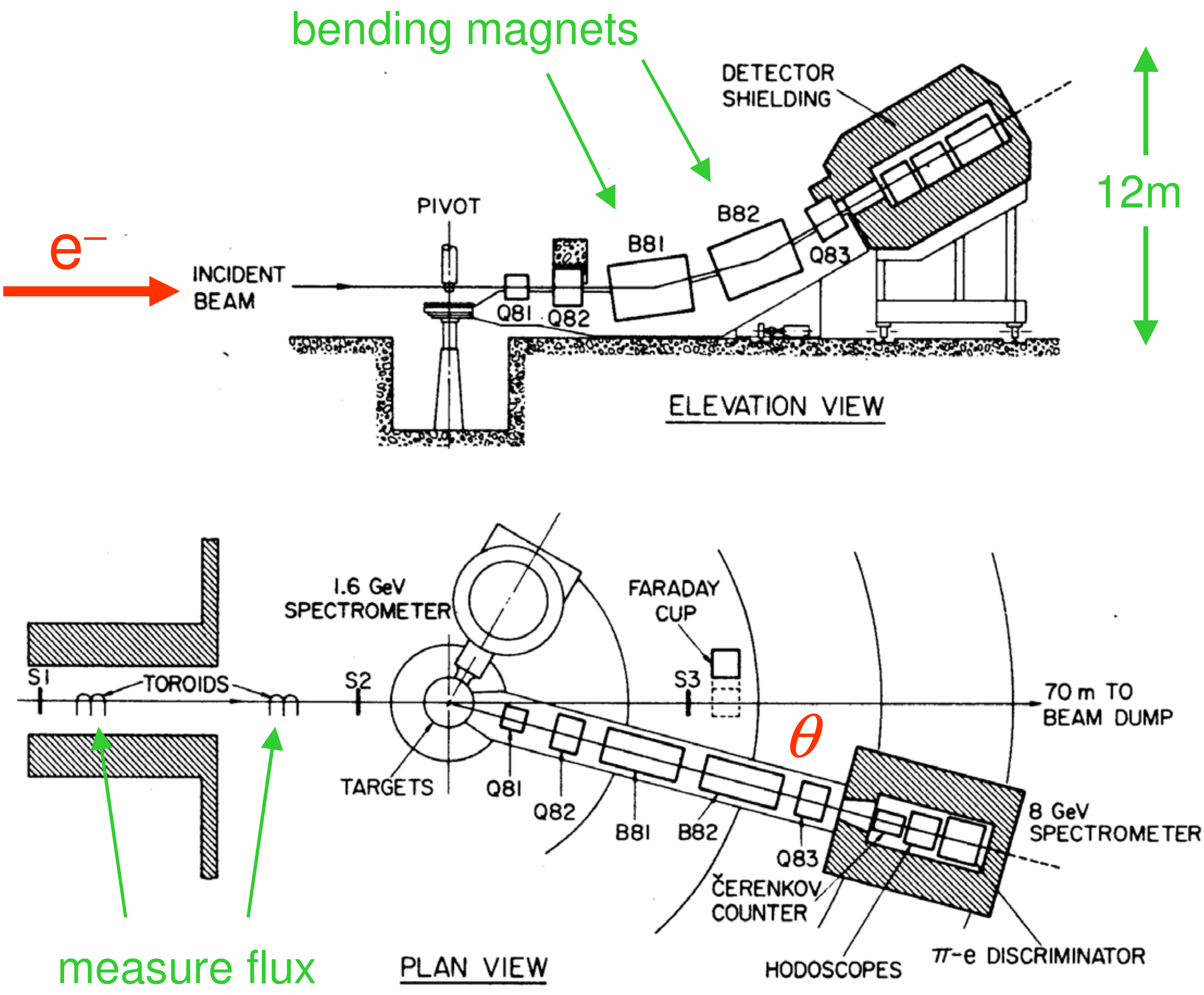


E.B.Hughes et al.,
Phys Rev **139** (1965) B458



◇ To go to higher values of q^2 :

The SLAC 8 GeV Spectrometer



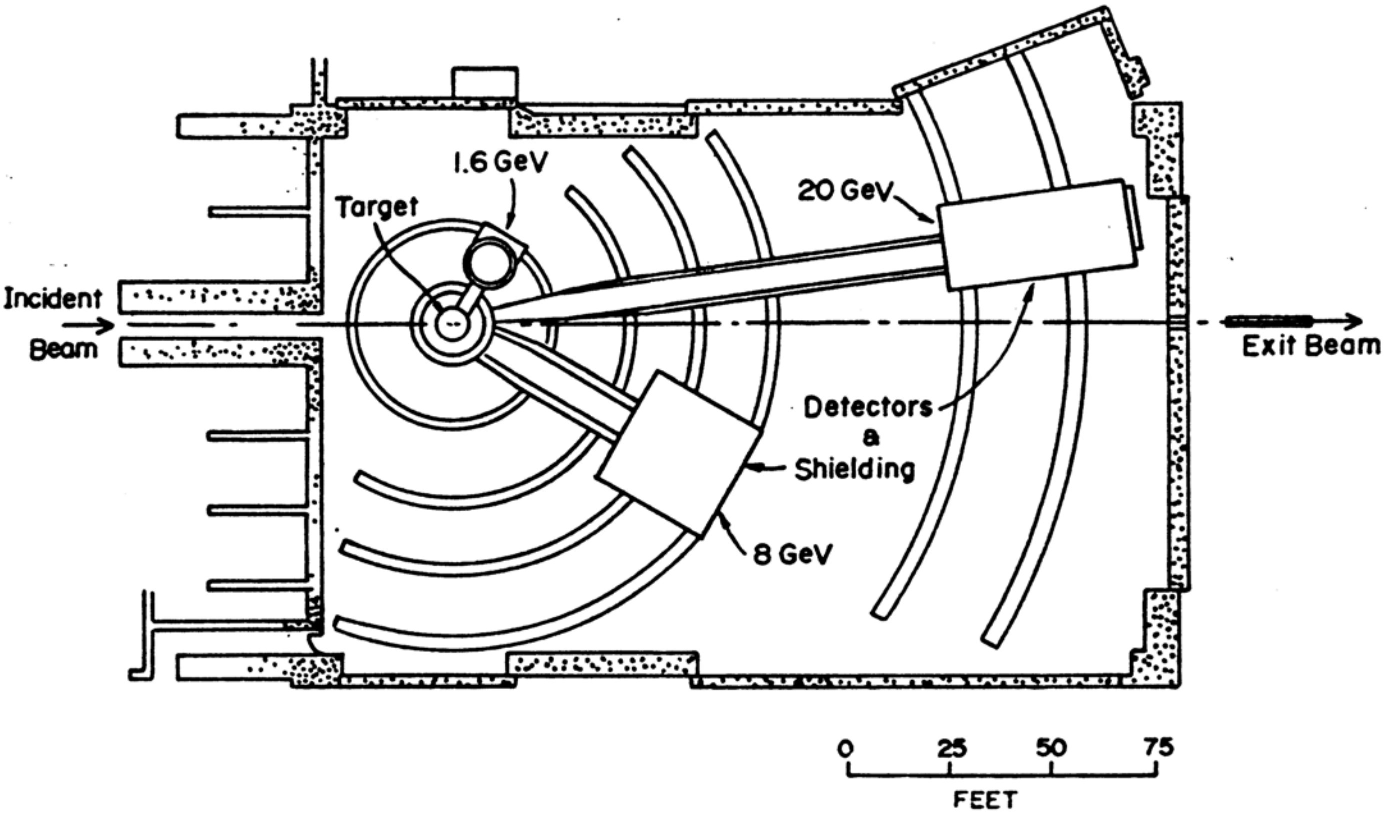
P.N.Kirk et al., Phys Rev **D8** (1973) 63

Use electron beam from SLAC LINAC:

$$5 < E_{beam} < 20 \text{ GeV}$$

… followed by ...

The SLAC 20 GeV spectrometer



- Allowed measurements of $G(q^2)$ up to $q^2 \approx 30\ \text{GeV}^2$

- Also used for extensive studies of deep-inelastic scattering

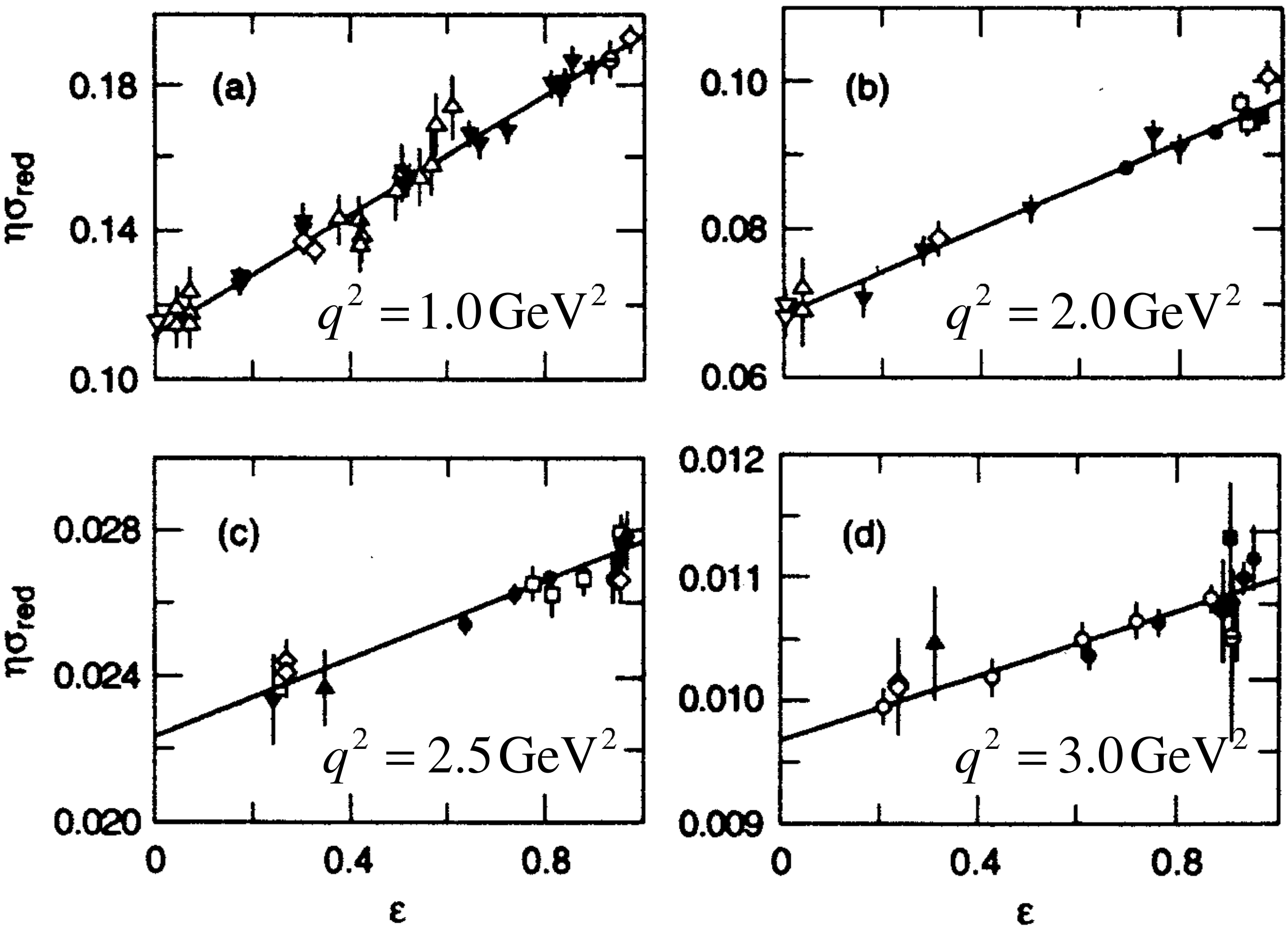
  (proton target breaks up – see later)

SLAC $e^{\pm}$ LINAC

SLAC 8 GeV and 20 GeV Spectrometers

Rosenbluth plots at higher q^2 :

R.C.Walker et al., Phys. Rev. **D49** (1994) 5671



$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{spin0}} \times \frac{1}{1+\tau}\left[G_{\mathrm{E}}^2 + \frac{\tau}{\varepsilon} G_{\mathrm{M}}^2\right]$$

$$\frac{1}{\varepsilon} \equiv 1 + 2(1+\tau)\tan^2\theta/2$$

(compilation of 8 GeV spectrometer plus earlier data)

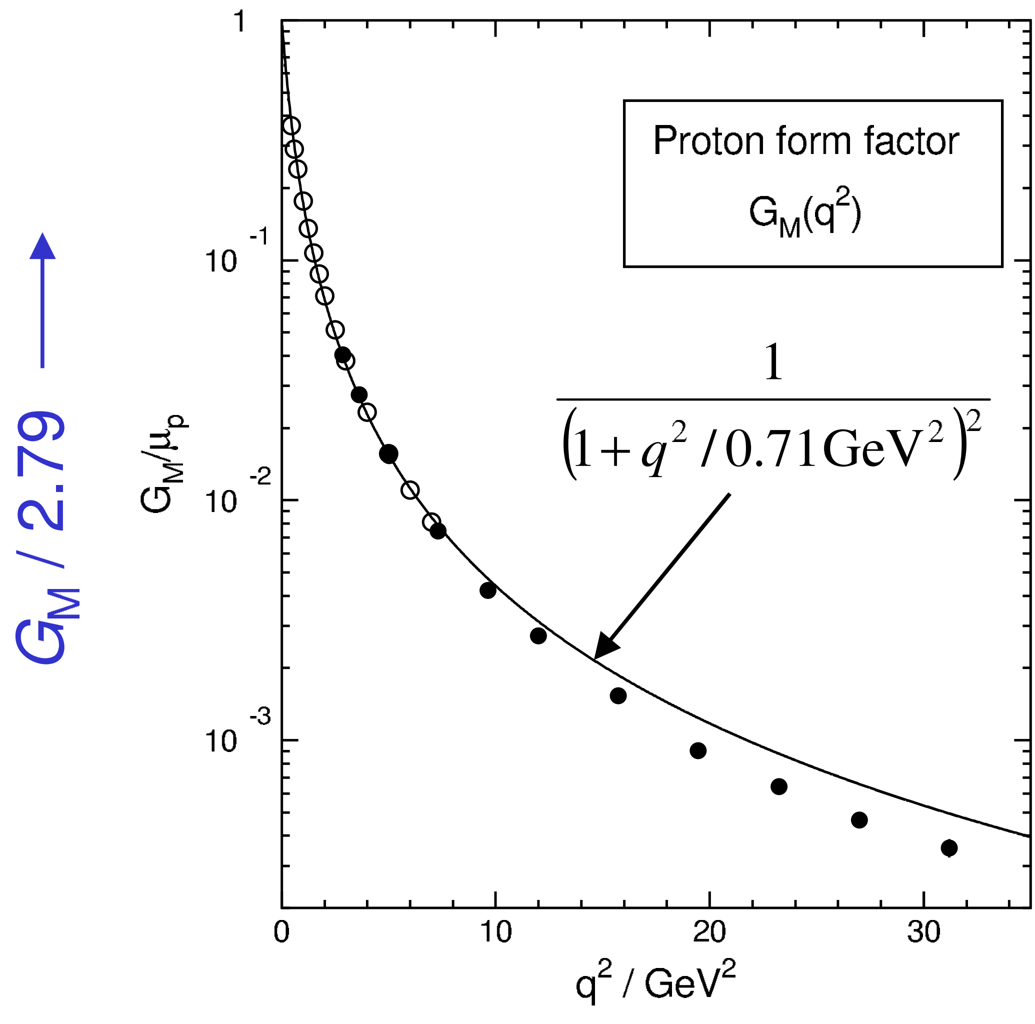
- At even higher q^2 : $\tau \gg 1 \quad G_E^2 \ll \tau G_M^2$

$\Rightarrow$ contribution from $G_E(q^2)$ becomes small and hard to measure

$\Rightarrow$ usually, extract $G_M(q^2)$ alone, assuming $G_E(q^2)$ and $G_M(q^2)$ have same shape :

$$G_E^p(q^2) = G_M^p(q^2)/2.79$$

- *e.g.* $G_M(q^2)$, up to $q^2 \approx 30$ (GeV)^2 :



R.C.Walker et al., Phys. Rev. **D49** (1994) 5671

A.F.Sill et al., Phys. Rev. **D48** (1993) 29

Form Factors: Summary

- Form factors fall rapidly with q^2
  - find good fit to data with "dipole" form :

$$G_E^p(q^2) \approx \frac{G_M^p(q^2)}{2.79} \approx \frac{G_M^n(q^2)}{-1.91} \approx \frac{1}{\left(1 + q^2/0.71\,\text{GeV}^2\right)^2}$$

- Take FT to find spatial distributions :

  $\Rightarrow$ charge and magnetic moment distributions of form

  $$\rho(r) \approx \rho_0 e^{-r/a} \quad \text{with } a \approx 0.24 \text{ fm}$$

  $\Rightarrow$ r.m.s. charge radius $\approx$ 0.80 fm

(examples sheet)

N.B. does not immediately follow that a finite size proton is composite …

- 1960's: the proton was still assumed to be an elementary particle
- strong interaction effects ("pion clouds") could then give it a finite extent