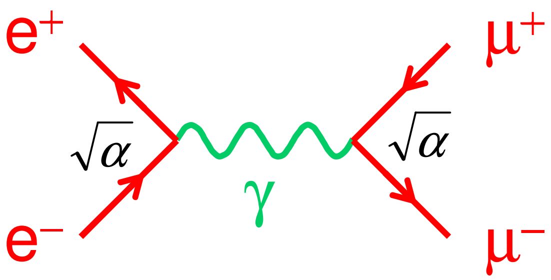
QED Calculations

1) Draw all possible Feynman diagrams

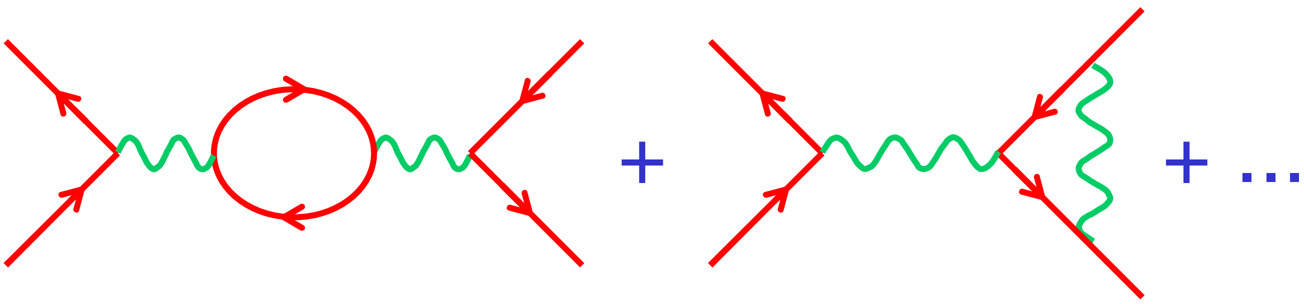
e.g. $e^+e^- \rightarrow \mu^+\mu^-$

only one diagram in leading order:



$\sigma \sim O(\alpha^2)$

many diagrams in second order:



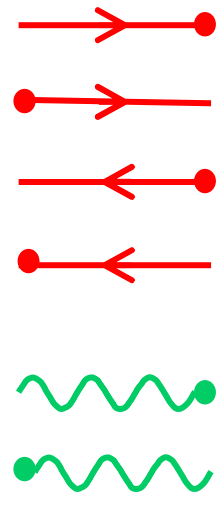
2) Use Feynman rules to find amplitude M_i for each diagram :

- propagator factor for each internal line
  (i.e. each internal virtual particle)
- spinor for each external line
  (i.e. each real incoming or outgoing particle)
- vertex factor for each vertex

QED Feynman Rules

External lines

spin 1/2	incoming particle	u(p)	
	outgoing particle	$\bar{u}(p)$	
	incoming antiparticle	$\bar{v}(p)$	
	outgoing antiparticle	v(p)	
spin 1	incoming photon	$\varepsilon^{\mu}(p)$	
	outgoing photon	$\varepsilon^{\mu}(p)^{*}$	



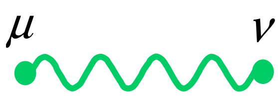
Internal lines

spin 1/2

$$\frac{i(\gamma^{\mu} p_{\mu} + m)}{p^2 - m^2}$$

(propagators)

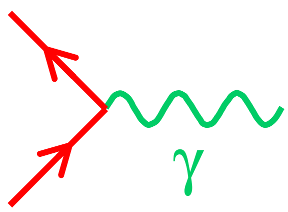
spin 1 (photon)



$$\frac{-ig^{\mu\nu}}{p^2}$$

Vertex factors

spin 1/2



$ie\gamma^{\mu}$

$$e = \sqrt{4\pi\alpha}$$

for particle of charge −e

(plus some additional rules for spin 0, higher orders etc.)

Product of above factors gives: $-iM_{fi}$

3) Sum the amplitudes M_i for each diagram ...

$$M_{\mathrm{fi}} = M_1 + M_2 + M_3 + \cdots$$

… and square:

$$\left|M_{\mathrm{fi}}\right|^2 = \left|M_1 + M_2 + M_3 + \cdots\right|^2$$

$\Rightarrow$ perturbation series in powers of α

4) Calculate decay rate or cross section ...

e.g. for 2-body decay a → 1 + 2 :

$$\Gamma = \frac{p^*}{32\pi^2 m_a^2} \times \int \left|M_{\mathrm{fi}}\right|^2 \times \mathrm{d}\cos\theta \, \mathrm{d}\phi$$

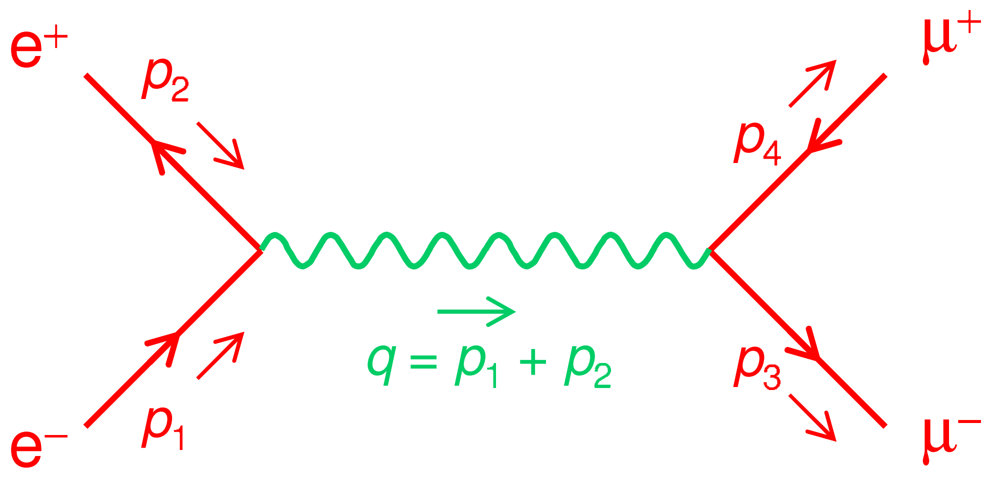
e.g. for scattering a + b → 1 + 2 in cms :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} = \frac{1}{64\pi^2 s} \cdot \frac{p_{\mathrm{f}}^*}{p_{\mathrm{i}}^*} \left|M_{\mathrm{fi}}\right|^2$$

e.g. for high energy elastic scattering in lab :

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{E_1 M} \right)^2 \left|M_{\mathrm{fi}}\right|^2$$

$e^+ e^- \to \mu^+ \mu^-$



p_i are physical particle or antiparticle 4-momenta

Feynman rules give: (Handout 4.1)

$$-iM_{\mathrm{fi}} = \overline{\mathrm{v}}(p_2) \cdot ie\gamma^{\mu} \cdot \mathrm{u}(p_1)$$ electron current

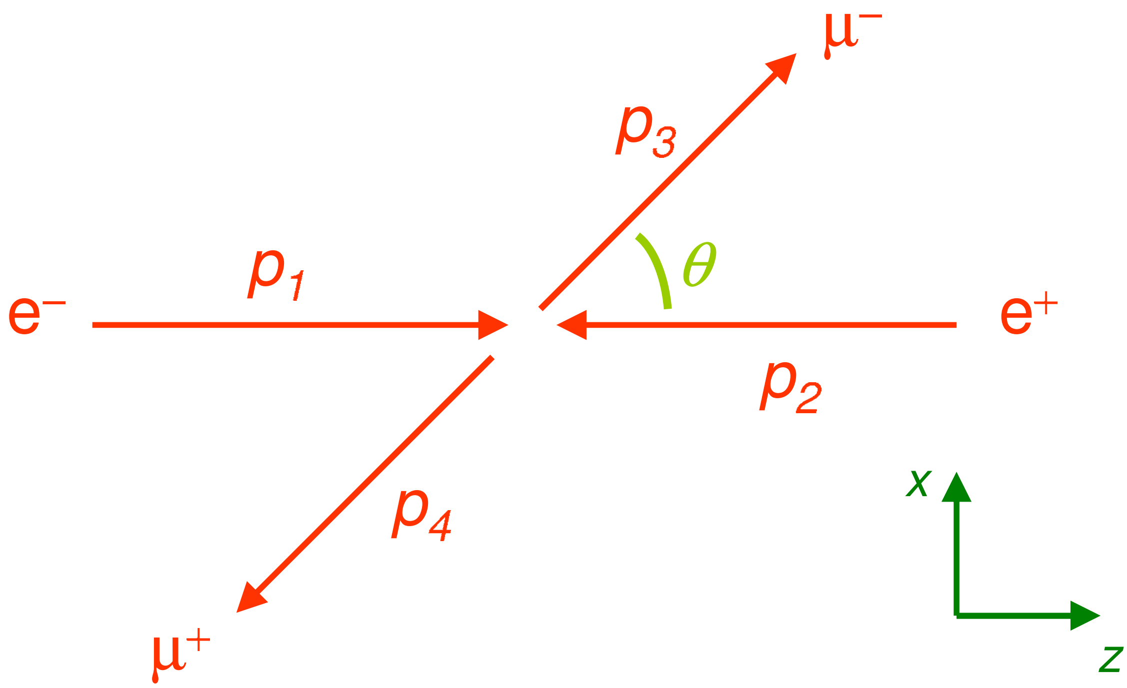
$$\times \frac{-ig_{\mu\nu}}{q^2}$$ photon propagator

$$\times \overline{\mathrm{u}}(p_3) \cdot ie\gamma^{\nu} \cdot \mathrm{v}(p_4)$$ muon current

$$\Rightarrow \boxed{M_{\mathrm{fi}} = \frac{-e^2}{(p_1+p_2)^2} \mathrm{g}_{\mu\nu} \cdot \overline{\mathrm{v}}(p_2)\gamma^{\mu}\mathrm{u}(p_1) \cdot \overline{\mathrm{u}}(p_3)\gamma^{\nu}\mathrm{v}(p_4)}$$

… a number which can be worked out once the spin states are specified

- Evaluate M_{fi} in centre of mass frame ...



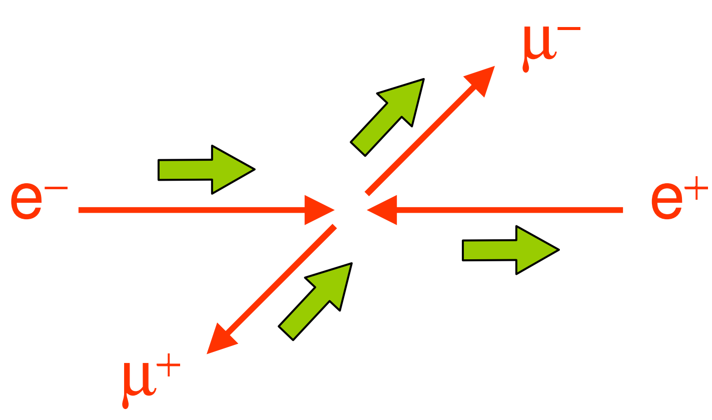
… at extreme relativistic energies:

$$p_1 = (E, 0, 0, E) \qquad p_3 = (E, E\sin\theta, 0, E\cos\theta)$$

$$p_2 = (E, 0, 0, -E) \qquad p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

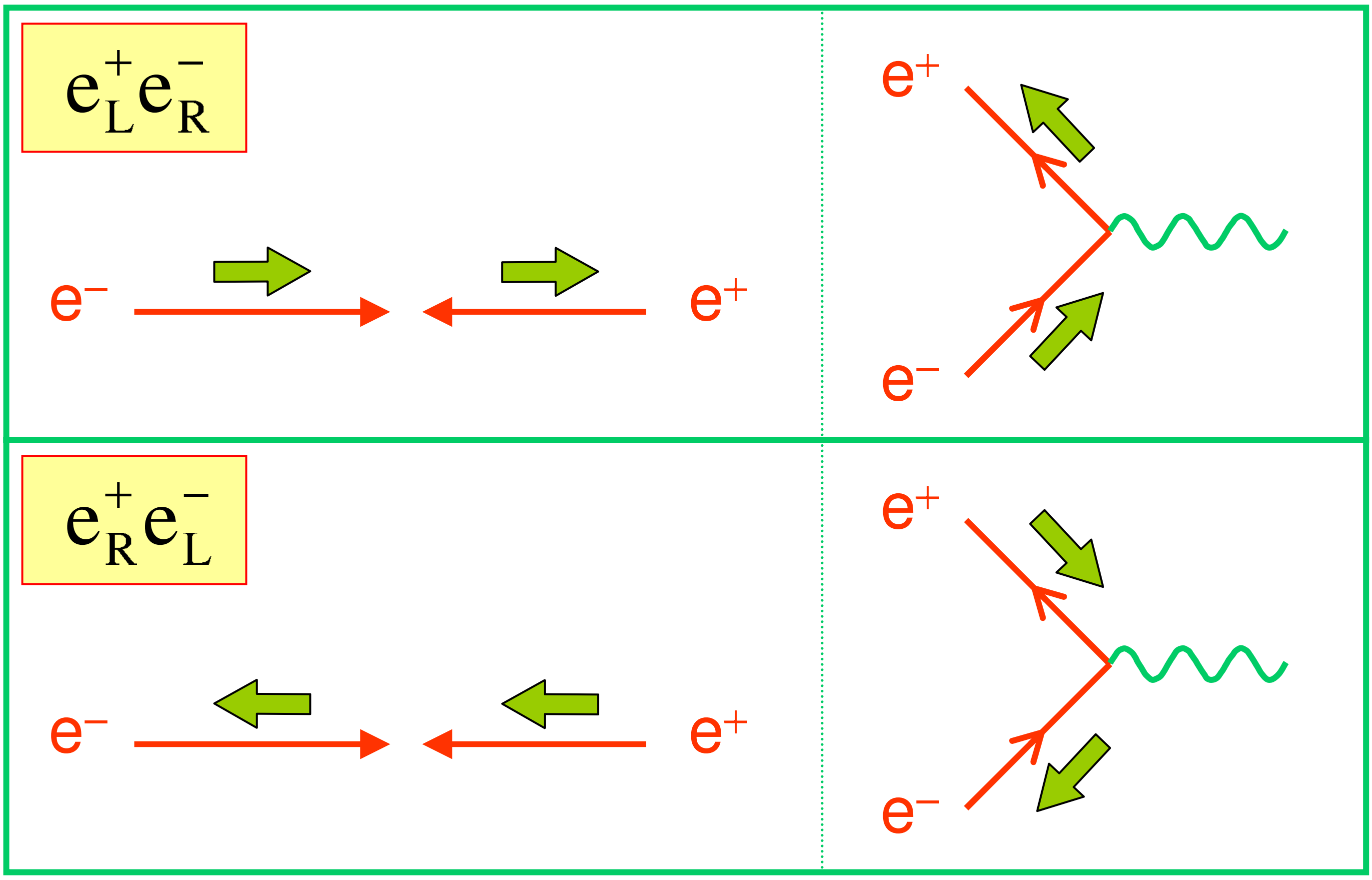
- 16 possible helicity configurations ...

e.g. $e^+_L e^-_R \rightarrow \mu^+_L \mu^-_R$

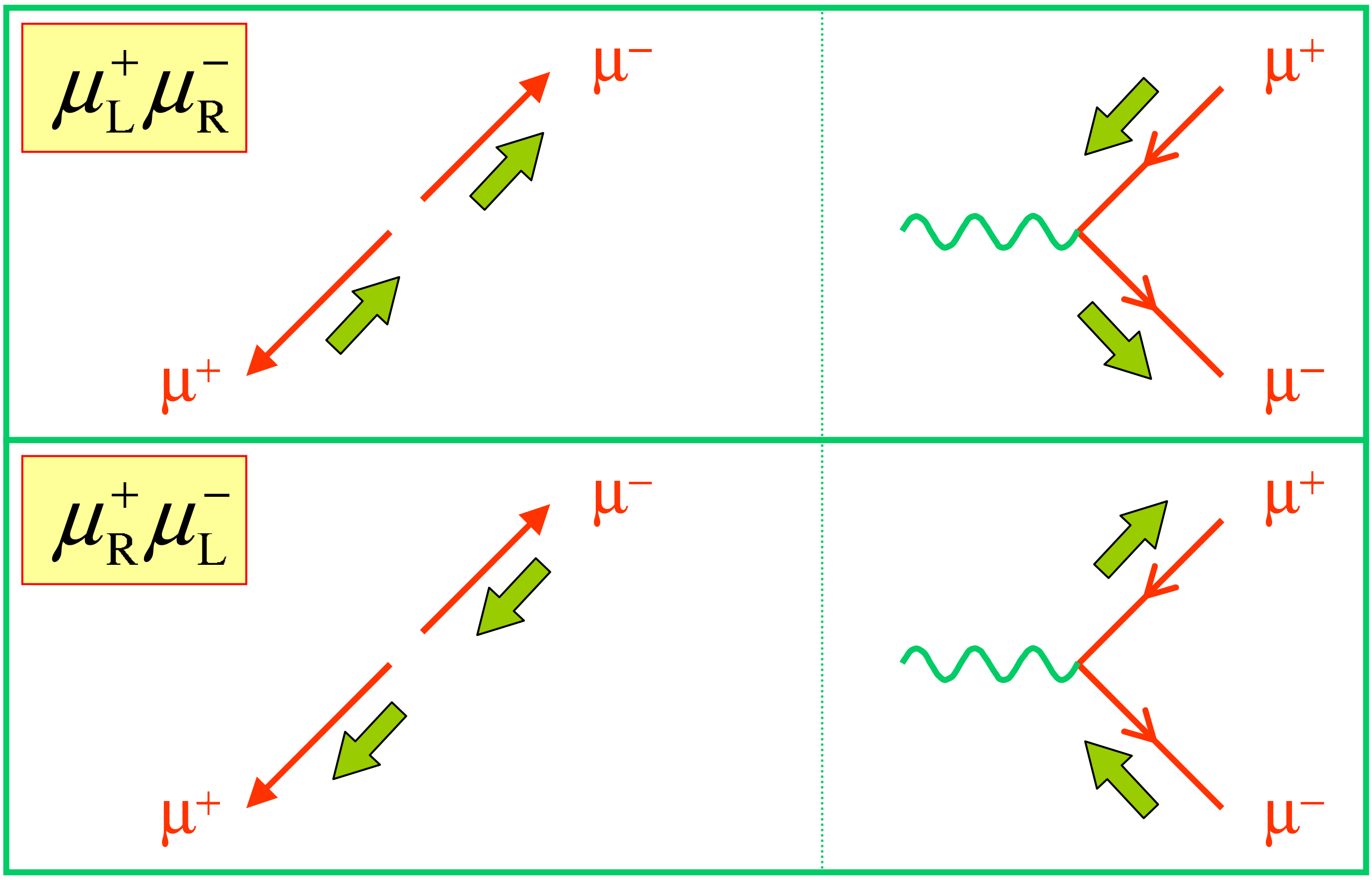


… but, in the relativistic limit, only 4 of them will turn out to be allowed

Electron current is non-zero only for :



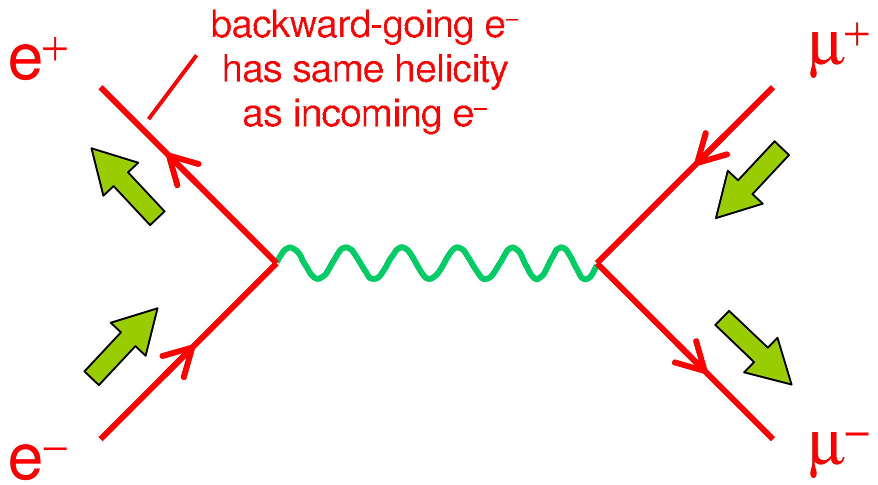
Muon current is non-zero only for :



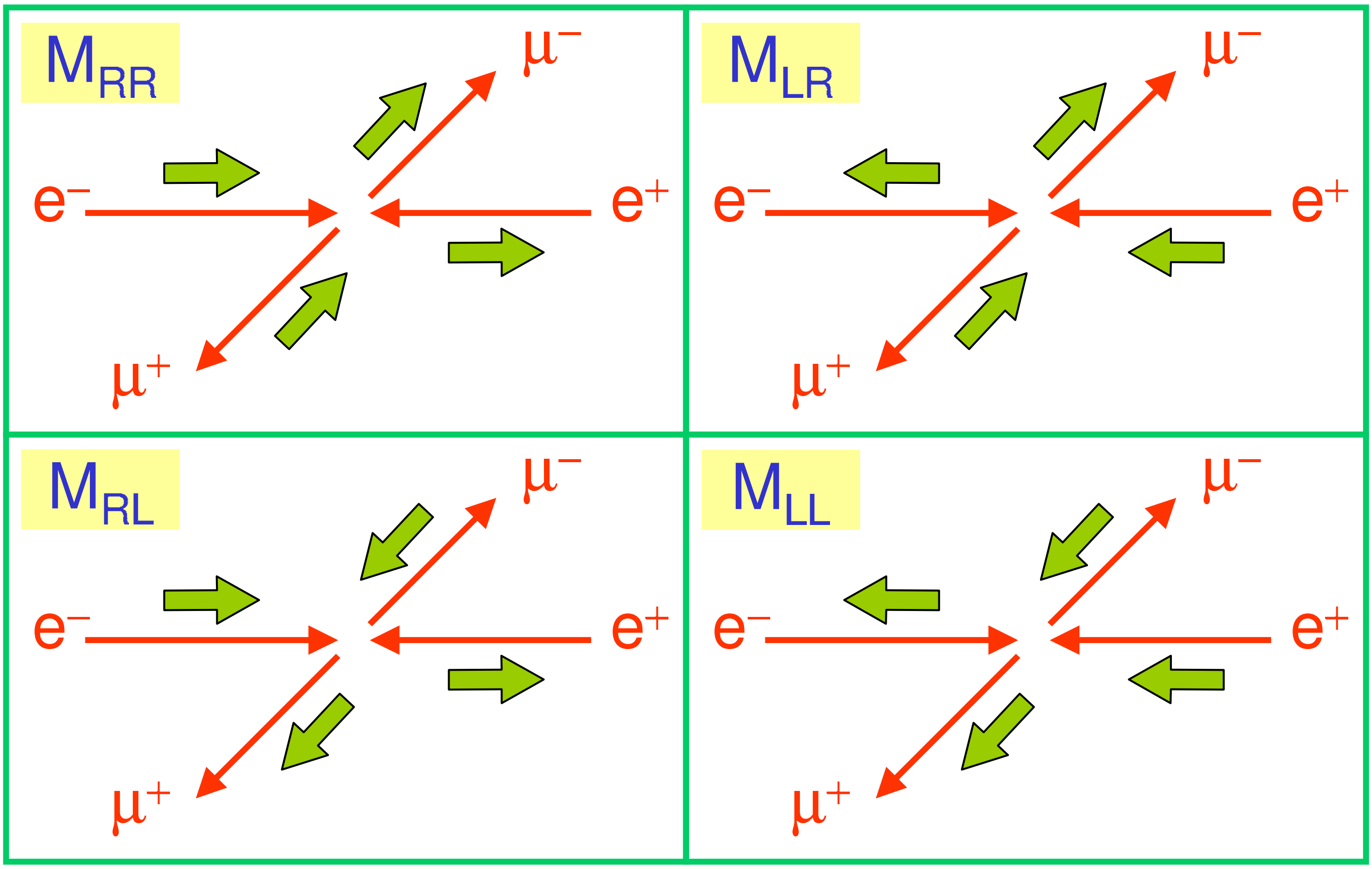
- This is helicity conservation in QED ...

  … in the high energy limit, the particle helicity at each vertex is preserved

e.g. for M_{RR}:

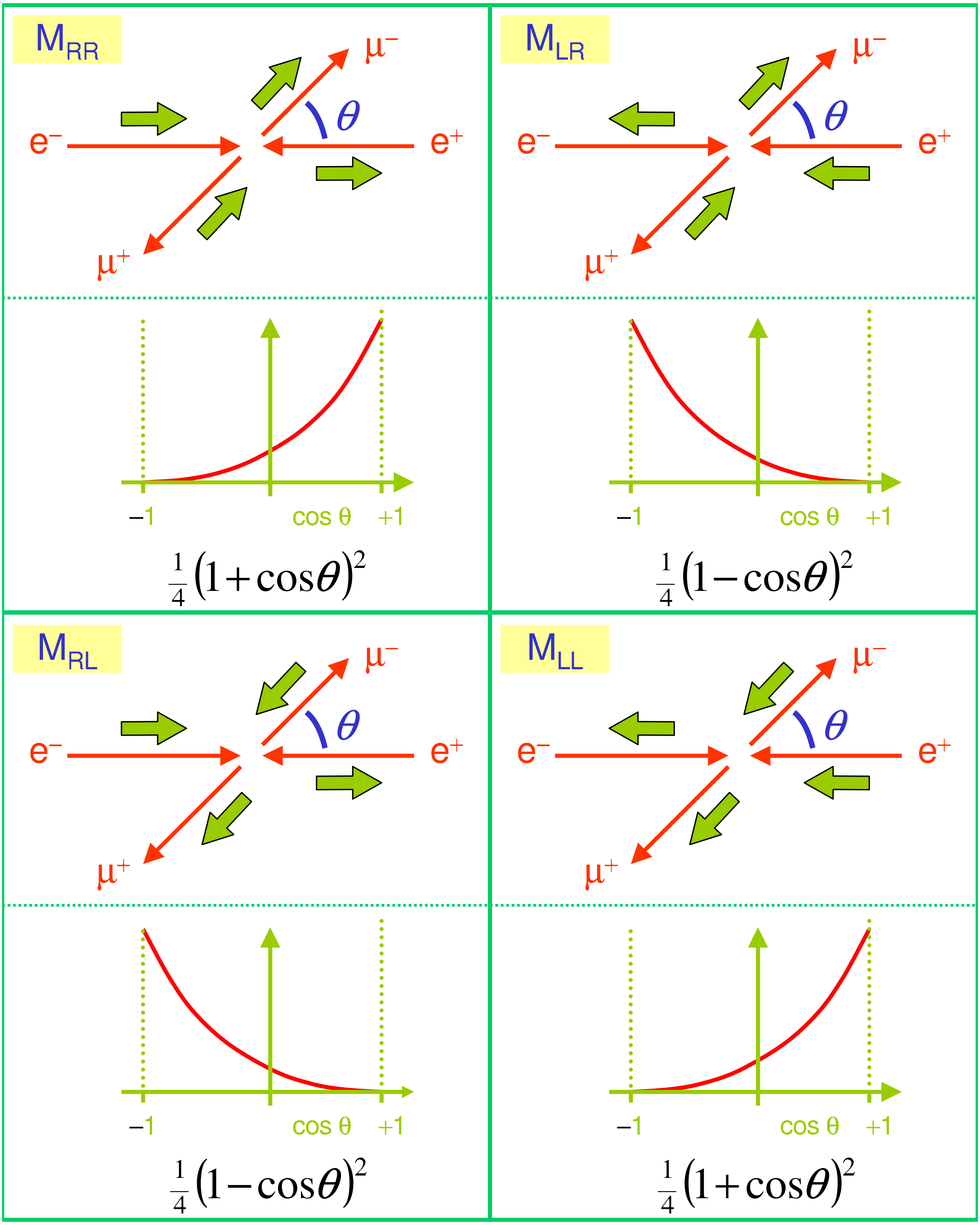


- The allowed helicity configurations are:



(Total spin projection is always $S_z = \pm 1$)

$e^+e^- \to \mu^+\mu^-$ cross sections

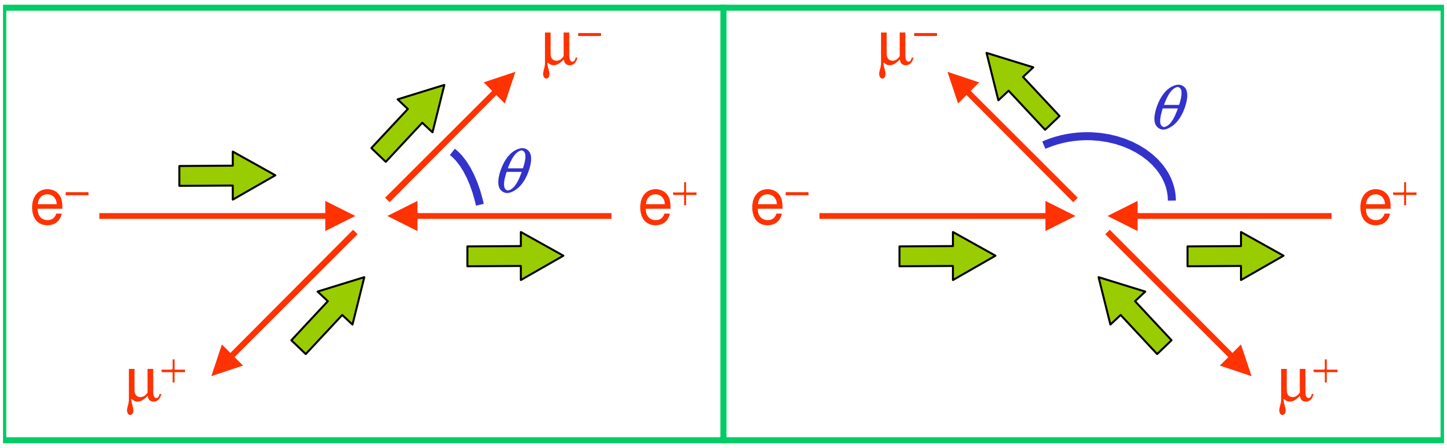


- The θ dependence can be understood from $S=1$ wavefunction overlap

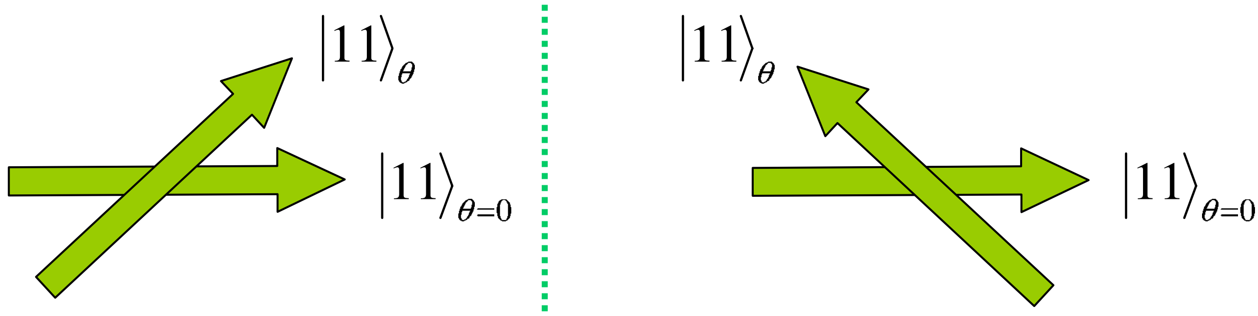
(Handout 4.8)

e.g. for $e^+_L e^-_R \to \mu^+_L \mu^-_R$:

$\underline{\cos\theta > 0}$ $\underline{\cos\theta < 0}$



favoured disfavoured



large overlap ... small overlap ...

... of spin wave functions

$\left|\langle 11(\theta)|11(\theta=0)\rangle\right|^2$ gives probability: $\boxed{\frac{1}{4}(1\pm\cos\theta)^2}$

Unpolarised cross section

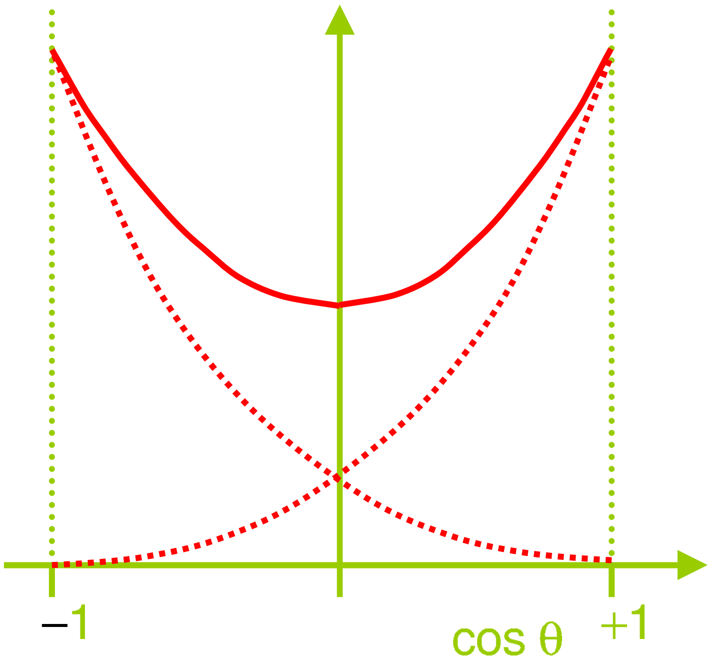
(Handout 4.6)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \left\langle |M_{\mathrm{fi}}|^2 \right\rangle = \frac{1}{2}\cdot\frac{1}{2}\cdot \sum_{\text{all spins}} |M_{\mathrm{fi}}|^2$$

average over e⁻ spins — average over e⁺ spins

equal mix of each helicity configuration

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta)}$$



Total $e^+e^- \to \mu^+\mu^-$ cross section is:

$$\boxed{\sigma = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\mathrm{d}\Omega = \frac{4\pi\alpha^2}{3s}}$$

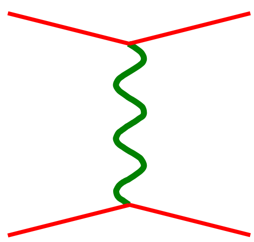
For $e^+e^- \to f\bar{f}$:

$$\boxed{\sigma = \frac{4\pi\alpha^2}{3s}\cdot N_c \cdot Q_f^2}$$

where Q_f is charge of fermion

$N_c = 3$ for $q\bar{q}$ final state

except $e^+e^- \to e^+e^-$

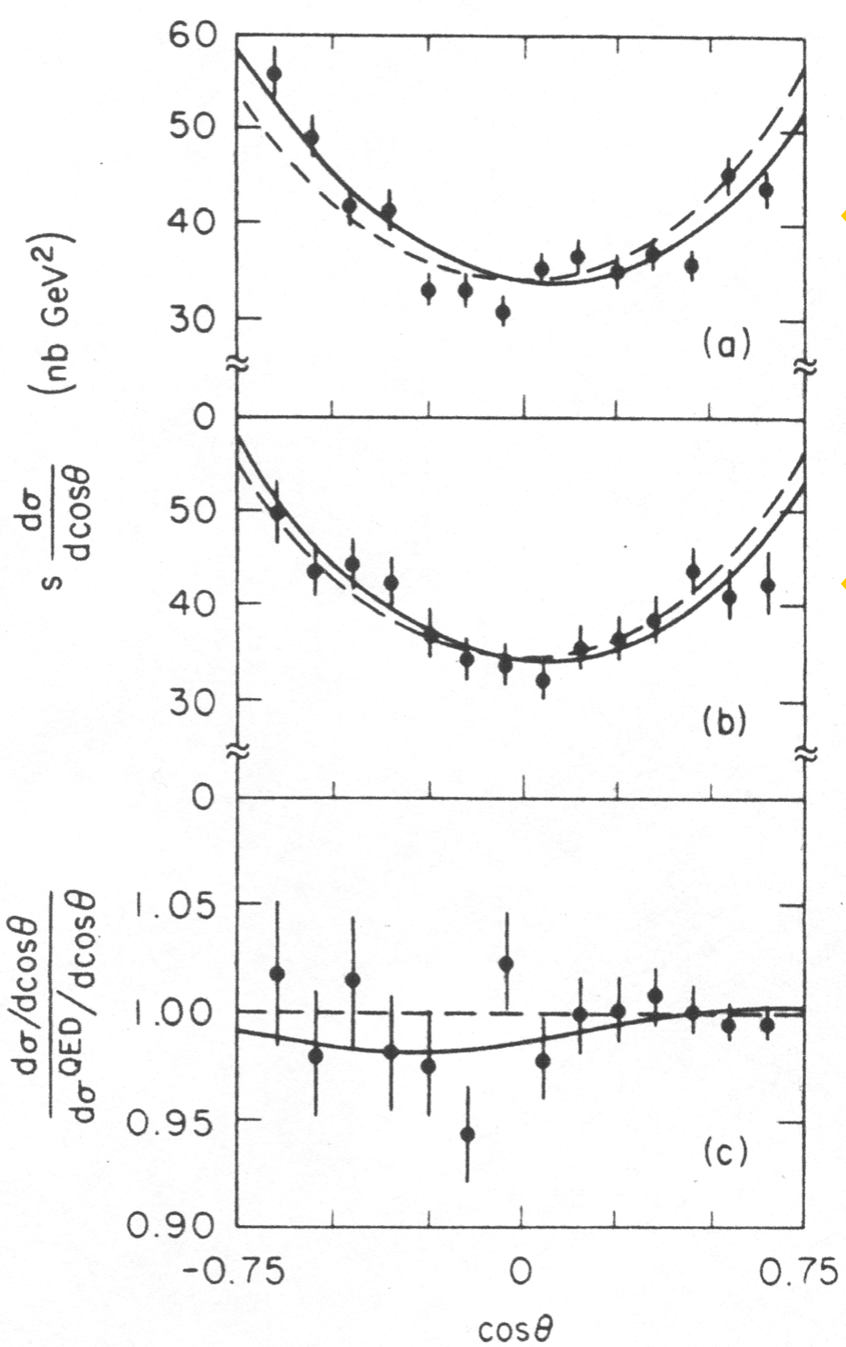


(extra diagram)

e^+e^- cross sections

$$\frac{d\sigma}{d\cos\theta}$$ at $\sqrt{s} = 29$ GeV :

Mark II Expt, SLAC
M.E.Levi et al.,
Phys Rev Lett **51** (1983) 1941



$e^+e^- \to \mu^+\mu^-$

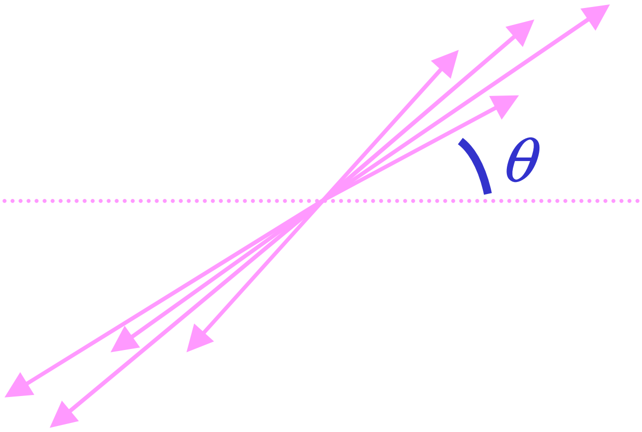
$e^+e^- \to \tau^+\tau^-$

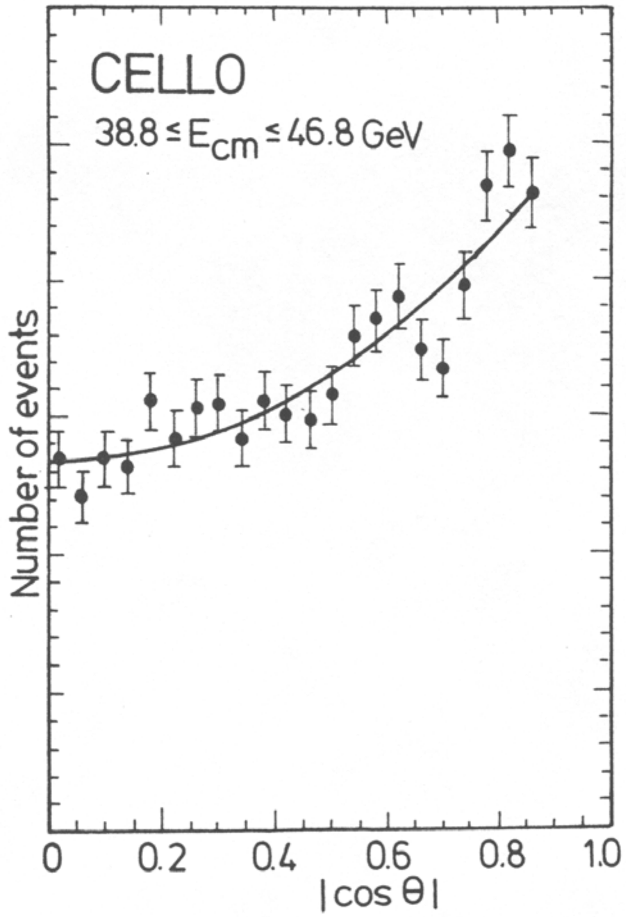
Angular distribution becomes slightly asymmetric in higher order pure QED or when weak interactions included

pure QED, $O(\alpha^3)$

QED plus Z^0 contribution

$e^+e^- \to q\bar{q}$





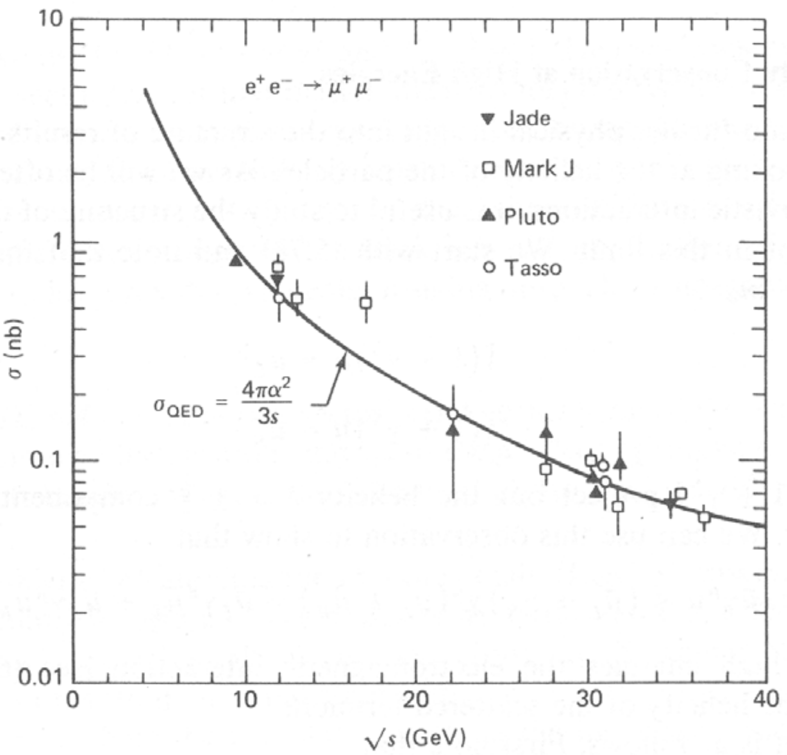
H.J.Behrend et al., Phys Lett **183B** (1987) 400

can't tell q-jet from $\bar{q}$-jet

$\Rightarrow$ can't tell sign of cos θ

$\sigma(e^+e^- \to \mu^+\mu^-)$ vs energy :

$$\sigma = \frac{4\pi\alpha^2}{3s}$$

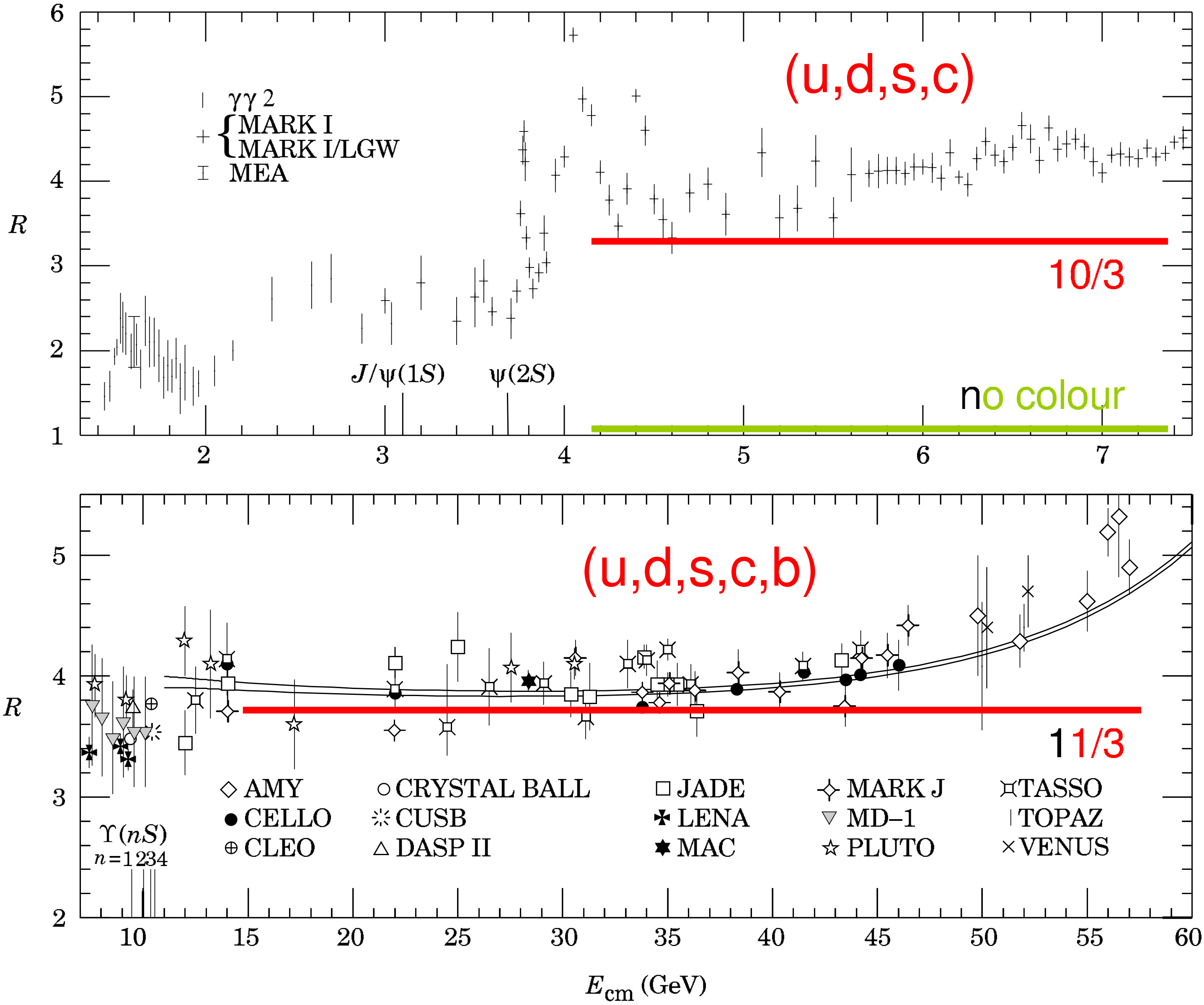


(Halzen + Martin)

e^+e^- ratio R

(Handout 4.7)

$$R \equiv \frac{\sigma\left(e^+e^- \rightarrow \text{hadrons}\right)}{\sigma\left(e^+e^- \rightarrow \mu^+\mu^-\right)} = 3\sum_f Q_f^2$$



Curve includes higher order QCD corrections: $R \propto 1 + \frac{\alpha_s}{\pi}$

plus Z^0 contribution (see later)

Log scales and a broader energy range :

(hep-ph/03121 1)